Mechanics

- Rigid bodies (Statics, Dynamics)
- Deformable bodies (a.k.a. Solid mech)
- Fluid Mech.

Fundamental concepts & principles

1) Law of parallelogram for adding two vectors (i.e., forces, accelerations, linear momentum) acting on particles. This becomes polygon rule when three or more vectors are added.

2). Principle of transmissibility: static or dynamic equilibrium of a rigid body is unaltered if a force is replaced by one acting at a different point but having same magnitude and line of action.

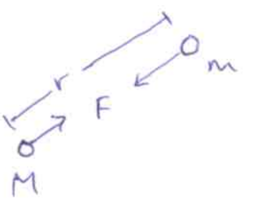
3). Newtons laws:

(i) If $\sum \underline{F} = 0$ on a particle, it will <u>continue</u> to remain at rest or move with uniform speed along a straight line.

(ii) $\sum \underline{F} = m\underline{a}$ for a particle

(iii) The forces of action & reaction between contacting bodies are equal in magnitude & opposite in direction.

4) Newtons law of Gravitation



$F = \frac{GMm}{r^2}$ → action at a distance.

$\Rightarrow W = mg = \frac{GMm}{R^2}$, R = dist from earths center (depend on altitude and latitude since earth not spherical)

These principles (1)-(4) are based on experimental evidence.
Newtonian mechanics fails when $v \to c$ (speed of light).

5) Idealizations: (a) Continuum (b) Rigid body (c) Point force (d) Particle.

6) Equal vectors : $\underline{V}_1 = \underline{V}_2$ if both have same magnitude and direction.

Equivalent vectors: Two vectors are equivalent if they produce the same "effect"

eg. 

If F_1 & F_2 produce same moment about fixed end, they are equivalent in this sense.

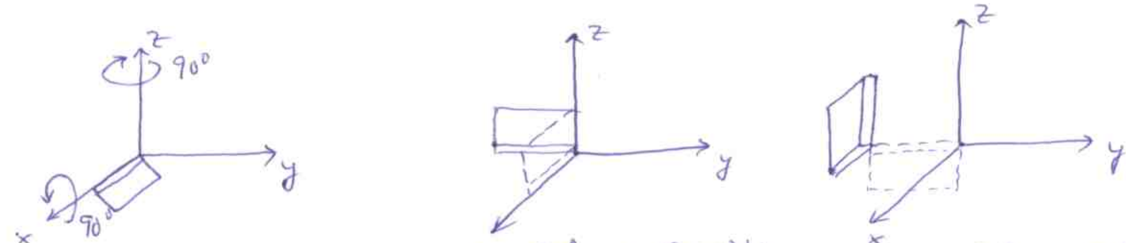
7) Solution method : Draw FBD's, ie isolate the body/particle/system from its surroundings/supports. Then use Newtons laws for solution

8) Vector addition

$$\underline{P} + \underline{Q} = \underline{Q} + \underline{P} \quad \text{(commutative)}$$

$$\underline{P} + (\underline{Q} + \underline{S}) = (\underline{P} + \underline{Q}) + \underline{S} \quad \text{(associative)}.$$

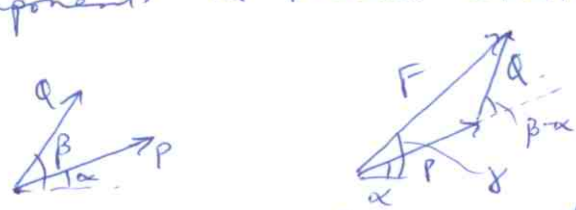
Note: Finite angles are not vectors since their addition is non-commutative & non-associative.



$90^\circ \underline{i} + (-90^\circ)\underline{k}$ $\qquad$ $-90^\circ \underline{k} + 90^\circ \underline{i}$.

9) Resolution of Force into components.

(i) Components in known arbitrary directions

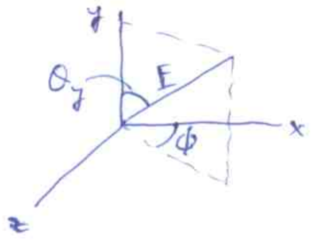


Use triangle rule for summation and then sine or cosine rule to compute

eg: $|\underline{P}|$, $|\underline{Q}|$, α, β known, get $|\underline{F}|$ by cosine rule and γ by sine rule.

eg: $|\underline{F}|$ and its direction γ, and α, β known. Find $|\underline{P}|$, $|\underline{Q}|$ by sine rule.

(ii) Components in known orthogonal directions — take dot product.



$F_y = F\cos\theta_y$

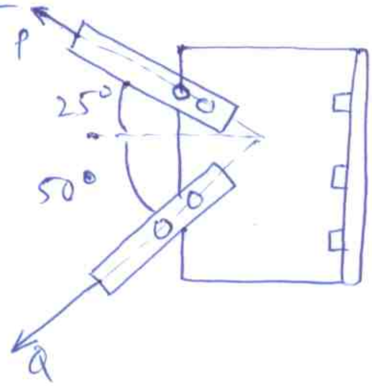
$F_x = F\sin\theta_y \cos\phi = F\cos\theta_x$

$F_z = F\sin\theta_y \sin\phi = F\cos\theta_z$

$\cos\theta_x$, $\cos\theta_y$, $\cos\theta_z$ are direction cosines.

$\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1.$

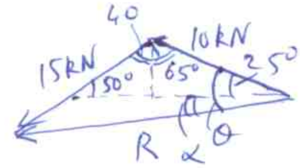
P-1



$P = 10\,kN$, $Q = 15\,kN$ given

Find: magnitude and direction of resultant force exerted on bracket.

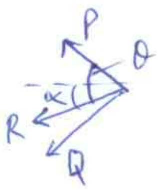
Method 1



$$R^2 = 15^2 + 10^2 - 2 \times 15 \times 10 \cos(40° + 65°) \Rightarrow R = 20.07\,kN$$

$$\frac{\sin\theta}{15} = \frac{\sin 105}{R} \Rightarrow \theta = 46.22°.$$

Method 2.



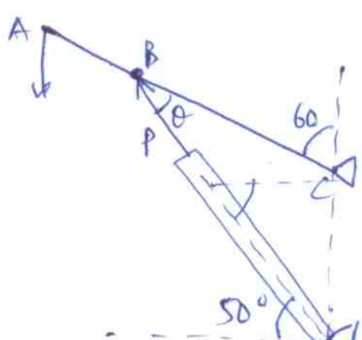
$$R = \left\{(10\cos 25 + 15\cos 50)^2 + (15\sin 50 - 10\sin 25)^2\right\}^{1/2}$$

$$= 20.07$$

$$\alpha = \tan^{-1}\left(\frac{15\sin 50 - 10\sin 25}{10\cos 25 + 15\cos 50}\right) = 21.22°$$

P.2.

ABC can be in equilibrium only if BD is in compression.



Given: Rectangular Cartesian comp of P taken normal to ABC is 750 N.

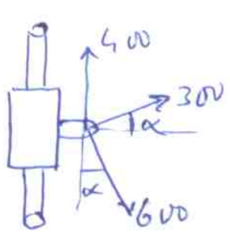
Find: P, and its rect Cartesian component along AC.

$$\theta = 180 - 30 - 130 = 20$$

$$P \sin 20 = 750 \Rightarrow P = 2192.85\,N$$

$$P \cos 20 = 2060.608\,N$$

P.3



Find α so that resultant force on collar is horizontal, and find the resultant.

$$\Rightarrow 400 + 300\sin\alpha = 600\cos\alpha.$$

$$\Rightarrow 16 + 9\sin^2\alpha + 24\sin\alpha = 36\cos^2\alpha$$

$$\Rightarrow 45\sin^2\alpha + 24\sin\alpha - 20 = 0$$

$$\sin\alpha = \left(-24 \pm \sqrt{24^2 + 4 \times 20 \times 45}\right)/90 = -0.9846, 0.4514$$

Take $\alpha > 0$ as shown, ie $\sin\alpha = 0.4514$, ie $\alpha = 26.83°$ ◄

$R = 300\cos\alpha + 600\sin\alpha = 538.516$ ◄

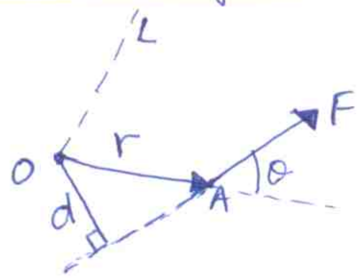
P.4. For the box supported by cables as shown, determine the angle between cables AB and AD.

$\underline{r}_{AB} = 1125\,\underline{j} + 700\,\underline{i}$, $\underline{r}_{AD} = 1125\,\underline{j} - 650\,\underline{i} + 450\,\underline{k}$

$$\cos\theta = \frac{\underline{r}_{AB}\cdot\underline{r}_{AD}}{|\underline{r}_{AB}|\,|\underline{r}_{AD}|} = \frac{1125^2 - 700 * 650}{\sqrt{1125^2+700^2}\,\sqrt{1125^2+650^2+450^2}} = 0.4449$$

$\theta = 63.58°$ ◄

Moment of a force about a point.



$$\underline{M}_0 = \underline{r} \times \underline{F}$$
$$= rF\sin\theta = rd$$

Two forces $(\underline{F}, \underline{F}')$ are statically equivalent iff

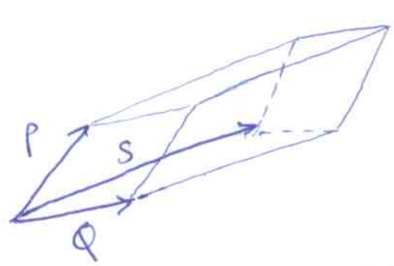
$\underline{F} = \underline{F}'$ and $\underline{M}_0 = \underline{M}_0'$

$$\underline{M}_0 = M_x \underline{i} + M_y \underline{j} + M_z \underline{k} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ x & y & z \\ F_x & F_y & F_z \end{vmatrix}$$

$$M_x = yF_z - zF_y$$
$$M_y = zF_x - xF_z$$
$$M_z = xF_y - yF_x$$

Moment of a force about a line (axis).

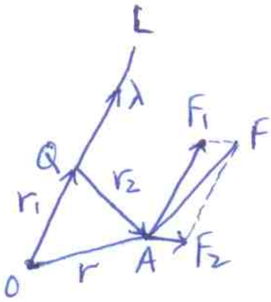
Mixed triple product is $\underline{S} \cdot (\underline{P} \times \underline{Q})$ = vol. of parallelopiped.



Now $M_{OL} = \underline{\lambda} \cdot \underline{M}_0 = \underline{\lambda} \cdot (\underline{r} \times \underline{F})$, $\underline{\lambda}$ = unit vector along OL

(see above fig)

$$= \begin{vmatrix} \lambda_x & \lambda_y & \lambda_z \\ x & y & z \\ F_x & F_y & F_z \end{vmatrix}$$



$$M_{OL} = \underline{\lambda} \cdot \left[(r_1 + r_2) \times (\underline{F}_1 + \underline{F}_2) \right]$$

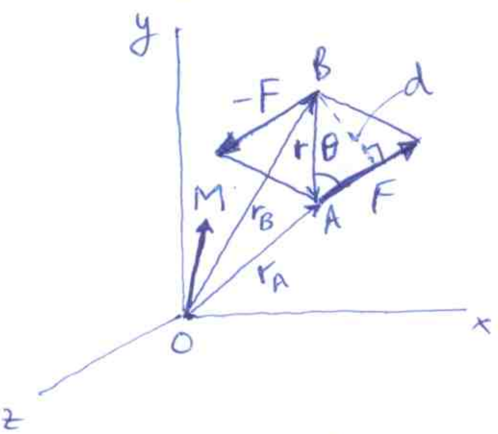
$$= \underline{\lambda} \cdot (\underline{r}_2 \times \underline{F}_2)$$

(other triple products vanish since they involve coplanar vectors).

So M_{OL} represents the ability of $\underline{F}$ to rotate the body about OL, wherein only contribution comes from $\underline{F}_2$ (ie component of $\underline{F}$ normal to axis OL).

Thus M_x, M_y, M_z, ie comps of $\underline{M}$, are the moments due to $\underline{F}$ about the x, y, z, axes respectively.

Couple.



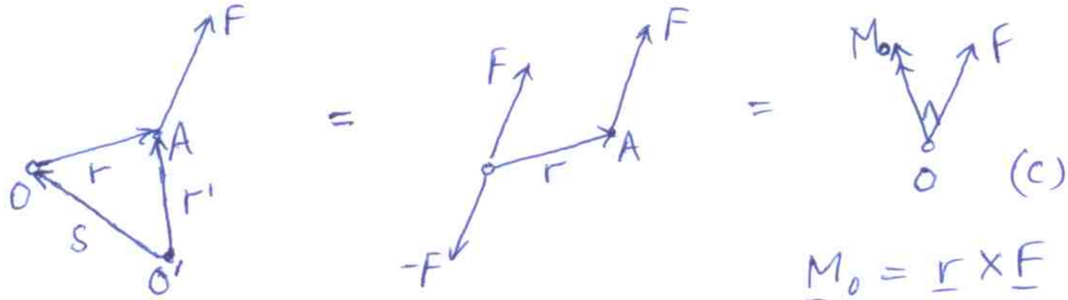
$$\underline{M} = \underline{r}_A \times \underline{F} + \underline{r}_B \times (-\underline{F}) = \underline{r} \times \underline{F}$$

$$M = rF\sin\theta = Fd$$

$\because \underline{r}$ is independent of origin O, couple is a free vector.

Two couples contained in same or parallel planes and having same moment (strength) M are equivalent (statically).

Resolution of a given force system into a force at O and a couple.



(a) (b) (c)

$$\underline{M}_O = \underline{r} \times \underline{F}$$

$$\underline{M}_{O'} = \underline{r}' \times \underline{F} = (\underline{s} + \underline{r}) \times \underline{F} = \underline{M}_O + \underline{s} \times \underline{F}$$

For a system of forces,

$$\underline{R} = \sum \underline{F}, \quad \underline{M}_O^R = \sum \underline{M}_O = \sum (\underline{r} \times \underline{F})$$

with $\underline{M}_O$, $\underline{F}$ replaced by M_O^R, R in above fig (c).

and $\underline{M}_{O'}^R = \underline{M}_O^R + \underline{s} \times \underline{R} \longrightarrow (*)$

Two system of forces are equivalent iff

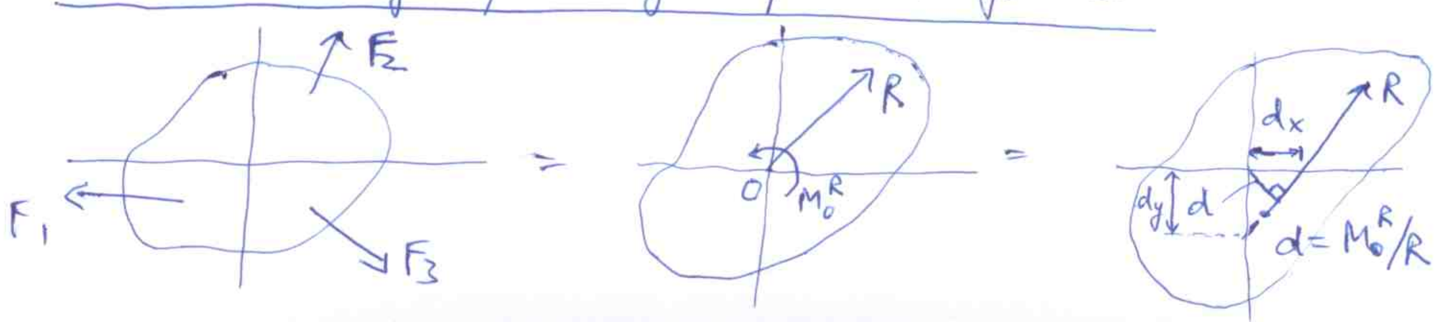
$$\sum \underline{F} = \sum \underline{F}', \quad \sum \underline{M}_O = \sum \underline{M}_O'.$$

If so, then $\sum \underline{M}_{O'} = \sum \underline{M}_{O'}'$ is assured, (see (*)) so the point O can be arbitrary when testing for equivalence.

Reduction of a system of concurrent forces.

Can be reduced to single force $\underline{R} = \sum \underline{F}$

Reduction of system of coplanar forces.



$R_x = \sum F_x$, $R_y = \sum F_y$, $M_z^R = M_0^R = \sum M_0$

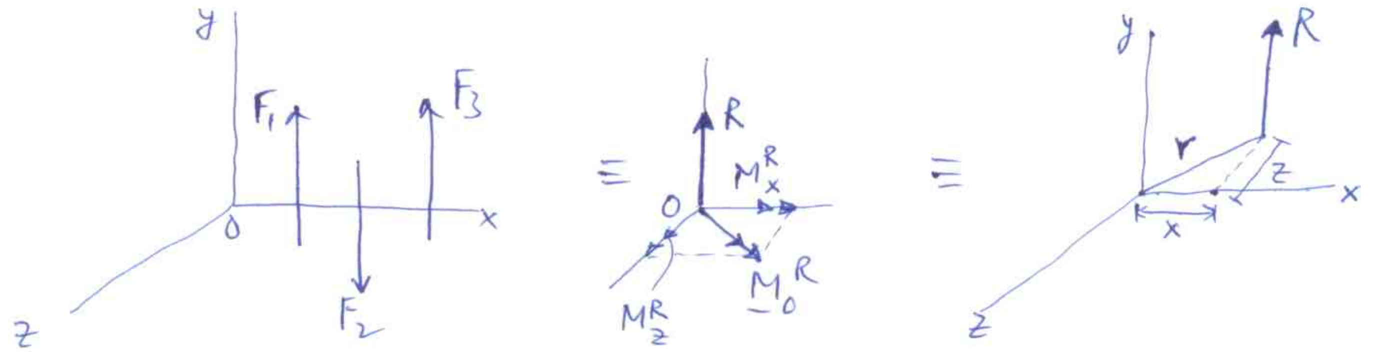
Require, $M_0^R = \underline{r} \times \underline{R} = xR_y - yR_x$ → eqn of line of action of $\underline{R}$

$y=0,\ x = d_x = M_0^R / R_y$

$x=0,\ y = d_y = M_0^R / R_x$

Note: Easy to show that physical line of action of $\underline{R}$ is indep of choice of 0.

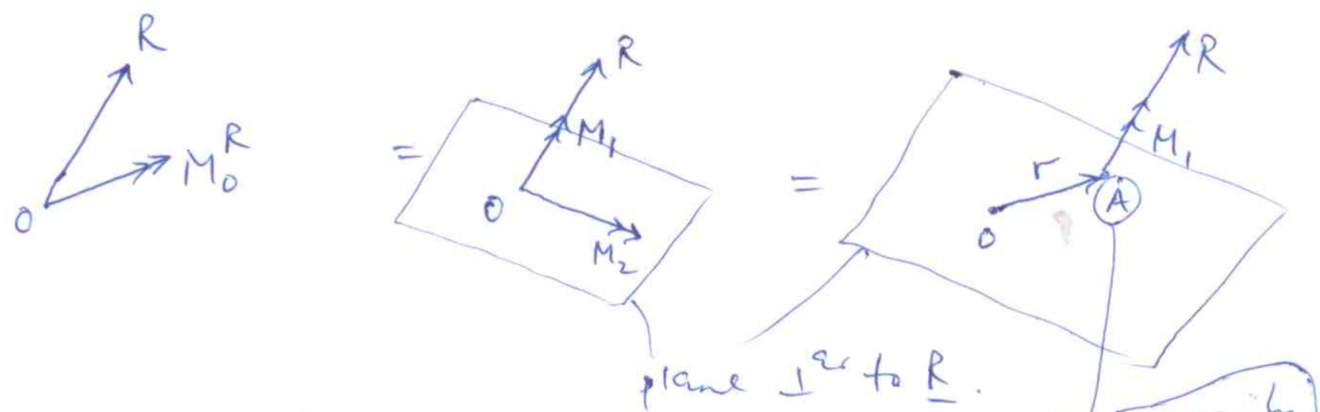
Reduction of a system of parallel forces.



$$\underline{r} \times \underline{R} = \underline{M}_0^R \Rightarrow -zR_y = M_x^R,\quad xR_y = M_z^R$$
$$(x\underline{i} + z\underline{k}) \times R_y\underline{j} = M_x^R \underline{i} + M_z^R k$$

Reduction to Wrench

This reduction is always possible



$$\left.\begin{aligned} \underline{M}_1 &= p\underline{R} \\ M_1 &= \frac{\underline{R} \cdot \underline{M}_0^R}{R} \end{aligned}\right\} \Rightarrow p = \frac{M_1}{R} = \frac{\underline{R} \cdot \underline{M}_0^R}{R^2}$$

Require $\underline{M}_1 + \underline{r} \times \underline{R} = \underline{M}_0^R$

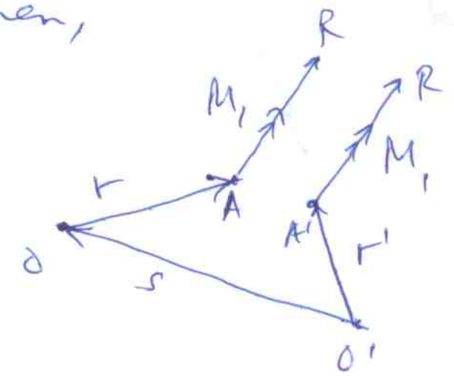
$\Rightarrow p\underline{R} + \underline{r} \times \underline{R} = \underline{M}_0^R$ → solve 3 scalar eqns for $\underline{r} = (x, y, z)$

If we took O' as the point of initial reduction instead of O, the wrench and its location in space would remain unchange, as seen below,

$$\underline{R} \cdot \underline{M}_{O'}^{R} = \underline{R} \cdot (\underline{M}_O^R + \underline{s} \times \underline{R}) = \underline{R} \cdot \underline{M}_O^R$$

Hence p is invariant to choice of O.

Then,



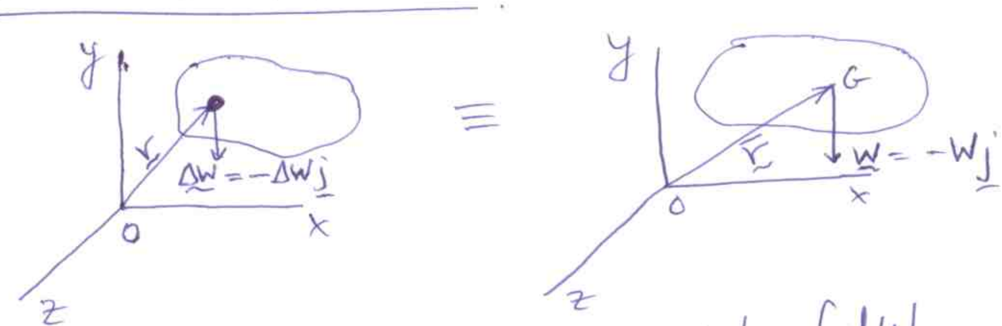
$$\left.\begin{aligned} p\underline{R} + \underline{r} \times \underline{R} &= M_O^R \\ pR + \underline{r}' \times \underline{R} &= M_{O'}^R \end{aligned}\right\}$$

$$(r' - r) \times R = s \times R$$

$\Rightarrow r' - r = s$, so A, A' are same point.

(see over)

Distributed Forces.



$$\Sigma F: \quad -W\underline{j} = \int -dW\underline{j} \Rightarrow W = \int dW$$

$$\Sigma M_O: \quad \bar{\underline{r}} \times (-W\underline{j}) = \int \underline{r} \times (-dW\underline{j})$$

$$\Rightarrow \bar{r}W \times (-\underline{j}) = \left(\int \underline{r}\, dW\right) \times (-\underline{j})$$

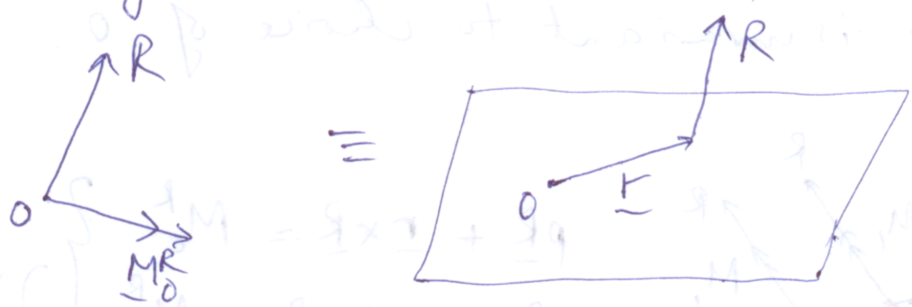
$$\Rightarrow \bar{\underline{r}}W = \int \underline{r}\, dW$$

ie $\bar{x}W = \int x\,dW$, $\bar{y}W = \int y\,dW$, $\bar{z}W = \int z\,dW$

Composite bodies. $\bar{X}\sum W_i = \sum \bar{x}_i W_i$, $\bar{Y}\sum W_i = \sum \bar{y}_i W_i$

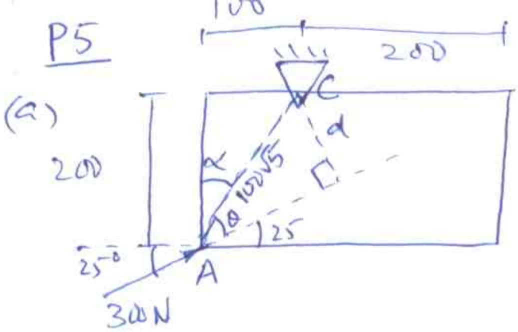
$\bar{Z}\sum W_i = \sum \bar{z}_i W_i$, summations over ith body

Note: if pitch, $p = 0$, it means that the 3-D force couple system can be reduced to a single force $\underline{R}$ acting at $\underline{r}$, ie $\underline{M}_1 = 0$, i-e,



P5

(a)



$d = 100\sqrt{5}\sin\theta$

$\alpha = \tan^{-1} 0.5 = 26.565°$, $\theta = 90 - \alpha - 25°$

$\theta = 38.4349°$, $d = 130.999$

$M_c = 300\,d = 41699.91$ N.mm.

(b) $\underline{M}_c = \underline{r} \times \underline{F} = (-100\underline{i} - 200\underline{j}) \times 300(\cos 25\,\underline{i} + \sin 25\,\underline{j})$

$= 300(-100\sin 25 + 200\cos 25)\underline{k} = 41699.91\underline{k}$

(c) $M_c = (300\cos 25 * 200 - 300\sin 25 * 100) = 41699.91$

(d) $M_c = 300\sin\theta * 100\sqrt{5} = 41699.91$ N-m

P6 $\underline{F} = 360 \dfrac{(-100\underline{j} + 200\underline{k} + 200\underline{i})}{100\sqrt{9}} = 120(2\underline{i} + 2\underline{k} - \underline{j})$

(a) $\underline{r} = 100\underline{j}$

$\underline{M}_0 = 100\underline{j} \times 120(2\underline{i} - \underline{j} + 2\underline{k}) = 12000(2\underline{i} - 2\underline{k})$ N-mm

$= 240(\underline{i} - \underline{k})$ N-m

$= 240\sqrt{2}\underbrace{\left(\frac{\underline{i}}{\sqrt{2}} - \frac{\underline{k}}{\sqrt{2}}\right)}_{\text{unit vector}}$ N.m.

(b) $\underline{r} = (60\underline{j} - 75\underline{k})$

$\underline{M}_D = \underline{r} \times \underline{F} = (60\underline{j} - 75\underline{k}) \times 120(2\underline{i} - \underline{j} + 2\underline{k})$

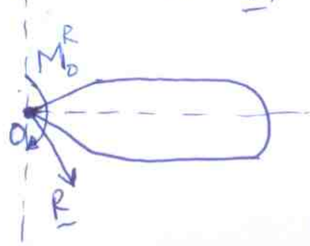
$= 120([60*2 + 75*(-1)]\underline{i} + [(-75)*2]\underline{j} + [(-60)*2]\underline{k})$

$\underline{M}_D = 120(45\underline{i} - 150\underline{j} - 120\underline{k}) = 236.75\underbrace{\frac{(45\underline{i} - 150\underline{j} - 120\underline{k})}{197.29}}_{\text{unit vector}}$ N.m

P.7. System of coplanar forces.

$\underline{R} = (-100\underline{j}) + (0.8\underline{i} - 0.6\underline{j})100 + (-100\underline{j}) + (0.6\underline{i} + 0.8\underline{j})100$

$= \underline{T}_1 + \underline{T}_2 + \underline{T}_3 + \underline{T}_4 = 100(1.4\underline{i} - 1.8\underline{j})$ kN.



$\underline{M}_0^R = \sum_{i=1}^{4} \underline{r}_i \times \underline{F}_i = \begin{bmatrix} (-100*24) + (-100*0.6*144 \\ -100*0.8*24) \\ +(-100*224) + (+100*0.8*184 \\ +100*0.6*24) \end{bmatrix}\underline{k}$

$\underline{M}_0^R = -19200\underline{k}$ kNm

For simplest resultant,

$$M_o^R \underline{k} = \underline{r} \times \underline{R} = (x R_y - y R_x)\,\underline{k}$$

$$\Rightarrow +19200 = +180x + 140y \rightarrow$$ eqn of line of action of $\underline{R}$ for simplest resultant (*)

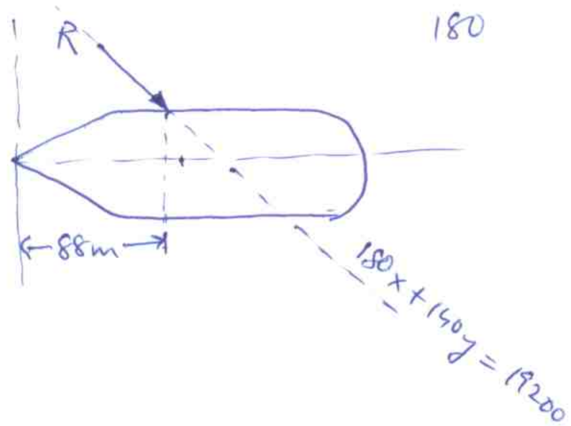
Since the single boat will provide a push of $\underline{R}$, it will be applied on the upper side. Point of application is intersection of str. line (*) and eqn of hull, ie, either $y = \frac{24}{64}x$ or $y = 24$.

Try intersect with $y = \frac{24}{64}x$

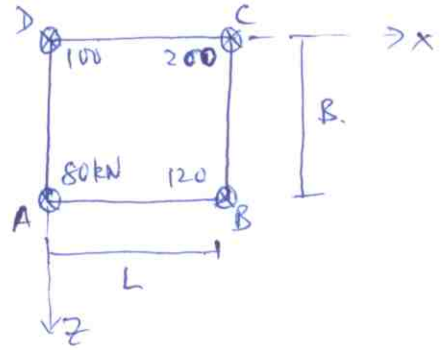
$$\Rightarrow x = \frac{19200}{180 + 140 \times \frac{24}{64}} = 82.58\,m > 64\,m$$ so cannot intersect there.

Try intersect with $y = 24$

$$\Rightarrow x = \frac{19200 - 140 \times 24}{180} = 88\,m$$



P.8. $\underline{R} = -(100 + 80 + 120 + 200)\,\underline{j} = -500\,\underline{j}$



$$\underline{M}_D^R = \underline{i}(80B + 120B) + (-200L - 120L)\,\underline{k}$$

Require $\underline{M}_D^R = \underline{r} \times \underline{R} = (x\,\underline{i} + z\,\underline{k}) \times R_y\,\underline{j}$

$$\Rightarrow 200B = 500z, \quad z = \frac{2}{5}B = 0.4B$$

$$-320L = -500x, \quad x = 0.64L$$

So $\underline{R}$ applied at $(x, z) = (0.64L, 0.4B)$

Let additional load $P(\downarrow)$ be located at $(x,z) = (\xi, \eta)$. (8)

$$\underline{R} = -(500+P)\,\underline{j}$$

$$\underline{M}_D^R = (200B + P\eta)\,\underline{i} + (-320L - P\xi)\,\underline{k}$$

We Require $\underline{M}_D^R = \left(\frac{L}{2}\underline{i} + \frac{B}{2}\underline{k}\right) \times (-500-P)\,\underline{j}$

$$\Rightarrow 200B + P\eta = (500+P)\frac{B}{2} \quad \Rightarrow P = \frac{50B}{\eta - B/2} \rightarrow (i) \Rightarrow \eta > \frac{B}{2} \text{ for } P>0$$

$$-(320L + P\xi) = -(500+P)\frac{L}{2} \quad \Rightarrow P = \frac{70L}{\frac{L}{2}-\xi} \rightarrow (ii) \Rightarrow \frac{L}{2} > \xi \text{ for } P>0$$

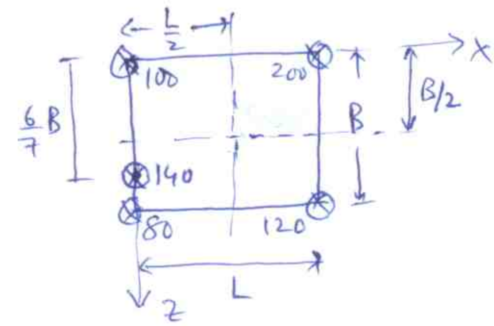
$$\Rightarrow 50B\left(\frac{L}{2} - \xi\right) = 70L\left(\eta - \frac{B}{2}\right) \longrightarrow (iii)$$

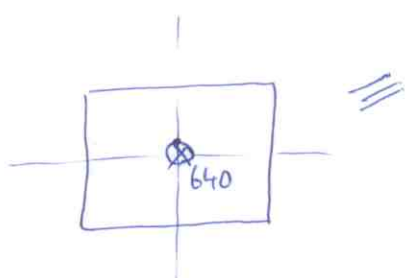
Least value of P:

(i) $\Rightarrow$ $P = 100$ for $\eta = B \longrightarrow$ (iii) $\Rightarrow \xi = \frac{L}{2} - \frac{7}{10}L \rightarrow$ gives $P<0$

(ii) $\Rightarrow$ $P = 140$ for $\xi = 0 \longrightarrow$ (iii) $\Rightarrow \eta = \frac{60}{70}B \rightarrow$ gives $P>0$.

So $P = 140$, $\eta = \frac{6}{7}B$, $\xi = 0$. ◀





P. 9. Distributed load: $R_1 = 10 * 12 + \frac{1}{2}(6)(20) = 180\,N$

$$\bar{x}_1 = \frac{(10)(12)(6) + \frac{1}{2}(6)(20)(10)}{180} = 7.333\cdots$$

O is the left end of beam.

$$M_O = (180)\left(\frac{22}{3}\right) + (100)(3) - 300$$

Require, $M_O = R\bar{x}$, $R = \underset{R_1}{180} + \underset{R_2}{100} = 280$

$$\Rightarrow \bar{x} = \frac{1320}{280} = 4.714\ m$$

P.10. $p = \rho g(d-x)$

$$R = \int_0^{225} p\, 225\, dx = 225 \rho g \int_0^{225} (d-x)\, dx = 225 \rho g \left(225 d - \frac{225^2}{2}\right)$$

center of pressure on valve $= \bar{x} = \int_0^{225} (p\, 225\, dx)\, x \Big/ R$

$$\bar{x} = \frac{\cancel{225 \rho g} \left(d \frac{x^2}{2} - \frac{x^3}{3}\right)\Big|_0^{225}}{\cancel{225 \rho g} \left(225 d - \frac{225^2}{2}\right)} = \frac{d \frac{(225)^2}{2} - \frac{225^3}{3}}{225 d - \frac{225^2}{2}}$$

For valve opening, $\bar{x} = h = 100$.

$$\Rightarrow (225)(100) d - \left(\frac{225^2}{2}\right)(100) = d \frac{(225)^2}{2} - \frac{225^3}{3}$$

$d = 450\,mm$

For any depth > 450 mm the c.o.p. will shift above hinge A and valve will open. For $d < 450$, the reaction at B prevents valve opening.

P.11 $\underline{R}_1$ (due to vertical pressure + self wt on horizontal part) $= \left\{ \rho g (0.4)(a+0.4)(0.5) + \frac{500\,(a+0.4)}{(a+0.4+0.5)} \right\} \underline{j}$

$\bar{x}_1 = \frac{(a+0.4)}{2}$ → measured horizontally from A

$\underline{R}_2$ (due to self wt of slanted part) $= \left\{ \frac{500\,(0.5)}{(a+0.4+0.5)} \right\} \underline{j}$

$\bar{x}_2 = a + 0.4 + \frac{0.3}{2}$ → measured horizontally from A.

$\underline{R}_3$ (due to inclined pressure on slanted part) $= \left\{ \int_0^{0.4} \rho g x \left(0.5 * \frac{5}{4} dx\right) \right\} \left(\frac{4}{5}\underline{i} - \frac{3}{5}\underline{j}\right)$

$$\bar{x}_3 = \frac{\int_0^{0.4} \rho g x \left(0.5 * \frac{5}{4} dx\right) \frac{5}{4} x}{|\underline{R}_3|} = \frac{\frac{5}{4} \frac{x^3}{3}\Big|_0^{0.4}}{\frac{x^2}{2}\Big|_0^{0.4}} = \left(\frac{5}{4}\right)\left(\frac{2}{3}\right)(0.4)$$

$\bar{x}_3 = \frac{1}{3}\,m$ (measured along incline) from D

Can get it directly from triangular load distribution along incline.

Find simplest resultant of $\underline{R}_1, \underline{R}_2, \underline{R}_3$

$$\underline{R} = \sum_{i=1}^{3} \underline{R}_i = -\left[\rho g (0.2)(a+0.4) + 500\right] \underline{j} + \rho g \left(\frac{2.5}{4}\right)\left(\frac{0.16}{2}\right)\left(\frac{4}{5}\underline{i} - \frac{3}{5}\underline{j}\right)$$

$\underline{M}_0^R = \underline{r} \times \underline{R}$ required

$$\Rightarrow \overbrace{-\left(\rho g (0.2)(a+0.4) + \frac{500(a+0.4)}{(a+0.4+0.5)}\right)\frac{(a+0.4)}{2}}^{\text{due to } \underline{R}_1}$$

$$\underbrace{-\frac{500(0.5)}{a+0.4+0.5}\left(a+0.4+\frac{0.3}{2}\right)}_{\text{due to } \underline{R}_2} \underbrace{-\rho g(0.05)\left(\frac{4}{5}*\frac{0.4}{3}+\frac{3}{5}[0.4+0.1+a]\right)}_{\text{due to } R_3}$$

$$= (x\underline{i} + y\underline{j}) \times \underline{R}$$

For min 'a' we need line of action of $\underline{R}$ to pass thru B (i.e, condition when it is just about to rotate about B by loosing contact with A).

$\Rightarrow$ put $x = a$, $y = 0$

and use $\rho g = 9.81 * 1000 = 9810$.

$$9810(0.2)\frac{(a+0.4)^2}{2} + \frac{500}{(a+0.9)(2)}\left([a+0.4]^2 + a + 0.4 + 0.15\right)$$

$$+ 9810(0.05)\left(\frac{6.1}{15} + \frac{3}{5}a\right) = \left[9810(0.2)(a+0.4) + 500 + 9810(0.05)\frac{3}{5}\right] a$$

$$\Rightarrow 1962(a^2 + 0.8a + 0.16)(a+0.9) + 500(a^2 + 0.8a + 0.16 + a + 0.55)$$

$$+ 981\left(\frac{6.1}{15}a + \frac{2.7}{5}a + \frac{3}{5}a^2 + \frac{5.49}{15}\right) = (1962(a+0.4) + 500 + 294.3) * (2a^2 + 1.8a)$$

$$\Rightarrow a^3(1962) + a^2(3335.4 + 500 + 588.6) + a(1726.56 + 900 + 928.68)$$

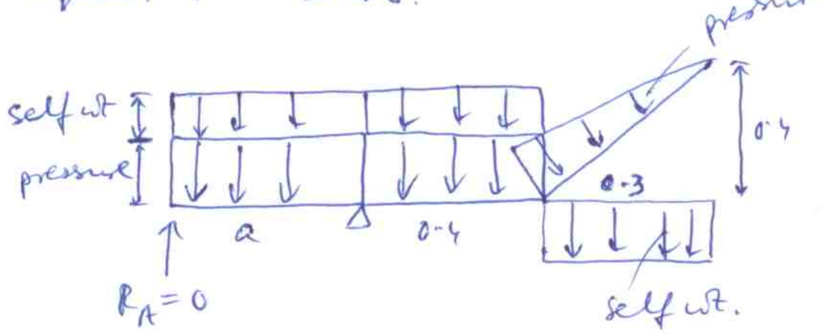
$$+ 996.574 = a^3(3924) + a^2(6689.8) + a(2842.38)$$

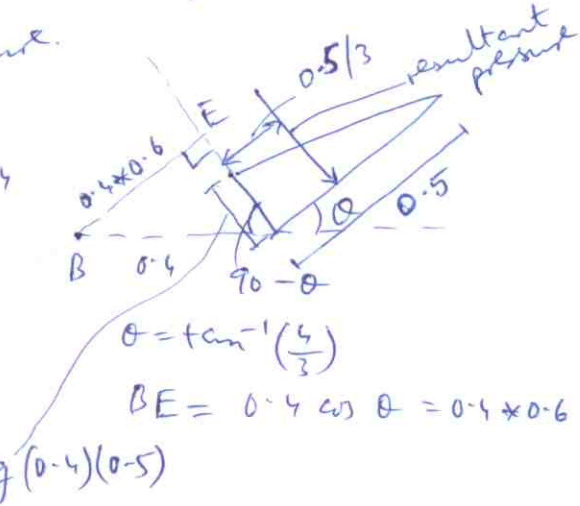
$$\Rightarrow a^3(-1962) + a^2(-2265.8) + a(712.86) + 996.574 = 0$$

$$a = \underbrace{\cancel{-0.77}}_{\text{discard}},\ \underline{\underline{0.6423}},\ \underbrace{\cancel{-1.0271}}_{\text{discard}}$$

Done using equilibrium approach.

Take moments about hinge B and put reaction at A equal to zero.





$$\theta = \tan^{-1}\left(\frac{4}{3}\right)$$

$$BE = 0.4 \cos\theta = 0.4 * 0.6$$

$\Sigma M_B = 0$ with $R_A = 0$ yields,

$$-\rho g(0.4)(0.5)(a)\left(\frac{a}{2}\right) + \rho g(0.4)(0.5)(0.4)\left(\frac{0.4}{2}\right)$$

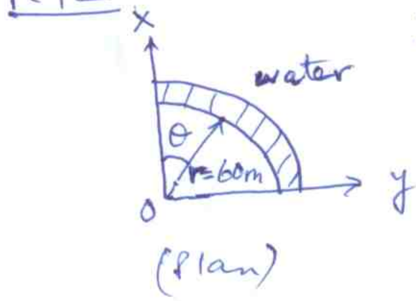
$$-\frac{500}{a+0.9}\left(\frac{a^2}{2}\right) + \frac{500}{a+0.9}\left(\frac{0.4^2}{2}\right) + \frac{500}{a+0.9}(0.5)\left(\frac{0.3}{2} + 0.4\right)$$

$$+\frac{\rho g(0.4)(0.5)}{2}(0.5)\left(\frac{0.5}{3} + 0.4*0.6\right) = 0$$

$$\Rightarrow a^3(-1962) + a^2(-2265.8) + a(712.86) + 996.574 = 0$$

i.e, we get same polynomial eqn as by first method

P. 12



Dam is 40 m height.

$$\underline{dR} = \rho g z\,(r\,d\theta\,dz)\left(-\cos\theta\,\underline{i} - \sin\theta\,\underline{j}\right)$$

$$\underline{R} = \int \underline{dR} = \rho g r \int_0^{\pi/2} \left(-\cos\theta\,\underline{i} - \sin\theta\,\underline{j}\right) d\theta \int_0^{40} z\,dz$$

$$= (9810)(60)\left(\frac{40^2}{2}\right)\left(-\sin\theta\Big|_0^{\pi/2}\underline{i} + \cos\theta\Big|_0^{\pi/2}\underline{j}\right)$$

$$= 47088 * 10^4\,(-\underline{i} - \underline{j})\ \text{N} = 6.659 * 10^8 * \left(\frac{-\underline{i}-\underline{j}}{\sqrt{2}}\right)\ \text{N}.$$

$\underline{\bar{r}} \times \underline{R} = \underline{M}_0^R$ (moment equivalence requirement).

$$(r\cos\bar\theta\,\underline{i} + r\sin\bar\theta\,\underline{j} + \bar z\,\underline{k}) \times \underline{R} = \int \underline{r} \times \underline{dR}$$

$$47088 * 10^4\left(-r\cos\bar\theta\,\underline{k} + r\sin\bar\theta\,\underline{k} - \bar z\,\underline{j} + \bar z\,\underline{i}\right)$$

$$= \cancel{\int_0^{\pi/2}\int_0^{40} -\rho g z r^2\,d\theta\,dz\,\frac{\sin 2\theta}{2}\,\underline{k}} + \cancel{\int_0^{\pi/2}\int_0^{40} \rho g z r^2\,d\theta\,dz\,\frac{\sin 2\theta}{2}\,\underline{k}}$$

$$+ \int_0^{\pi/2}\int_0^{40} \rho g z r\,d\theta\,dz\left(-z\cos\theta\,\underline{j} + z\sin\theta\,\underline{i}\right)$$

$$= \rho g r\,\frac{40^3}{3}\left(-\sin\theta\Big|_0^{\pi/2}\underline{j} - \cos\theta\Big|_0^{\pi/2}\underline{i}\right)$$

$$= \rho g r\,\frac{40^3}{3}\left(-\underline{j} + \underline{i}\right)$$

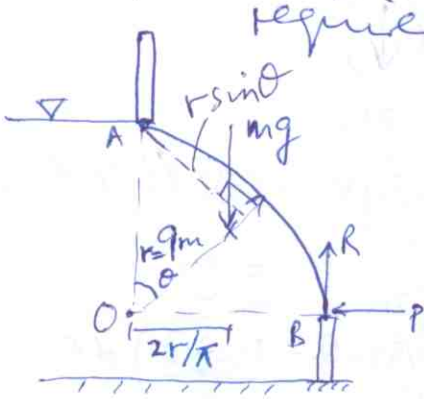
$$\left.\begin{aligned} \Rightarrow \cos\bar\theta &= \sin\bar\theta \Rightarrow \bar\theta = \pi/4 \\ \bar z &= \frac{\rho g r}{47088 * 10^4}\,\frac{40^3}{3} = 26.66 \end{aligned}\right\}$$

Shorter method:
Can get this directly from triangular distribution of pressure load (ie centroid is $\frac{2}{3}$ * height = 26.66) and at $\theta = 45°$ due to symmetry of x, y components of pressure for the quarter circle.

$$\underline{dR} = \rho g (40) * \left(\frac{40}{2}\right) r\,d\theta\left(-\cos\theta\,\underline{i} - \sin\theta\,\underline{j}\right)$$

$$\Rightarrow R = \rho g\,\frac{40^2}{2}(60)\,[-i - j]$$

P. 13. The equivalent force-couple system is a force in the plane of the figure and zero moment about pt. A. If R=0 at pt. P then P will be min force required to keep gate closed.



$$\Rightarrow \sum M_A = 0$$

$$\Rightarrow 6800 g \frac{2r}{\pi} + Pr - \int_0^{\pi/2} \rho g\, r(1-\cos\theta)(6)(r\,d\theta)\, r\sin\theta = 0$$

$$\Rightarrow 6800(9.81)\left(\frac{2 \times 9}{\pi}\right) + P(9) - (9810)(6)(9^3)\int_0^{\pi/2}\left(\sin\theta - \frac{\sin 2\theta}{2}\right) d\theta = 0$$

$$\Rightarrow (6800)(9.81)\left(\frac{18}{\pi}\right) + 9P - (9810)(6)(9^3)\left[\frac{\cos 2\theta}{4} - \cos\theta\right]_0^{\pi/2} = 0$$

$$\left[\frac{\cos 2\theta}{4} - \cos\theta\right]_0^{\pi/2} = \frac{-1-1}{4} - (0-1) = 1/2$$

$$\Rightarrow P = 2.341 \times 10^6 \text{ N}$$

Note: This is same as 'equilibrium' approach.
Note: Can also do by $\sum M_o = 0$ (see over)

P. 14 $\underline{R}_1 = -100\underline{k}$, $\underline{R}_2 = 100\underline{k}$, $\underline{R}_3 = 400\underline{j}$, $\underline{R}_4 = 500\dfrac{(-4\underline{i}+4\underline{j}+8\underline{k})}{4\sqrt{6}}$

$$\underline{R} = \sum_{i=1}^{4} \underline{R}_i = 400\underline{j} + 500\left(-\frac{\underline{i}}{\sqrt6} + \frac{\underline{j}}{\sqrt6} + \frac{2\underline{k}}{\sqrt6}\right)$$

$$= -\frac{500}{\sqrt6}\underline{i} + \left(400 + \frac{500}{\sqrt6}\right)\underline{j} + \frac{1000}{\sqrt6}\underline{k}$$

$$\underline{M}_A^R = \cancel{-(100)(20)\underline{i}} + \cancel{(100)(20)\underline{i}} + (100)(4)\underline{j} + (-4\underline{i} + 20\underline{j}) \times \underline{R}_4$$

$$= 400\underline{j} + \frac{500}{\sqrt6}(-4\underline{k} + 8\underline{j} + 20\underline{k} + 40\underline{i}) = \frac{20000}{\sqrt6}\underline{i} + \left(400 + \frac{4000}{\sqrt6}\right)\underline{j} + \frac{8000}{\sqrt6}\underline{k}$$

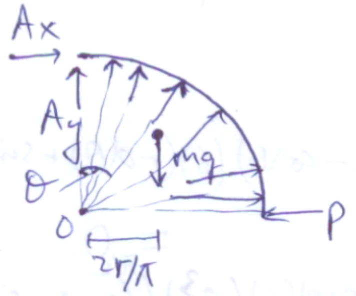
$$\underline{M}_B^R = (100)(4)\underline{j} + (-4\underline{i}) \times \underline{R}_4 = 400\underline{j} + \frac{500}{\sqrt6}(-4\underline{k} + 8\underline{j})$$

$$= \left(400 + \frac{4000}{\sqrt6}\right)\underline{j} - \frac{2000}{\sqrt6}\underline{k}$$

Simplest resultant in BCDE plane is a wrench.
Note that in the theory of 'wrench' we took the

P-13 Alternative method.

$$\sum M_0 = 0 \Rightarrow A_x(r) + mg\left(\frac{2r}{\pi}\right) = 0$$



$$A_x = -mg\left(\frac{2}{\pi}\right) (\ ).$$

$$\sum F_x = 0 \Rightarrow A_x - P + \int_0^{\pi/2} \rho g r(1-\cos\theta)(6)(r\,d\theta)\sin\theta = 0$$

$$\Rightarrow P = A_x + (9810)(6)(9^2)\int_0^{\pi/2}\left(\sin\theta - \frac{\sin 2\theta}{2}\right)d\theta$$

$$P = A_x + (9810)(6)(9^2)\left[\frac{\cos 2\theta}{4} - \cos\theta\right]_0^{\pi/2} = \frac{-1-1}{4} - (0-1) = 1/2$$

$$= 2341.3 \text{ kN}.$$

point of application of the wrench in the plane $\perp^{ar}$ to $\underline{R}$. However, due to principle of transmissibility, the wrench would intersect BCDE plane and thus can be located in BCDE plane as follows:

$$p = \frac{\underline{R} \cdot \underline{M}_B^R}{R^2} = \frac{\left(400+\frac{500}{\sqrt{6}}\right)\left(400+\frac{4000}{\sqrt{6}}\right) - \left(\frac{1000}{\sqrt{6}}\right)\left(\frac{2000}{\sqrt{6}}\right)}{573299.3162}$$

$$= 1.560872$$

Now $p\underline{R} + \underline{r} \times \underline{R} = \underline{M}_B^R \longrightarrow (1)$.

$\underline{r} \to (x\underline{i}+z\underline{k})$ instead of $(x\underline{i}+y\underline{j}+z\underline{k})$ as done in theory, in class.

Equate $\underline{i}$, $\underline{j}$, $\underline{k}$ comp's of (1)

$\underline{i}: \; p\left(-\frac{500}{\sqrt{6}}\right) - \left(400+\frac{500}{\sqrt{6}}\right)z = 0 \Rightarrow z = -0.52739 \text{ m}$

$\underline{j}: \; p\left(400+\frac{500}{\sqrt{6}}\right) - \frac{1000}{\sqrt{6}}x - \frac{500}{\sqrt{6}}z = 400+\frac{4000}{\sqrt{6}} \longrightarrow (*)$

$\underline{k}: \; p\left(\frac{1000}{\sqrt{6}}\right) + \left(400+\frac{500}{\sqrt{6}}\right)x = -\frac{2000}{\sqrt{6}} \Rightarrow x = -2.40633 \text{ m}$

put x, z in (*) and you get an identity, as expected. Hence simplest resultant in BCDE plane is $\underline{R}$ and $\underline{M} = p\underline{R}$ acting at x, z as given above measured w.r.t. B.