- **CE 603: Numerical Methods**

HW-5: Due date: November 4, 2011

1. Consider the mass-spring system in Fig.1 where dry friction is present between the block and the horizontal surface. The frictional force has a constant magnitude μ mg (μ is the coefficient of friction) and always opposes the motion. The differential equation for the motion of the block can be expressed as:

$$\ddot{y} = -\frac{k}{m} y - \mu g \frac{\dot{y}}{|\dot{y}|}$$

where y is measured from the position where the spring is unstretched. If the block is released from rest at $y = y_0$, verify by numerical integration using **rungeKutta4** and **rungeKutta45** that the next positive peak value of y is $y_0 - 4\mu \text{mg}/k$ (this relationship can be derived analytically). Use $k = 3000$ N/m, $m = 6$ kg, $\mu = 0.5$, $g = 9.80665\, m/s^2$, and $y_0 = 0.1$m.

2. The differential equation describing the motion of the mass-spring system in Fig. 2 is:

$$\ddot{y} + \frac{c}{m}\dot{y} + \frac{k}{m} y = 0$$

where $m = 2$ kg, $c = 460$ N.s/m, and $k = 450$ N/m. The initial conditions are $y(0) = 0.01$ m, and $\dot{y}(0) = 0$. (a) Show that this is a stiff problem and determine the value of h that you would use in numerical integration with the non-adaptive Runge-Kutta method. (c) Carry out the integration from $t = 0$ to $t = 0.2$ s with the chose h and plot $\dot{y}$ versus t.

3. The simple supported beam of length L in Fig. 3 is resting on an elastic foundation of stiffness $k\, N/m^2$. The displacement v of the beam due to the uniformly distributed load of intensity w_0 N/m is given by the solution of the boundary value problem.

$$E I \frac{d^4 u}{d x^4} + k u = w_0,\ v|_{x=0} = \frac{d^2 u}{d x^2} = u|_{x=L} = \frac{d^2 u}{d x^2}|_{x=L} = 0$$

The non-dimensional form of the problem is:

$$\frac{d^4 y}{d \xi^4} + \gamma y = 1,\ y|_{\xi=0} = \frac{d^2 y}{d \xi^2} = y|_{\xi=1} = \frac{d^2 y}{d \xi^2}|_{\xi=1} = 0$$

where

$$\xi = \frac{x}{L},\ y = \frac{E I}{w_0 L^4} u,\ \gamma = \frac{k L^4}{E I}.$$

Solve this problem by a finite difference method with $\gamma = 10^5$ and plot y versus ξ.

4. The simply supported beam in Fig. 4 carries a uniform load of intensity w_0 and the tensile force N. The differential equation for the vertical displacement u can be shown to be:

$$\frac{d^4 v}{d x^4} - \frac{N}{E I}\frac{d^2 v}{d x^2} = \frac{w_0}{E I}$$

where EI is the bending rigidity. The boundary conditions are $u = d^2 u / d x^2 = 0$ at $x = 0$ and $x = L$. Changing the variables to $\xi = \frac{x}{L}$ and $y = \frac{EI}{w_0 L^4} u$ transforms the problem to the dimensionless form

$$\frac{d^4 y}{d\xi^4} - \beta \frac{d^2 y}{d\xi^2} = 1, \quad \beta = \frac{N L^2}{EI}$$

$$y|_{\xi=0} = \frac{d^2 y}{d\xi^2}|_{\xi=0} = y|_{\xi=1} = \frac{d^2 y}{d\xi^2}|_{x=1} = 0.$$

Determine the maximum displacement if (a) $\beta = 1.65929$, and (b) $\beta = -1.65929$ (N is compressive). Use shooting method to solve the problem.

6. The equation of motion of a harmonic oscillator subjected to a periodic loading is given as follows:

$$m\ddot{x} + c\dot{x} + kx = F_0 \sin \omega t,$$

subject to initial conditions,

$$x(0) = x_0, \quad \dot{x}(0) = v_0$$

For, $m = 1\,\text{kg}$, $c = 0.5\,N - s/m$, $k = 1\,N/m$, $F_0 = 1\,N$, $\omega = 2\,s^{-1}$, $x_0 = 0$, $v_0 = 0.05\,m/s$,
obtain the steady state amplitude A and phase shift ϕ for steady-state value of $x(t)$ using Newmark-Beta method for:

(i) $\gamma = 1/2, \beta = 1/6$, (ii) $\gamma = 1/4, \beta = 1/4$, and (iii) $\gamma = 2/3, \beta = 1/4$.

Use appropriate time-step h. Compare the answer with the actual analytical answer.

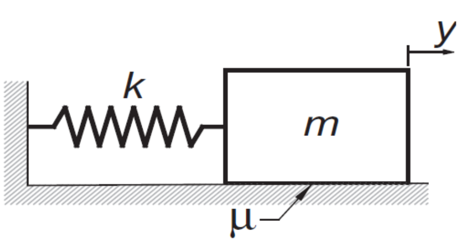
$$A_0 = \frac{F_0}{k}\left(\left(1 - \frac{m\omega^2}{k}\right)^2 + \left(\frac{c\omega}{k}\right)^2\right)^{-1/2}, \text{ and } \phi = -\tan^{-1}\left(\frac{c\omega/k}{1 - m\omega^2/k}\right).$$

7. Solve the boundary value problem:

$$y''' + y y'' = 0, \quad y(0) = y'(0) = 0, \quad y'(\infty) = 2.$$

using shooting method and plot $y(x)$ and $y'(x)$. This problem arises in determining the velocity profile of boundary in incompressible flow (Blasius solution).

Fig.1



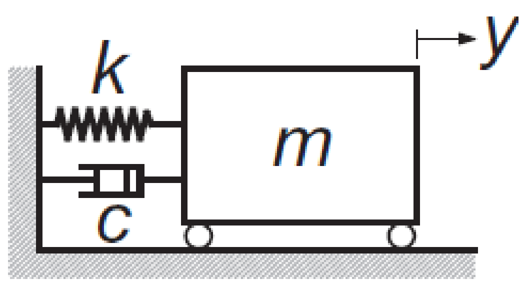


Fig.2

Fig.3

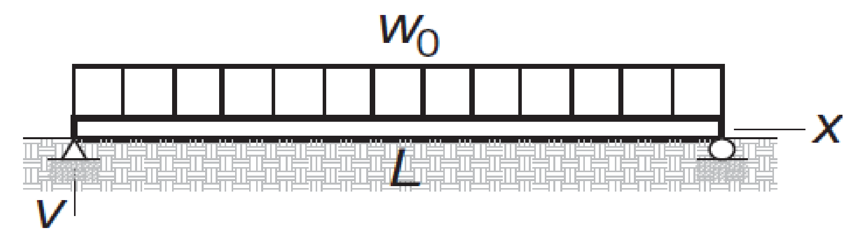


Fig. 4