CE 620: Finite Element Method, HW-4/5

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Notes:(i) Insulated boundary means no flux, i.e., $\bar{q} = 0$. (ii) Point source corresponds to a δ function and a similar process for 1D as described in HW-3 should be used here.

Q1. Construct the shape functions for the five-node triangular element as shown in Figure 1, which has quadratic shape functions along two sides and linear shape functions along the third. Be sure that your shape functions for all nodes are linear between nodes 1 and 2. Use triangular coordinates and express your answer in terms of triangular coordinates.

Q2. Given a nine-node rectangular element as shown in Figure 2.

1. Construct the element shape functions by the tensor product method.
2. If the temperature field at nodes A and B is 1 °C and zero at all other nodes, what is the temperature at $x = y = 1$?
3. Consider the three-node triangular element ABC located to the right of the nine-node rectangular element. Will the function be continuous across the edge AB? Explain.

Q3. Consider the four-node isoparametric element. Show that ${}^{\partial N_1}/_{\partial x}$ at the origin $\xi = \eta = 0$ is given by $\frac{\partial N_1}{\partial x} = \frac{y_{24}}{2A}$ and that $J = \det(J^e(0,0)) = {}^{A}/_{4}$, where A is the area of the quadrilateral and $y_{24} = y_2 - y_4$ for the element.

Q4. A finite element mesh consisting of a rectangular and a triangular element is shown in Figure 3. The dimensions of the plate are in meters. A constant temperature $\bar{T} = 10$ °C is prescribed along the boundary $y = 0$. A constant and linear boundary flux as shown in Figure 3 is applied along the edges $y = x + 2$ and $x = 0$, respectively. The edge $x = 2$ is insulated. A point source $P = 10$ is applied at $(0, 2)$ m. The material is isotropic with $k = 1$ W °C^{-1} for element 1 and $k = 2$ W °C^{-1} for element 2. Compute the nodal temperatures and fluxes at the two element center points.

Q5. Consider a triangular panel as shown in Figure 4 of two isotropic materials with thermal conductivities $k_1 = 4$ W°C^{-1} and $k_2 = 8$ W°C^{-1}. All dimensions are in meters. A constant temperature $\bar{T} = 10$ °C is prescribed along the edge BC. The edge AB is insulated and a linear distribution of flux, $\bar{q} = 15x$ W.m^{-1}, is applied along the edge AC. Point source $P = 45$ W is applied at $x = 3$ and $y = 0$. For the finite element mesh, consider two triangular elements, ABD and BDC. Carry out calculation manually and find the temperature and flux distributions in the plate.

Q6. Consider a chimney constructed of two isotropic materials: dense concrete ($k = 2.0$ W °C^{-1}) and bricks ($k = 0.9$ W °C^{-1}). The temperature of the hot gases on the inside surface of the chimney is 140 °C, whereas the outside is exposed to the surrounding air, which is at 10 °C. The dimensions of the chimney (in meters) are shown below. Solve this problem using FEniCS using a fine enough mesh. What is the maximum temperature in bricks and in concrete, and where does it occur. Also, what is the maximum magnitude of heat flux and where does it occur? Can you simplify the computational effort by exploiting the symmetry of the problem? What would be the boundary conditions in that case?

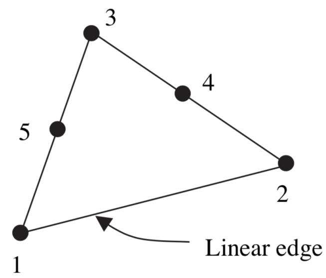

Figure 1: Five-node triangular element of Figure 1.

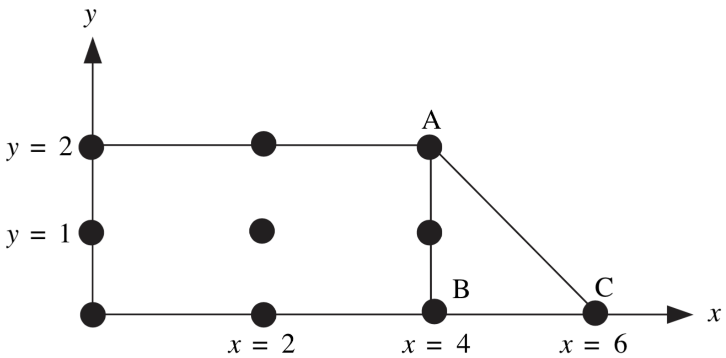

Figure 2: Nine-node rectangular and adjacent three-node triangular element.

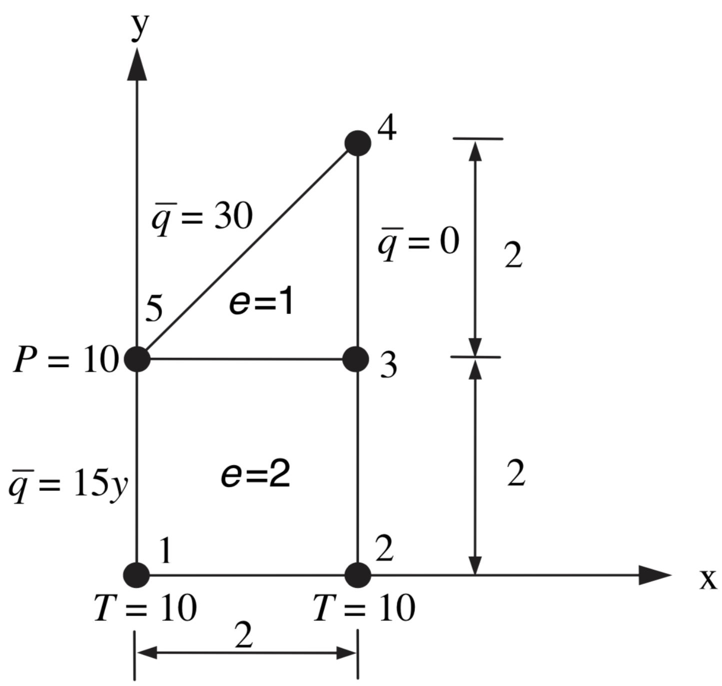

Figure 3: Nine-node rectangular and adjacent three-node triangular element.

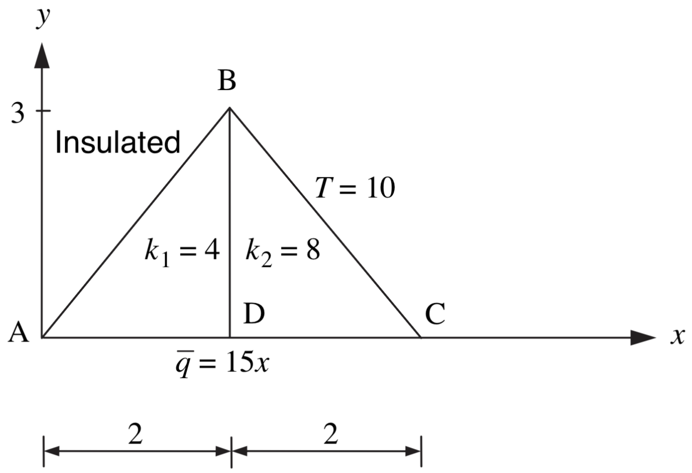

Figure 4: Nine-node rectangular and adjacent three-node triangular element.

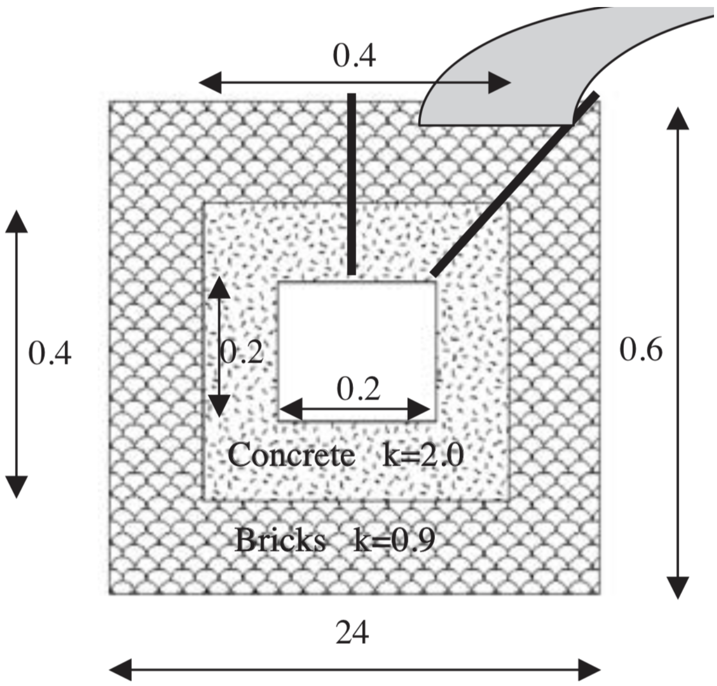

Figure 5: Chimney cross-section.