CE 620: Finite Element Method, HW-3

uploaded on: 11 Feb 2020, **submission due:** 02 Mar 2020

Notation: (i) n_{el} = number of elements and (ii) n_{en} = number of nodes per element.

1. Consider a heat conduction problem shown in Figure 1. The dimensions are in meters. The bar has a constant unit cross section, constant thermal conductivity $k = 5$ W°C^{-1}m^{-1} and a linear heat source s as shown in Figure 1. The boundary conditions are $T(x = 1) = 100$ °C and $T(x = 4) = 0$ °C. Divide the bar into two elements ($n_{\text{el}} = 2$) as shown in the Figure. Note that element 1 is a three-node (quadratic) element ($n_{\text{en}} = 3$), whereas element 2 is a two-node ($n_{\text{en}} = 2$) element.

(a) State the strong form representing the heat flow and solve it analytically. Find the temperature and flux distributions.
(b) Construct the element source matrices and assemble them to obtain the global source matrix. Note that the boundary flux matrix is zero.
(c) Find the temperature distribution using the FEM. Sketch the analytical (exact) and the finite element temperature distributions.
(d) Find the flux distribution using the FEM. Sketch the exact and the finite element flux distributions.

2. Given the one-dimensional elasticity problem as shown in Figure 2. The bar is constrained at both ends (A and C). Its cross-sectional area is constant ($A = 0.1$ m^2) on segment AB and varies linearly $A = 0.05(x - 1)$m^2 on BC. The Young's modulus is $E = 2 \times 10^7$ Pa. A distributed load $b = 10$Nm^{-1} is applied along the left portion of the bar AB and point force $P = 150$ N acts at point B. The geometry, material properties, loads and boundary conditions are given in Figure 2a. Use a three-node element on AB($n_{\text{en}} = 3$) and a two-node element on BC ($n_{\text{en}} = 2$) as shown in Figure 2b. The dimensions in Figure 2 are in meters.

(a) Construct the element body force matrices and assemble them to obtain the global force matrix.
(b) Construct the element stiffness matrices and assemble them to obtain the global stiffness matrix.
(c) Find and sketch the finite element displacements.
(d) Find and sketch the finite element stresses.

3. Consider a three-node quadratic element in one dimension with unequally spaced nodes as shown in Figure 3.

(a) Obtain the $\mathbf{B}^e$ matrix.
(b) Consider an element with $x_1 = 0$, $x_2 = 1/4$ and $x_3 = 1$. Evaluate strain ϵ in terms of u_2 and u_3 ($u_1 = 0$).
(c) If you evaluate $\mathbf{K}^e$ by one-point quadrature using $\mathbf{B}^{eT}\mathbf{E}^e\mathbf{A}^e\mathbf{B}^e$ for same coordinates as in (b) and constrain node 1 (i.e. $u_1 = 0$), is $\mathbf{K}^e$ invertible? Assume EA to be constant over the element.
(d) If $u(x)$ in part (b) is given by $x_i^2/2$ at node-i, does it mean that $\epsilon = x$ for the given element?

Note on how to include point load in FEA.
Point load of magnitude P at a location x_0 on the body is in fact body force of the form $b(x) = P\delta(x - x_0)$, where $\delta(x - x_0)$ corresponds to Dirac-Delta function centered at location x_0. Hence, the expression for weak form corresponding to the body force becomes

$$\int_0^L w(x)b(x)dx = \int_0^L w(x)P\delta(x - x_0)dx = Pw(x_0).$$

Since in FEA, $w(x) \approx w^h(x) = \langle w \rangle \{N(x)\}$, the quantity $w(x_0) \approx w^h(x_0) = \langle w \rangle \{N(x_0)\}$. Here $\{N(x)\}$ correspond to global shape functions at every node.

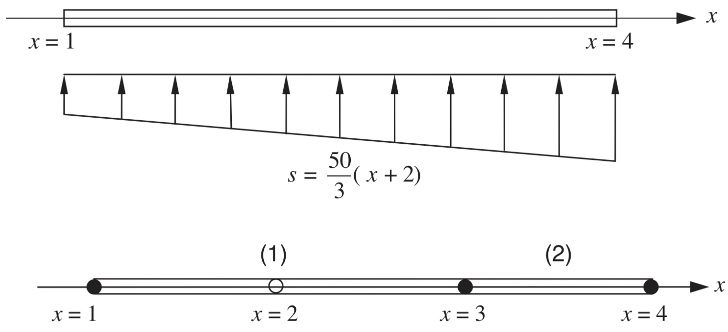

Figure 1: Heat conduction problem and its finite element mesh below.

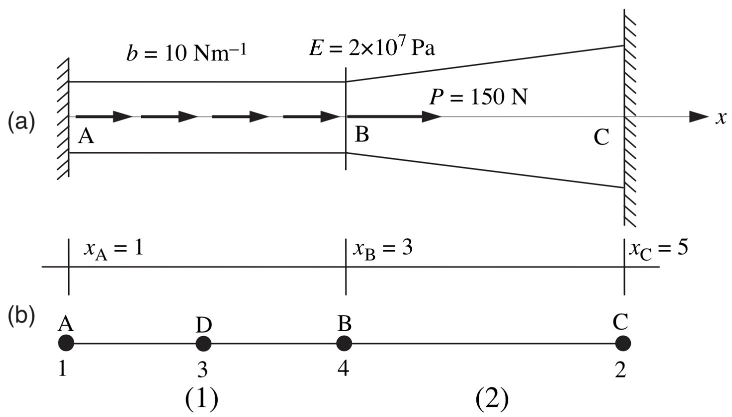

Figure 2: (a)Geometry,materialproperties,loads and boundary conditionsfor a bar with a variable cross-sectional area (b) the finite element model.

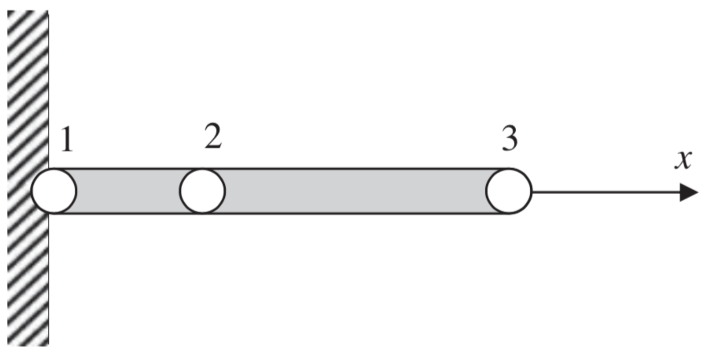

Figure 3: