CE 620: Finite Element Method, HW-2

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Note: In the questions below, any quantity $\mathbf{A}$ or $\mathbf{A}^T$ in **bold** indicates a matrix. If that corresponds to $\langle A\rangle$ or $\{A\}$ or $[A]$ is clear from the context.

1. Consider a five-node element in one dimension. The element length is 4, with node 1 at $x = 2$, and the remaining nodes are equally spaced along the x axis.

(a) Construct the shape functions for the element.

(b) The temperatures at the nodes are given by $T_1 = 3°\text{C}$, $T_2 = 1°\text{C}$, $T_3 = 0°\text{C}$, $T_4 = -1°\text{C}$, $T_5 = 2°\text{C}$. Find the temperature field at $x = 3.5$ using shape functions constructed in (a).

2. Consider a four-node cubic element in one dimension. The element length is 3 with $x_1 = -1$; the remaining nodes are equally spaced.

(a) Construct the element shape functions.

(b) Find the displacement field in the element when

$$\mathbf{d} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} = 10^{-3}\begin{bmatrix} 1 \\ 0 \\ 1 \\ 4 \end{bmatrix}$$

(c) Evaluate the $\mathbf{B}^e = {}^{d\mathbf{N}^e}/_{dx}$ matrix and find the strain for the above displacement field.

(d) Plot the displacement $u(x)$ and strain $\epsilon(x)$.

(e) Find the strain field when the nodal displacements are $\mathbf{d}^{eT} = [1111]$. Why is this result expected?

3. Use Gauss quadrature to obtain exact values for the following integrals. Verify by analytical integration.

$$\text{(a)} \quad \int_0^4 (x^2+1)(x-2)dx$$

$$\text{(b)} \quad \int_{-1}^1 (\xi^4 + 2\xi^2)d\xi.$$

What is the correct order n_g of Gauss quadrature that needs to be employed in each of the integrals above? Use order of integration one more or one less than the exact order and check how the accuracy is affected. The table of weights and integration points for Gauss quadrature is provided in Figure 1.

4. Use three-point Gauss quadrature to evaluate the following integrals. Compare to the analytical integral.

$$\text{(a)} \quad \int_{-1}^1 \frac{\xi}{\xi^2+1}d\xi$$

$$\text{(b)} \quad \int_{-1}^1 \cos^2 \pi\xi d\xi.$$

n_{gp}	Location, ξ_i	Weights, W_i
1	0.0	2.0
2	$\pm 1/\sqrt{3} = \pm 0.5773502692$	1.0
3	± 0.7745966692	0.555 555 5556
	0.0	0.888 888 8889
4	± 0.8611363116	0.347 854 8451
	± 0.3399810436	0.652 145 1549
5	± 0.9061798459	0.236 926 8851
	± 0.5384693101	0.478 628 6705
	0.0	0.568 888 8889
6	± 0.9324695142	0.171 324 4924
	± 0.6612093865	0.360 761 5730
	± 0.2386191861	0.467 913 9346

Figure 1: Position of Gauss points and corresponding weights.