CE 620: Finite Element Method, HW-1

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1. Figure 1 shows a beam of length L and flexural rigidity EI which is built-in at $z = 0$ and simply supported at the intermediate point $z = 3L/4$. Use the Rayleigh-Ritz method with one degree of freedom to estimate the deflection at the free end.

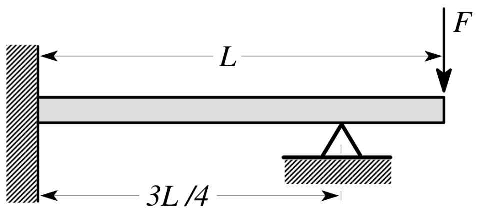

Figure 1:

2. Given the strong form for the heat conduction problem in a circular plate:

$$\begin{aligned} k\frac{d}{dr}\left(r\frac{dT(r)}{dr}\right) + rs &= 0, \quad 0 < r < R \\ \text{natural boundary condition: } \frac{dT}{dr}(r=0) &= 0, \\ \text{essential boundary condition: } T(r=R) &= 0, \end{aligned}$$

where R is the total radius of the plate, s is the heat source per unit length along the plate radius, T is the temperature and k is the conductivity. Assumed that k, s, and R are given constants.

(a) Construct the weak form for the above strong form.
(b) Use quadratic trial (candidate) solutions of the form $T = \alpha_0 + \alpha_1 r + \alpha_2 r^2$ and weight functions of the same form to obtain a solution of the weak form.
(c) Solve the differential equation with the boundary conditions and show that the temperature distribution along the radius is given by

$$T = \frac{s}{4k}(R^2 - r^2).$$

3. Show that the weak form of

$$\begin{aligned} \frac{d}{dx}\left(AE\frac{du}{dx}\right) + 2xA &= 0 \text{ on } 1 < x < 3, \\ \sigma(1) = \left(E\frac{du}{dx}\right)_{x=1} &= 0.1, \\ u(3) &= 0.001 \end{aligned}$$

is given by

$$\int_1^3 \frac{dw}{dx}AE\frac{du}{dx}dx = -0.1(wA)_{x=1} + \int_1^3 2xAw dx \quad \forall w \text{ with } w(3) = 0.$$

Now, consider a trial (candidate) solution of the form $u(x) = \alpha_0 + \alpha_1(x-3) + \alpha_2(x-3)^2$, and a weight function of the same form. Obtain a solution of the weak form above. Check the equilibrium equation in the strong form in the problem; is it satisfied? Check the natural boundary condition; is it satisfied? Assume that A and E are constants.