CE 603: Numerical Methods

Homework 3

Due date: 6th September

1. Compute the zero of the function $y(x)$ from the following data:

x	0.2	0.4	0.6	0.8	1.0
y	1.150	0.855	0.377	−0.266	−1.049

using natural cubic spline interpolant. You have to use reverse interpolation —x and y are interchanged, and with y arranged in the increasing order—and find the value of x when $y = 0$.

2. Density of air ρ varies with elevation h in the following manner:

h (km)	0	3	6
ρ (kg / m^3)	1.225	0.905	0.652

Express $\rho(h)$ as a quadratic function using Lagrange's and Newton's method. Verify that both polynomials are the same.

3. Three tensile tests were carried out on an aluminium bar. In each test the strain was measured at the same value of stress. The results were:

Stress (MPa)	34.5	69.0	103.5	138.0
Strain (Test 1)	0.46	0.95	1.48	1.93
Strain (Test 2)	0.34	1.02	1.51	2.09
Strain (Test 3)	0.73	1.10	1.62	2.12

where the units of strain are mm/m. Use linear regression to estimate the moulus of elasticity of the bar (modulus of elasticity = stress/strain).

Assuming that the third test was performed on an inferior machine, so that its results carry only half the weight of the other tests, obtain the modulus of elasticity using weighted linear regression.

4. Let $f(x) = a\,x^b$ be the least squares fit of the data $(x_i,\ y_i)$, $i = 0, 1, \ldots, n$, and let $F(x) = \ln a + b \ln x$ be the least-squares fit of $(\ln x_i,\ \ln y_i)$. Prove that $R_i \approx r_i / y_i$, where the residuals are $r_i = y_i - f(x_i)$ and $R_i = \ln(y_i) - F(x_i)$. Assume that $r_i \ll y_i$.