Equilibrium of Rigid Bodies.

$\sum \underline{F} = \underline{0}, \quad \sum \underline{M}_0 = \underline{0}$ (ie eqvt force-couple system vanishes).

If $\sum \underline{M}_0 = 0 \Rightarrow \sum \underline{M}_{0'} = 0$ $\left(\because \sum \underline{M}_{0'} = \underbrace{\sum \underline{M}_0}_{=0} + \underline{r}_{0'0} \times \underbrace{\sum \underline{F}}_{=0} \right)$

FBD's : (i) Seperate body/system from ground and other interconnected bodies/systems.

Show all external forces only:

(ii) Show all reactions – their line of action, if known, should be correctly shown. Else, show them as components.

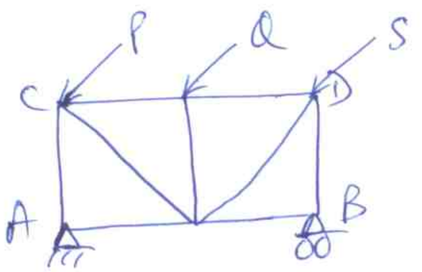
(iii) Applied forces should be shown with correct direction.

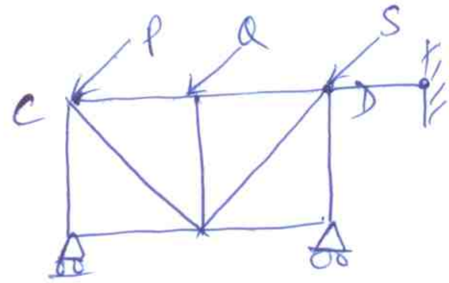
(iv) Work with FBD of one body or of system, as per convenience. When working with FBD of system, do not show interconnecting forces in FBD as they are internal forces.

(V) Maintain sign consistency between FBD's and equil eqns.

Note: The above equil eqns are applicable to system of rigid bodies, since interconnecting forces and moments (due to friction in connections) will cancel out when summing above eqns over all RB's comprising the system.

(Egs)





$\sum F_x = 0,\ \sum M_A = 0,\ \sum M_B = 0$
or $\sum M_A = 0,\ \sum M_B = 0,\ \sum M_C = 0$
or $\sum F_x = 0,\ \sum F_y = 0,\ \sum M_A = 0$

} these sets are dependent (ie linear combo) of each other.

Working Tip: write equil eqns involving only one unknown in each eqn, so you can solve 'progressively' instead of 'simultaneously'.

Statically indeterminate reactions. Partial constraints.

(i) Completely constrained: RB cannot move under general loading.

(ii) Statically determinate: When number of equilibrium equations (for all RB's in the system) equals number of unknown reactions appearing in all FBD's of the system (comprising). Hence, by using eqns of statics alone we can determine all reactions (internal & external).

necessary not only

If system = one planar RB, then SD iff equil eqns = external reactions = 3.

If system = one 3-D RB, then SD iff equil eqns = ext reactions = 6.

(iii) Statically indeterminate: When unknown reactions exceed available equil eqns. Then we need mechanics of deformation to determine the excess reactions

(eg) Above truss with A, B both pinned. Then n = reactions = 4, $\Rightarrow n > 3$ so SID.

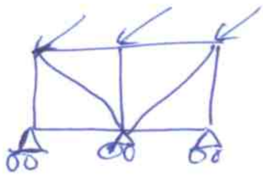
: Unknown reactions < Equil eqns

In that case equilibrium can't be maintained for general loading.

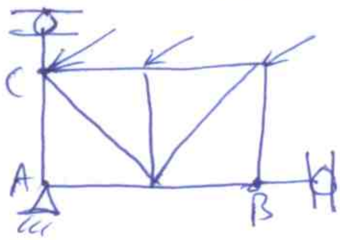
(eg) Above truss with A, B, both roller supports. Then $\sum F_x = 0$ not satisfied if loads have horizontal component.

<u>Improper constraints</u>: Here we can have static indeterminacy even though eqns = unknowns.

(eg)



$\sum F_x = 0$ not satisfied.
Can't find all reactions.



$\sum M_A = 0$ can't be satisfied since lines of action of R_B, R_C pass thru A. Hence truss rotates about A.

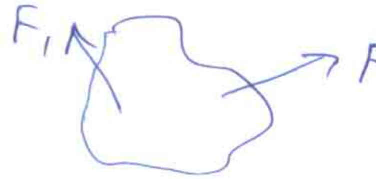
- A RB is improperly constrained (despite that we may have eqns $\leq$ unknowns) when supports are arranged such that reactions are either "<u>concurrent</u>" or parallel.
- Necc + suff condt for RB to be SD and completely constrained is that equil eqns = unknown reactions and supports do not yield "concurrent" or parallel reactions.

<u>Note</u>: (i) For planar RB "concurrent" $\Rightarrow$ concurrent at a point.

(ii) For 3-D RB's "concurrent" $\Rightarrow$ concurrent on a line. In that case, since all forces intersect a line, the comp of moment taken along the line, of all reaction forces about any pt on that line, is identically zero. So we have lost one equilibrium eq

Force Body - Equilibrium.

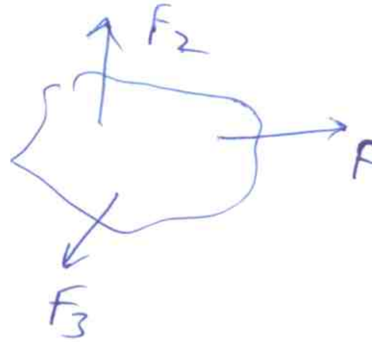
For $\Sigma \underline{F} = 0$, $\underline{F}_1 = -\underline{F}_2$



For $\Sigma \underline{M} = 0$, $\underline{F}_1$ and $\underline{F}_2$ should be collinear.

$\Rightarrow$ $\underline{F}_1 = -\underline{F}_2$ and both are collinear.

3-Force Body - Equilibrium



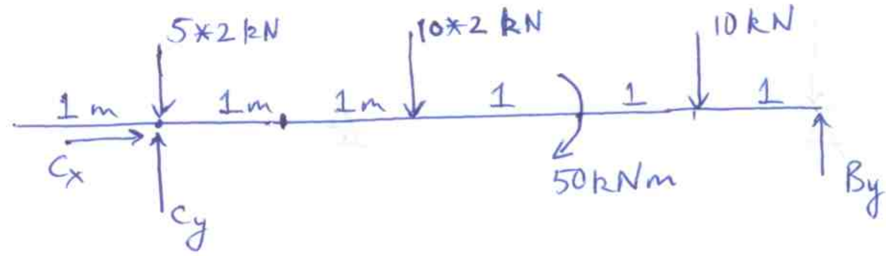
For $\Sigma \underline{F} = 0$ $\underline{F}_3$ must lie in same plane as $\underline{F}_1 + \underline{F}_2$, ie all three forces should be coplanar.

For $\Sigma \underline{M} = 0$, all should be concurrent.

$\Rightarrow$ Forces should be coplanar & concurrent.

However, they could also all be parallel, in special cases, and $\Sigma \underline{M}$ would still be zero.

P. 15



$$\Sigma F_x = 0 \Rightarrow C_x = 0$$

$$\Sigma M_C = 0 \Rightarrow 20*2 + 50 + 10*4 - B_y*5 = 0$$

$$\Rightarrow B_y = 26$$

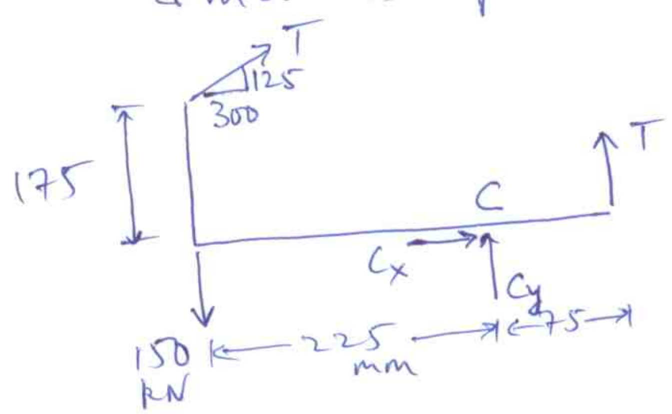
$$\Sigma M_B = 0 \Rightarrow 10*5 - C_y*5 + 20*3 - 50 + 10*1 = 0$$

$$\Rightarrow C_y = 14$$

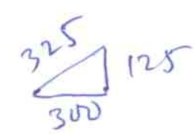
Check: $\Sigma F_y \overset{?}{=} 0 \Rightarrow 5*2 + 10*2 + 10 - B_y - C_y \overset{?}{=} 0$

Yes.

P. 16. Frictionless pulley & massless rope $\Rightarrow T_{AB} = T_{DB} = T$.



$\Sigma M_C = 0$:



$$150*225 + T*75 - T*\frac{125}{325}*225 - T*\frac{300}{325}*175 = 0$$

$$\Rightarrow T = 195$$

$$\Sigma F_y = 0 : T*\frac{125}{325} - 150 + C_y + T = 0$$

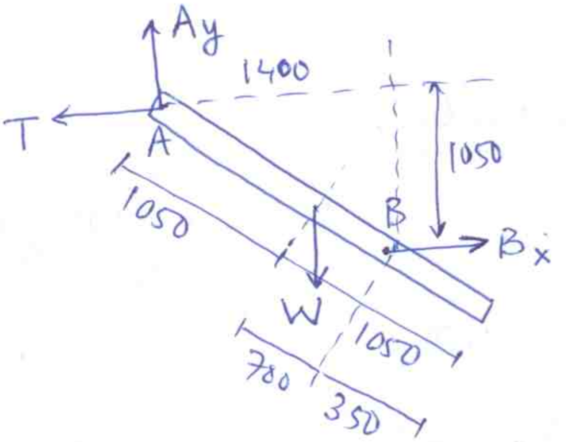
$$\Rightarrow C_y = -120 \quad (\text{ie } 120 \downarrow).$$

$$\Sigma F_x = 0 : T*\frac{300}{325} + C_x = 0$$

$$\Rightarrow C_x = -180 \quad (\text{ie } 180 \leftarrow)$$

P-17

(see over for alternate soln)



$\Sigma M_A = 0$:

$\underset{70 * 9.81}{W}\left(1400 * \frac{1050}{1750}\right) - B_x(1050) = 0$

$B_x = 549.36$ N

$\Sigma F_x = 0 \Rightarrow T - B_x = 0$

$T = 549.36$ N.

$\Sigma F_y = 0 \Rightarrow A_y - W = 0$

$A_y = W = 686.7$ N

Each roller supports half the respective reaction

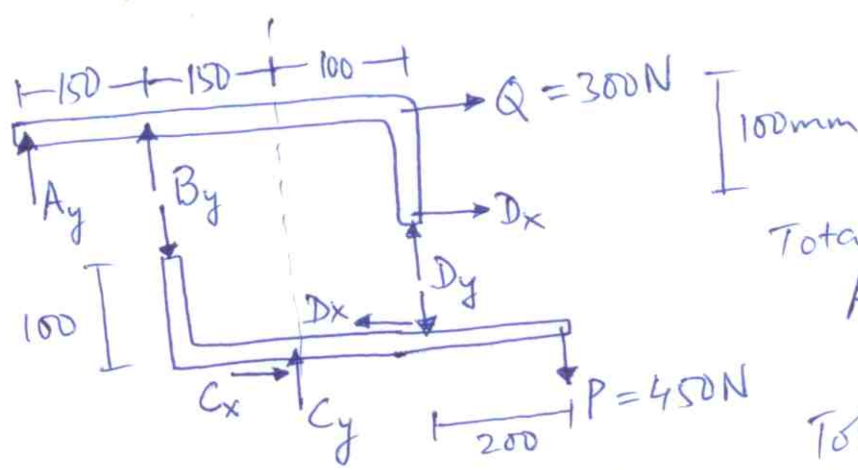
i.e rollers A support 343.35N

rollers B support 274.68N

each.

Note: Assumed that T applied at midpoint of upper edge (as given) so no moments due to T to cause out of plane motion/reactions, i.e it is a 2-D statics problem.

P.18.



Total unknowns:

$A_y, B_y, C_x, C_y, D_x, D_y$

$= 6$

Total eqns: 2 rigid bodies so 6 eqns of static equilibrium.

statically Determinate $\Leftarrow$

Upper body:

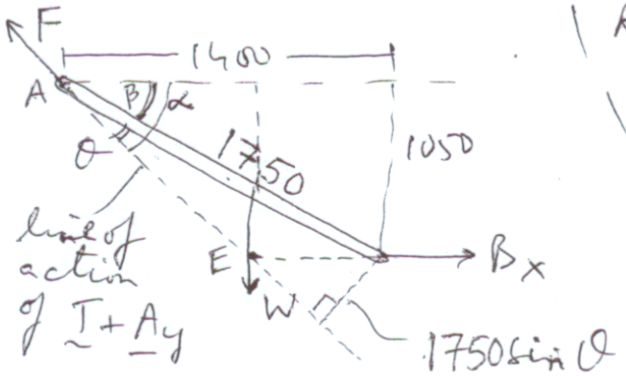
$\Sigma M_A = 0$: $B_y(150) + D_x(100) + D_y(400) = 0 \rightarrow (1)$

$\Sigma F_x = 0$: $D_x + Q = 0 \rightarrow (2)$

$\Sigma F_y = 0$: $A_y + B_y + D_y = 0 \rightarrow (3)$

P.17 alternate soln.

It is a 3-Force member.



$$AE = \sqrt{\left[1400\left(\frac{1050}{1750}\right)\right]^2 + 1050^2} = 1344.6569$$

not req

$$\theta = \alpha - \beta = \tan^{-1}\left(\frac{1050}{1400 * \frac{1050}{1750}}\right) - \tan^{-1}\frac{1050}{1400}$$

$$\Rightarrow \theta = 14.470294°$$

$$\alpha = 51.34019°$$

$$\Sigma M_B = 0: \; F(1750 \sin\theta) - W\left(1400 * \frac{700}{1750}\right) = 0$$

$$\Rightarrow F = 879.405$$

$A_y = F \sin\alpha = 686.7\,N = 70\,kg$ (same as W, as expected)

$T = F \cos\alpha = 549.36\,N$

$B_x = T$ (as before).

Lower body

$$\sum M_c = 0 : \quad B_y(150) - D_y(100) - P(300) = 0 \rightarrow (4)$$

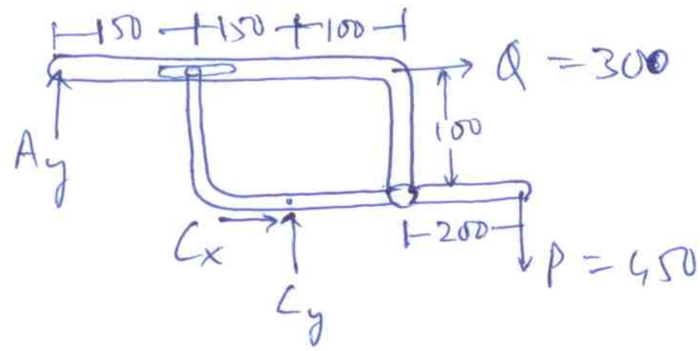
$$\sum F_x = 0 : \quad C_x - D_x = 0 \rightarrow (5)$$

$$\sum F_y = 0 : \quad B_y - C_y + D_y + P = 0 \rightarrow (6)$$

Can solve (1)–(6) for 6 unknowns. This is the straightforward way.

Quicker way:

Consider both bodies with supports A, C, isolated. This is legitimate FBD of system of RB's.



$\sum M_c = 0$:

$0 = A_y(300) + Q(100) + P(300)$

$\Rightarrow A_y = -550$ N ◄ (i)

$\sum F_y = 0 : A_y + C_y - P = 0$

$\Rightarrow C_y = 1000$ ◄ (ii)

$\sum F_x = 0 : C_x + Q = 0 \rightarrow$ (iii)

$C_x = -300$ ◄

Now from FBD of upper body:

(2) $\Rightarrow D_x = -300$ ◄

$$\sum M_D = 0 : A_y(400) + B_y(250) + Q(100) = 0 \rightarrow (iv)$$

$\Rightarrow B_y = 760$ ◄

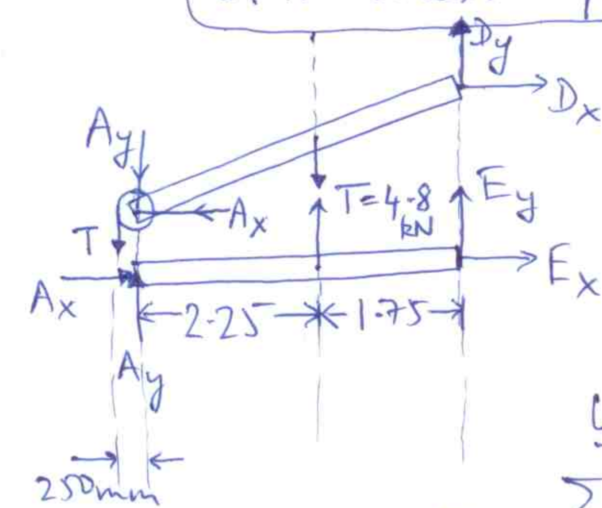
(3) $\Rightarrow D_y = -210$ ◄

Can easily verify that soln solves (1)–(6)

Note: (i)–(iv) are not independent of (1)–(6).

(i)–(iv) are linear combo's of (1)–(6)

P-19. Frictionless pulleys, (& massless rope) $\Rightarrow$ tension constant in rope.



6 unknowns ($A_x, A_y, E_x, E_y, D_x, D_y$) & 6 eqns (for 2 bodies) so statically determinate.

Upper link:

$\sum M_A = 0: \ T(0.25) - T(2.25) + D_y(4) - D_x(1.5) = 0 \rightarrow ①$

$\sum F_x = 0: \ -A_x + D_x = 0 \rightarrow ②$

$\sum F_y = 0: \ -T - A_y - T + D_y = 0 \rightarrow ③$

Lower link

$\sum M_A = 0: \ T(2.25) + E_y(4) = 0 \rightarrow ④$

$\sum F_x = 0: \ A_x + E_x = 0 \rightarrow ⑤$

$\sum F_y = 0: \ A_y + T + E_y = 0 \rightarrow ⑥$

Can solve ①–⑥ for 6 unknowns. This is straightforward way

Shorter way:

$\because$ only D_x, D_y, E_x, E_y required, use FBD of system of both links isolated from supports D, E.

$\sum M_E = 0 \Rightarrow D_x(1.5) - T(4.25) = 0 \rightarrow$ (i)

$\Rightarrow D_x = 13.6$ ◄

$\sum F_x = 0 \Rightarrow D_x + E_x = 0 \rightarrow$ (ii)

$\Rightarrow E_x = -13.6$ ◄

(4), ie FBD of lower link $\Rightarrow E_y = -2.7$ ◄

$\sum F_y = 0 \Rightarrow D_y + E_y - T = 0 \rightarrow$ (iii) $\Rightarrow D_y = 7.5$ ◄

Check: soln solves ① & ④.

ie, (i)–(iv) are dependent on ①–⑥.

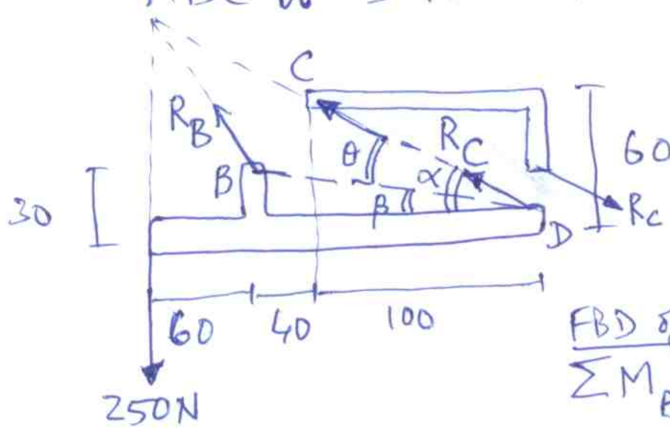
Get A_x, A_y from ②, ③ or ⑤, ⑥.

soln of A_x, A_y from these satisfy these $\because$ ③ + ⑥ $\equiv$ (iii)

② + ⑤ $\equiv$ (ii)

P.20 Shorter way (but prone to errors).

ABC is 3-Force member, CD is 2-Force member.



$BD = \sqrt{140^2 + 30^2} = 143.1782$

$\theta = \alpha - \beta$

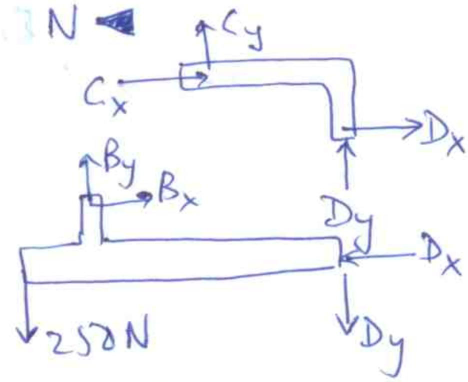
$\theta = \tan^{-1}\left(\frac{60}{100}\right) - \tan^{-1}\left(\frac{30}{140}\right) = 18.87^\circ$

FBD of 3-Force member:

$\sum M_B = 0 : 250(60) + R_c(BD \sin\theta) = 0$

$\Rightarrow R_c = -323.942$ N at $\alpha = 30.96375^\circ$ as shown,

or, $C_x = +277.77$ N, $C_y = -166.66$ N



$\sum F_y = 0 : B_y - 250 - \cancel{D_y}^{-C_y} = 0$

$B_y = 250 - C_y = 416.66$

$\sum F_x = 0 : B_x - \cancel{D_x}^{-C_x} = 0$

$\Rightarrow B_x + C_x = 0 \Rightarrow B_x = -277.77$ N.

Straightforward way → consider FBD's of upper & lower members (6 eqns, 6 unknowns).

Upper member:

$\sum F_x = 0 \Rightarrow C_x + D_x = 0 \longrightarrow (1)$

$\sum F_y = 0 \Rightarrow C_y + D_y = 0 \longrightarrow (2)$

$\sum M_c = 0 \Rightarrow D_x(60) + D_y(100) = 0 \longrightarrow (3)$

Lower member:

$\sum F_x = 0 \Rightarrow B_x - D_x = 0 \longrightarrow (4)$

$\sum F_y = 0 \Rightarrow B_y - 250 - D_y = 0 \longrightarrow (5)$

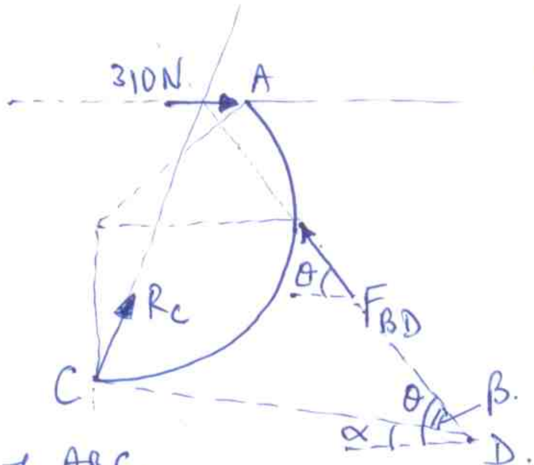
$\sum M_B = 0 \Rightarrow 250(60) - D_x(30) - D_y(140) = 0 \longrightarrow (6)$

Above soln solves (1) – (6)

Advice: Do by straightforward method, since chance of errors in signs of R_c, C_x, C_y, D_x, D_y.

R-21. 1st method (shorter).

BD is 2-Force member, ABC is 3-Force member.



$$\theta = \tan^{-1}\left(\frac{1.92}{0.56}\right) = 73.7397^\circ$$

$$CD = \sqrt{(1.4+0.56)^2 + (1.92-1.4)^2} = 2.0278 \text{ m}$$

$$\alpha = \tan^{-1}\left(\frac{0.52}{1.96}\right) = 14.8586^\circ$$

$$\beta = \theta - \alpha = 58.8818$$

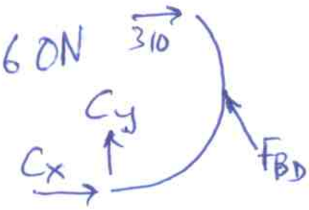
FBD of ABC

$$\sum M_c = 0 \Rightarrow F_{BD}(CD \sin\beta) - 310(1.4 + 1.4\sin 30) = 0$$

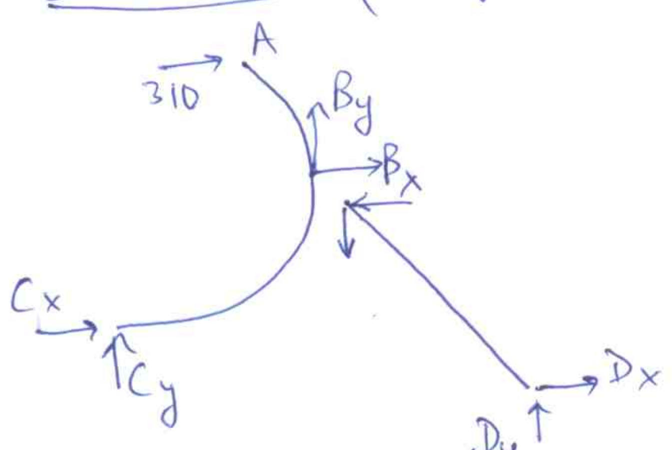
$$\Rightarrow F_{BD} = 375 \text{ N}$$

$$\sum F_x = 0 \Rightarrow C_x + 310 - F_{BD}\cos\theta = 0 \Rightarrow C_x = -205\text{N}$$

$$\sum F_y = 0 \Rightarrow C_y + F_{BD}\sin\theta = 0 \Rightarrow C_y = -360\text{N}$$



2nd method (straightforward, longer).



3 Force member.

$$\sum M_c = 0 \Rightarrow 310(1.4 + 1.4\sin 30) + B_x(1.4) - B_y(1.4) = 0 \rightarrow (1)$$

$$\sum F_x = 0 \Rightarrow C_x + B_x + 310 = 0 \rightarrow (2)$$

$$\sum F_y = 0 \Rightarrow C_y + B_y = 0 \rightarrow (3)$$

2 Force member.

$$\sum M_D = 0 \Rightarrow B_x(1.92) + B_y(0.56) = 0 \rightarrow (4)$$

$$\sum F_x = 0 \Rightarrow B_x + D_x = 0 \rightarrow (5)$$

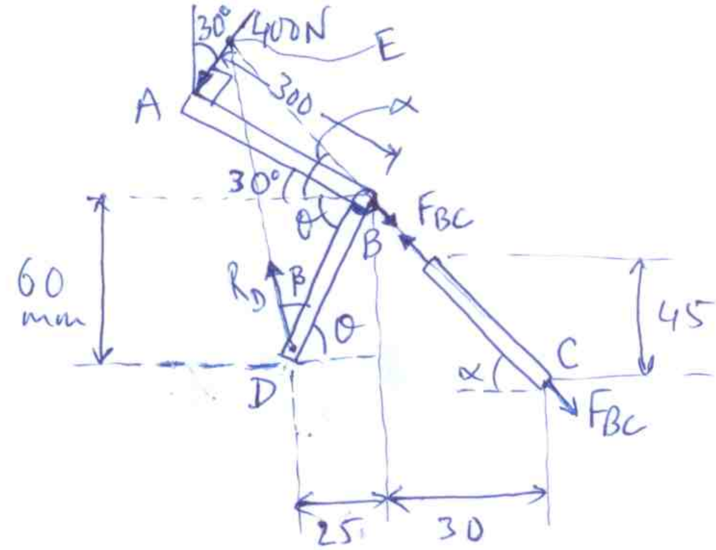
$$\sum F_y = 0 \Rightarrow B_y + D_y = 0 \rightarrow (6)$$

From 1st method

$$\left.\begin{aligned} \text{Now } B_x &= -F_{BD}\cos\theta = -105 \\ B_y &= F_{BD}\sin\theta = 360 \end{aligned}\right\} \rightarrow (a)$$

Now above soln + (a), solves (1) – (6).

<u>P.22</u> ABD is a 3-Force member, BC is 2-Force member.



$DB = \sqrt{60^2+25^2} = 65$

$\theta = \tan^{-1}\left(\frac{60}{25}\right) = 67.38°$

$\alpha = \tan^{-1}\left(\frac{45}{30}\right) = 56.31°$

$BE = \frac{\overset{\nearrow 300}{BA}}{\cos(\alpha-30)} = 334.67$

$DE = \sqrt{DB^2 + BE^2 - 2(DB)(BE)\cos(\alpha+\theta)}$
$= 374.65$

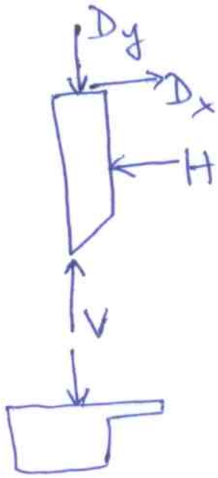
$$\beta = \sin^{-1}\left\{(BE)\frac{(\sin[\alpha+\theta])}{DE}\right\} = 48.01°$$

$$\sum M_B = 0 \Rightarrow 400\,(300) - R_D\,(BD\sin\beta) = 0$$

$$\Rightarrow R_D = 2483.86\ N$$

$$D_y = R_D\sin(\pi-\beta-\theta) = 2243.94\ N\ (\uparrow)$$

$$D_x = R_D\cos(\pi-\beta-\theta) = 1065.02\ N\ (\leftarrow).$$

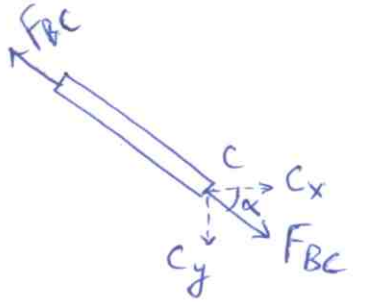


$\Rightarrow V = D_y$ (from FBD of cutting block)

$= 2243.94\ N$

(as shown in Fig)

$C_x = F_{BC}\cos\alpha$, $C_y = F_{BC}\sin\alpha$



For body ABD,

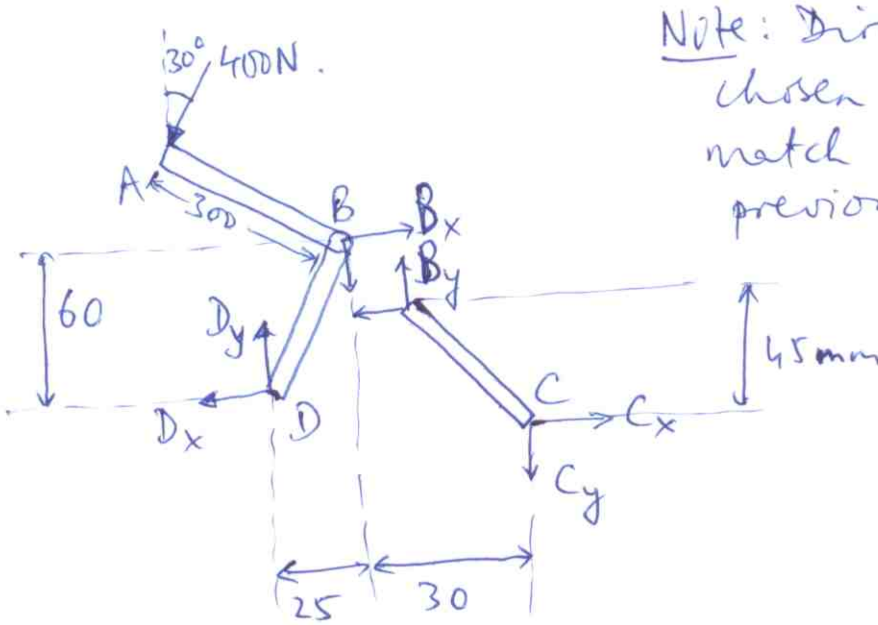
$$\sum F_x = 0 \Rightarrow -400\sin 30 - \overbrace{R_D\cos(\pi-\beta-\theta)}^{=D_x} + F_{BC}\cos\alpha = 0$$

$$\Rightarrow F_{BC}\cos\alpha = C_x = 1265.02\ (\rightarrow)$$

$$\sum F_y = 0 \Rightarrow -400\cos 30 + \underbrace{R_D\sin(\pi-\beta-\theta)}_{=D_y} - F_{BC}\sin\alpha = 0$$

$$\Rightarrow F_{BC}\sin\alpha = C_y = 1897.53(\downarrow)$$

By Straightforward approach.



<u>Note</u>: Directions of reactions chosen for convenience, ie to match signs with results of previous method.

<u>Upper FBD.</u>

$\Sigma M_B = 0: \quad 400(300) - D_x(60) - D_y(25) = 0 \longrightarrow (1)$

$\Sigma F_x = 0: \quad -400 \sin 30 + B_x - D_x = 0 \longrightarrow (2)$

$\Sigma F_y = 0: \quad -400 \cos 30 - B_y + D_y = 0 \longrightarrow (3)$

<u>Lower FBD</u>

$\Sigma F_x = 0 \Rightarrow B_y = C_y \longrightarrow (4)$

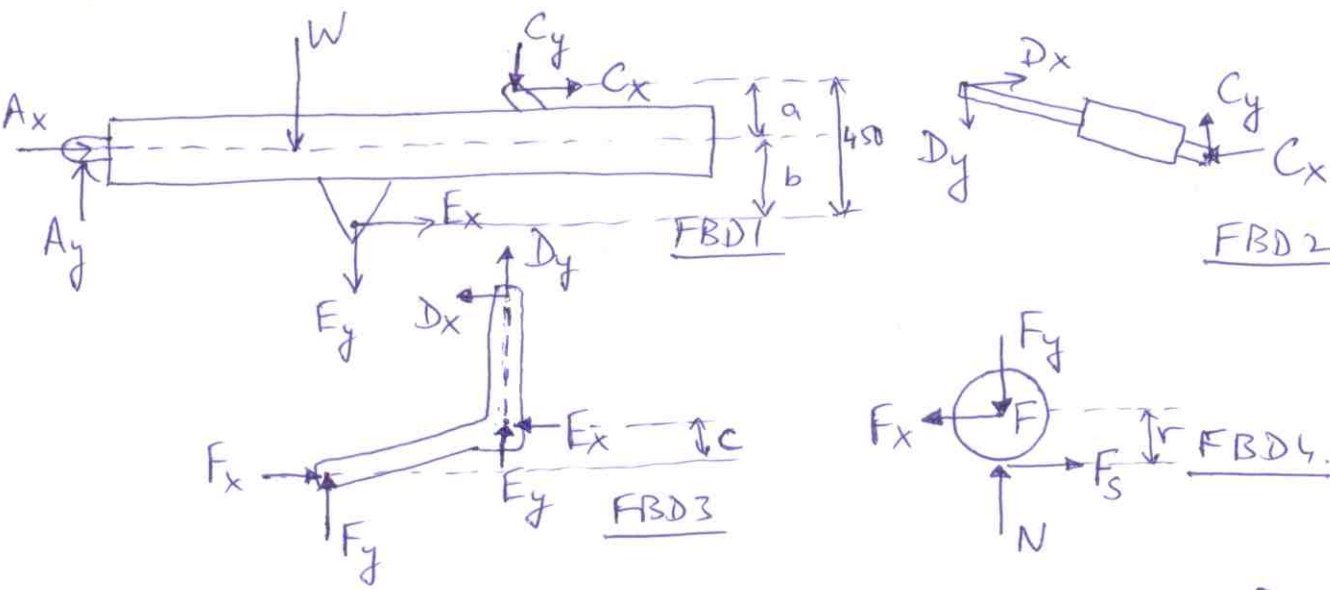
$\Sigma F_y = 0 \Rightarrow B_x = C_x \longrightarrow (5)$

$\Sigma M_B = 0 \Rightarrow C_y(30) - C_x(45) = 0 \longrightarrow (6)$

Now D_x, D_y, C_x, C_y from 1st method satisfy (1), (2), (3), (6), and the concept of 2-Force member for member BC is same as (4), (5).

<u>Note</u>: Trignometry involved in 1st method makes it less attractive despite the fact that (1)–(6) need to be solved simultaneously.

P.23 There are 4-bodies, ie, tow bar AGB, hydraulic cylinder CD, arm DEF, and wheel F. The arm DEF is pinned to cylinder at D, and tow bar at E, and wheel at F. The cylinder is also pinned to tow bar at C. When cylinder translates, arm DEF rotates about E. Then, since gravity would force wheel to touch the ground, the nett effect (of rotation of DEF due to cylinder translation) is that tow bar gets inclined, ie it rotates about A. FBD's are,
Note that end A is tow truck & end B is aircraft attachment.



Accounts: 12 unknowns ($A_x, A_y, C_x, C_y, D_x, D_y, E_x, E_y, F_x, F_y, N, F_s$)

12 eqns ($\because$ 4 R.B's and planar statics, ie, 4×3 eqns).

So system is Statically Determinate.

Equilibrium equations:

FBD 1 : $A_x + C_x + E_x = 0 \longrightarrow ①$

$A_y - W - C_y - E_y = 0 \longrightarrow ②$

(ΣM_A): $W(1150) + C_y(2025) + E_y(1350) + C_x(a) - E_x(b) = 0$. ③

FBD2 $D_x = C_x \rightarrow$ (4)

$D_y = C_y \rightarrow$ (5)

(ΣM_D): $C_x(100) - C_y(675) = 0 \rightarrow$ (6)

FBD3. $F_x - E_x - D_x = 0 \rightarrow$ (7)

$F_y + E_y + D_y = 0 \rightarrow$ (8)

(ΣM_E): $F_x(c) - F_y(500) + D_x(550) = 0 \rightarrow$ (9)

FBD4: $F_x = F_s \rightarrow$ (10)

$N = F_y \rightarrow$ (11)

(ΣM_F): $F_s(r) = 0 \rightarrow$ (12)

(10,12) $\rightarrow F_x = 0 \rightarrow$ (i)

(i),(4),(7) $\rightarrow C_x = -E_x \rightarrow$ (ii)

(i),(4),(9) $\rightarrow F_y = 1.1\, C_x \rightarrow$ (iii)

(iii),(8),(5) $\rightarrow E_y = -C_y - 1.1\, C_x \rightarrow$ (iv)

(iv),(ii),(3) $\rightarrow W(1150) + C_y(2025-1350) + C_x(-1.1*1350+450) = 0$ $\rightarrow$ (v)

(6),(v) $\rightarrow C_y = \dfrac{-W*1150}{675 - 6.75*1035} = 357.5$

$C_x = 2413.16$

$F_c = \sqrt{C_x^2 + C_y^2} = 2439.5$

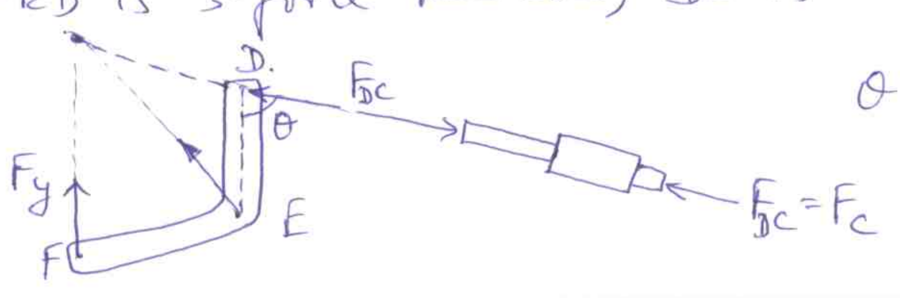
$E_x = -2413.6 (\rightarrow)$, $E_y = -3011.98 (\downarrow)$, $F_E = \sqrt{E_x^2 + E_y^2} = 3859.4$ N.

Force on each arm by pin at E is $\frac{F_E}{2} = 1929.72$ N.

(∵ pin E connected to two identical arms FED).

Shorter way.

FED is 3-force member, DC is 2-Force member.

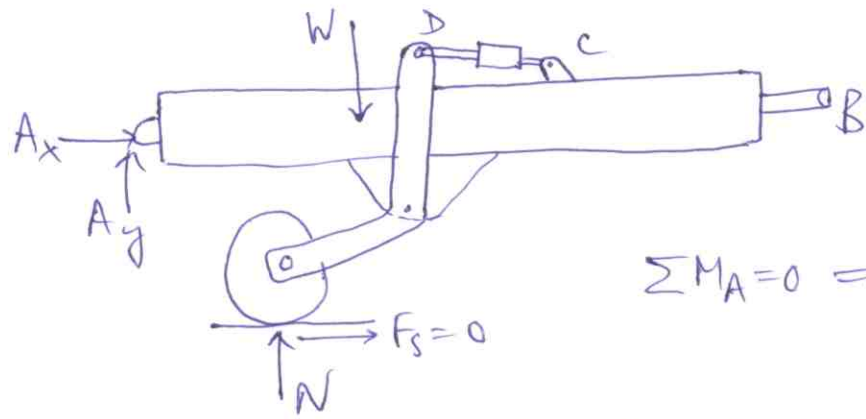


$\theta = \tan^{-1}\left(\frac{675}{100}\right) = 81.57°$

$$\Sigma M_E = 0 \Rightarrow F_y (500) - F_{DC}(DE \sin\theta) = 0 \longrightarrow (a)$$

Note: Equilibrium of wheel, implying $F_S = F_x = 0$ has been used/implied, as well as $N = F_y$.

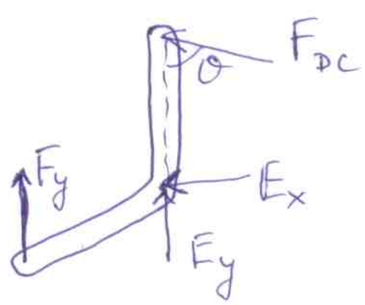
Consider FBD of entire system, ie isolated at A and from ground only



$$\Sigma M_A = 0 \Rightarrow W(1150) - \overset{F_y}{\cancel{N}}(850) = 0 \longrightarrow (b)$$

$$(a,b) \rightarrow F_{DC} = F_C = \underset{200g}{\cancel{W}}\left(\frac{1150}{850}\right)(500)\frac{1}{530 \sin\theta} = 2439.49\ \text{N}$$

$$F_y = 2654.47\ \text{N.}$$

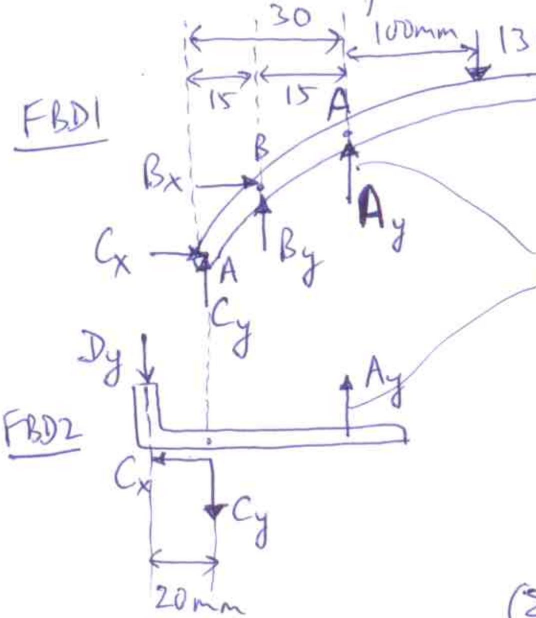


$$\Sigma F_x = 0 \Rightarrow E_x = -F_{DC}\sin\theta = -2413.2 (\rightarrow)$$

$$\Sigma F_y = 0 \Rightarrow E_y = -F_y - F_{DC}\cos\theta = -3011.97 (\downarrow)$$

So horizontal and vertical reactions at E on arm FED are half of E_x, E_y ($\because$ two identical arms FED)

P.24 Use symmetry.



6 unknowns $(A_y, B_x, B_y, C_x, C_y, D_y)$ and 2 RB's ie 6 eqns $\Rightarrow$ S.D.

directions must be consistent, based on symmetry.

FBD2

$C_x = 0 \longrightarrow (1)$

$A_y = D_y + C_y \longrightarrow (2)$

$(\Sigma M_D) : C_y(20) = A_y(50) \longrightarrow (3)$

FBD1

$A_y + B_y + C_y = 130 \longrightarrow (4)$

$B_x + C_x = 0 \longrightarrow (5)$

$(\Sigma M_B) : C_y(15) + 130(115) - A_y(15) = 0 \longrightarrow (6)$

($C_x = 0$ used here).

(2),(3) $\rightarrow D_y = A_y - C_y = -1.5 A_y$

(3),(6) $\rightarrow 1.5 * 15 * A_y = -130(115) \Rightarrow A_y = -664.44$

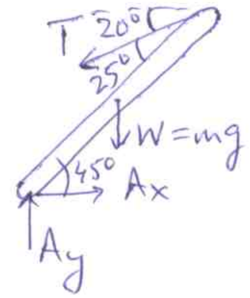
$\Rightarrow D_y = 1.5 * 664.44 = 996.67\,N.$

(Answer matches Shames P. 5.144).

Note: The horizontal distances from A to B and B to C are taken equal, since in Shames the portion ABC is shown as a straight portion (hence from symmetry, these distances are equal only if ABC is straight, otherwise they need to be explicitly given). This is same as Tute #2 problem, where these horizontal distances are explicitly given.

Note: Pliers are force multiplies, input = 130N, output = 996.67 N

P.25.



$$A_x - T\cos 20 = 0$$

$$A_y - W - T\sin 20 = 0$$

$$W(2\cancel{\cos 45}) + T\sin 20\,(4\cancel{\cos 45}) - T\cos 20\,(4\cancel{\sin 45}) = 0$$

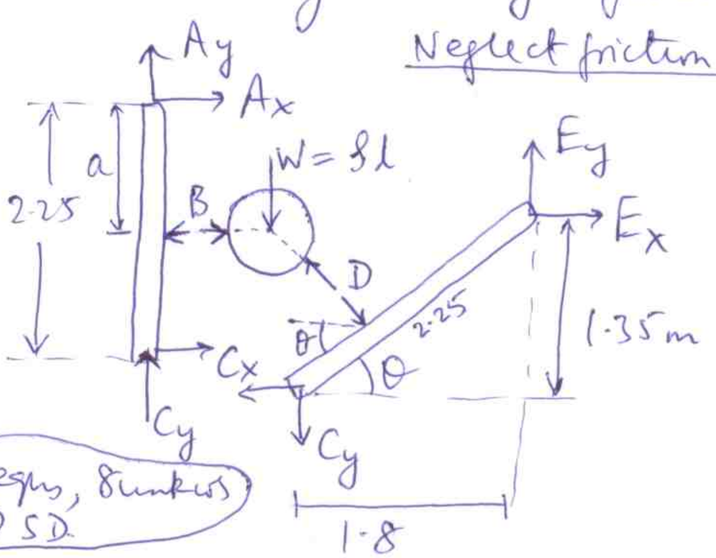
$$\Rightarrow T = \frac{2mg}{4(\cos 20 - \sin 20)} = 82.07\text{ N}$$

$$A_x = 77.12\text{ N},\quad A_y = 126.17\text{ N}.$$

Note: can also do as a 3-Force member problem.

P.26. Although ABC and CDE are (both) 3-force members, we cannot use this concept since direction of forces at A, C, E are not known. So do by straightforward approach.

Neglect friction at B, D.



8 eqns, 8 unknowns ⇒ SD.

$W = 4500 \times 4.8$

$\theta = \tan^{-1}(1.35/1.8) = 36.87°$

FBD of pipe:

$B - D\sin\theta = 0 \rightarrow ①$

$D\cos\theta - W = 0 \rightarrow ②$

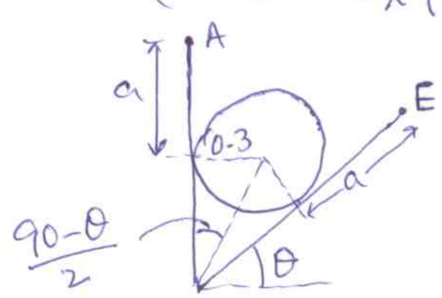
$\Rightarrow D = 27000\text{ N}$

$B = 16200\text{ N}$

FBD of AC: $A_y + C_y = 0 \rightarrow ③$

$A_x + C_x - B = 0 \rightarrow ④$

(ΣM_A): $C_x(2.25) - B(a) = 0 \rightarrow ⑤$



$$a = 2.25 - 0.3\cot\left(45 - \frac{\theta}{2}\right) = 1.65$$

$$⑤ \Rightarrow C_x = 11880\text{ N}$$

FBD of CE : $-C_x + E_x + D\sin\theta = 0 \rightarrow (6)$

$-C_y - D\cos\theta + E_y = 0 \rightarrow (7)$

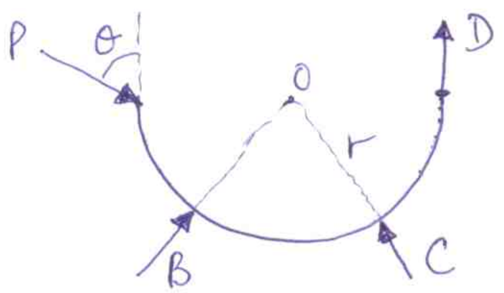
$(\Sigma M_c): D(0.6) + E_x(1.35) - E_y(1.8) = 0 \rightarrow (8)$.

$(6) \rightarrow E_x = -4320\ (\leftarrow)$.

$(8) \rightarrow E_y = 5760\ (\uparrow)$

$(7) \rightarrow C_y = -15840$ (direction opp to what is shown in FBD)

P.27 Frictionless cylinders $\Rightarrow$ only normal reactions at B and C.
Loss of equilibrium when either reactions at B or C are zero (ie they cant be negative).



$\Sigma M_O = 0 \Rightarrow Dr + (P\cos\theta)r = 0 \rightarrow (1)$

$\Rightarrow D = -P\cos\theta$

$\Sigma F_x = 0 \Rightarrow P\sin\theta + (B-C)\cos 60 = 0 \rightarrow (2)$

$\Sigma F_y = 0 \Rightarrow -P\cos\theta + (B+C)\sin 60 + D = 0 \rightarrow (3)$

$$\Rightarrow B = -\frac{P(\sin\theta\sin 60 - 2\cos\theta\cos 60)}{\sin 120^\circ}$$

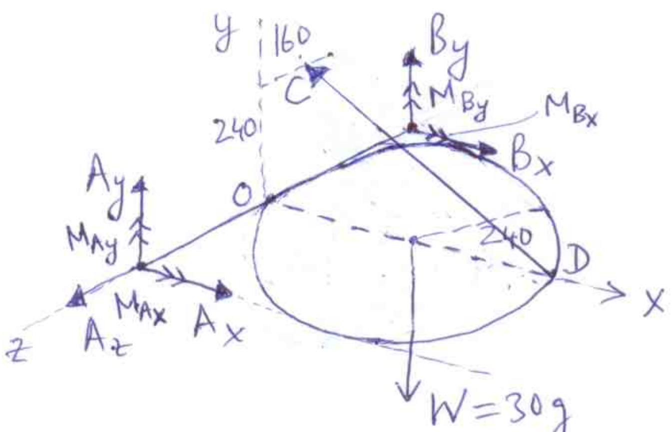
$$C = -\frac{P(-2\cos\theta\cos 60 - \sin\theta\sin 60)}{\sin 120^\circ}$$

$B < 0$ if $\tan\theta > 2\cot 60^\circ$

$C < 0$ if $\tan\theta < -2\cot 60^\circ \rightarrow$ not possible for θ acute.

$\Rightarrow \boxed{\theta > 49.1066^\circ}$ will cause loss of equilibrium

P.28. The pipe cover is connected to shaft via hinges. Since cable is eccentric by 160mm, tension has component shaft axis AB. Then, since bearing at B doesent provide axial thrust, the one at A should, otherwise shaft will slip out of bearings due to eccentricity of attachment point C. This axial thrust can be provided by end stops, on either side of bearing at A, fitting into radial grooves on the shaft.



$$\underline{T} = T\frac{(-480\underline{i} + 240\underline{j} - 160\underline{k})}{560}$$

$$\underline{T} = T\frac{(-6\underline{i} + 3\underline{j} - 2\underline{k})}{7}$$

Unknowns = ($A_x, A_y, A_z, B_x, B_y, T$,
10 = ($M_{Ax}, M_{Ay}, M_{Bx}, M_{By}$)

Eqns = 6 (3-D RB ⇒ 6 eqns).

So SID

$$\Sigma F_x: \quad A_x + B_x - \frac{6}{7}T = 0 \longrightarrow ①$$

$$\Sigma F_y: \quad A_y + B_y - W + \frac{3T}{7} = 0 \longrightarrow ②$$

$$\Sigma F_z: \quad A_z - \frac{2}{7}T = 0 \longrightarrow ③$$

$$\Sigma \underline{M}_0 = 0 = 240(B_y - A_y)\underline{i} + 240(A_x - B_x)\underline{j} - 240W\underline{k} + 480\,\underline{i}\times\underline{T} + (M_{Ax} + M_{Bx})\underline{i} + (M_{Ay} + M_{By})\underline{j}$$

$$\Rightarrow 240(B_y - A_y)\underline{i} + \left\{240(A_x - B_x) + 480\left(\tfrac{2}{7}\right)T\right\}\underline{j} + \left\{-240W + 480\left(\tfrac{3}{7}\right)T\right\}\underline{k} + (M_{Ax} + M_{Bx})\underline{i} + (M_{Ay} + M_{By})\underline{j} = 0$$

$$\Rightarrow 240(B_y - A_y) + (M_{Ax} + M_{Bx}) = 0 \longrightarrow ④$$

$$240(A_x - B_x) + \frac{960}{7}T + (M_{Ay} + M_{By}) = 0 \longrightarrow ⑤$$

$$-240W + \frac{1440}{7}T = 0 \longrightarrow ⑥$$

⑥ → $T = 343.35\text{ N}$ ◄ Can solve for A_z from ③ Cannot solve for other reactions.

One-step direct method.

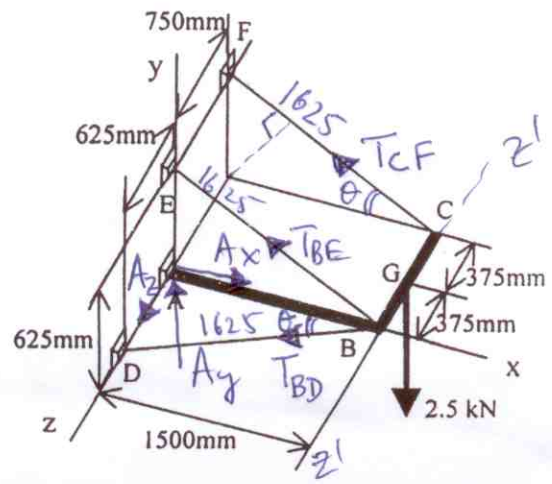
$\because$ we want only T, note that moment of all forces about AB axis involves on W and T, $\because$ reactions at A and B lie on axis AB. Now moment about AB axis is $\underline{k}$ comp of moment about any pt lying on AB axis, say pt. O.

$$\Rightarrow M^R_{AB} = 0 = \underline{k}.\left(-240W\underline{k} + 480\left(\tfrac{2}{7}\right)\underline{j} + 480\left(\tfrac{3}{7}\right)\underline{k}\right)$$

$$= -240W + 480\left(\tfrac{3}{7}\right)T = \underline{k} \text{ comp of } \underline{M}^R_O = 0$$

$$\Rightarrow -240W + 480\left(\tfrac{3}{7}\right)T = 0 \longrightarrow \text{same as (6)}.$$

P. 29.



FBD (as shown on fig).

$$\theta = \tan^{-1}\left(\frac{625}{1500}\right) = 22.6199°$$

$$\cos\theta = 1500/1625$$

$$\sin\theta = 625/1625$$

6 unknowns ($A_x, A_y, A_z, T_{CF}, T_{BE}, T_{BD}$) $\Rightarrow$ SD

Short way. (Consider moments about x, y, z axes)

$M_{XX} = 0$: only $\underline{T}_{CF}$ involved (apart from load).

$$\Rightarrow \left(T_{CF}\left(\frac{625}{1625}\right)(750)\right) - 2.5(375) = 0 \xrightarrow{(1)} T_{CF} = 3.25$$

(same as $\underline{i}.\left[-750\,\underline{k} \times T_{CF}\left(-\frac{1500}{1625}\underline{i} + \frac{625}{1625}\underline{j}\right)\right]$ vector approach)

$M_{ZZ} = 0$: only $\underline{T}_{BE}$ (and $\underline{T}_{CF}$) involved besides load.

$$\Rightarrow \left((T_{BE} + T_{CF})\,1500\sin\theta\right) - 2.5\,(1500) = 0 \xrightarrow{(2)} T_{BE} = 3.25$$

(same as vector approach, ie, $\underline{k}.\left[1500\,\underline{i} \times \{T_{BE} + T_{CF}\}\left(-\frac{1500}{1625}\underline{i} + \frac{625}{1625}\underline{j}\right)\right]$)

$M_{yy} = 0$: only T_{BD} (and T_{CF}) involved (not even load).

$\Rightarrow T_{BD}(1500 \sin\theta) - (T_{CF}\cos\theta)(750) = 0 \xrightarrow{3} \Rightarrow T_{BD} = 3.9$ ◀

$\sum F_x : A_x - (T_{BD} + T_{BE} + T_{CF})\cos\theta = 0 \xrightarrow{4} \Rightarrow A_x = 9.6$ ◀

$\sum F_y : A_y - 2.5 + (T_{BE} + T_{CF})\sin\theta = 0 \xrightarrow{5} \Rightarrow A_y = 0$ ◀

$\sum F_z : A_z + T_{BD}\sin\theta = 0 \xrightarrow{6} \Rightarrow A_z = -1.5$ ◀

can get it by observation when taking $M_{z'z'} = 0$ for which no other force or load participate.

<u>Long way:</u> (straightfwd approach).

$$\sum \underline{M}_A^R = 0 = 1500\,\underline{i} \times \left(T_{BE}[-\cos\theta\,\underline{i} + \sin\theta\,\underline{j}] + T_{BD}[-\cos\theta\,\underline{i} + \sin\theta\,\underline{k}]\right)$$
$$+ (1500\,\underline{i} - 375\,\underline{k}) \times (-W\,\underline{j})$$
$$+ (1500\,\underline{i} - 750\,\underline{k}) \times \left(T_{CF}[-\cos\theta\,\underline{i} + \sin\theta\,\underline{j}]\right)$$

$\underline{i}$: $-375W + T_{CF}(750\sin\theta) = 0$ → same as 1

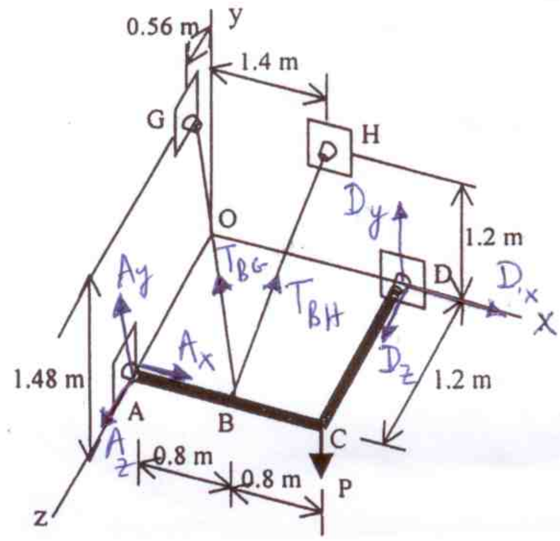
$\underline{j}$: $-T_{BD}(1500\sin\theta) + T_{CF}(750\cos\theta) = 0$ → same as 3

$\underline{k}$: $T_{BE}(1500\sin\theta) - 1500W + T_{CF}(1500\sin\theta) = 0$ → same as 2

$\sum \underline{F} = 0$ gives back 4, 5, 6.

<u>Moral:</u> The "straight fwd" approach is not really longer. Use it when all reactions (unknowns) are required. Use "short way" when only certain reactions (eg. tensions) are required.

P.30



Note: Due to single cable around hook at B, $T_{BG} = T_{BH} = T$

Unknowns $= A_x, A_y, A_z, B_x, B_y, B_z, T_{BG}, T_{BH}$ $= 7$

Equations $= 6$ (ie one 3-D, RB)

$\Rightarrow$ SID.

However, T_{BG} and T_{BH} can be found by statics.

Shorter (elegant) approach:

Moment about any axis $= 0$ for equilibrium.

$M_{AD} = 0$: only T_{BG}, T_{BH}, besides load, participate. $\because$ reactions at A, B won't participate in dot product

$$\Rightarrow \underline{e}_{AD} \cdot \underline{M}_A^R = 0$$ due to T_{BG}, T_{BH}, P only

$$\Rightarrow \underbrace{\frac{(1.6\underline{i} - 1.2\underline{k})}{2}}_{0} \cdot \left[0.8\underline{i} \times \left\{ T_{BG} \frac{\left[-0.8\underline{i} + 1.48\underline{j} - (1.2-0.56)\underline{k}\right]}{1.8} + T_{BH} \frac{\left((1.4-0.8)\underline{i} + 1.2\underline{j} - 1.2\underline{k}\right)}{1.8} \right\} + 1.6\underline{i} \times (-335\underline{j}) \right]$$

$$\Rightarrow (0.8\underline{i} - 0.6\underline{k}) \cdot \left[\left(T_{BG} \left\{ \frac{0.8 * 1.48}{1.8} \right\} + T_{BH} \left\{ \frac{0.8 * 1.2}{1.8} \right\} - 1.6 * 335 \right)\underline{k} + (??)\underline{j} \right] = 0$$

$$\Rightarrow \left[\left(-\frac{1.184}{3} \right) - 0.32 \right] T + 321.6 = 0 \longrightarrow (1)$$

$\Rightarrow T = 450\text{kN}$

$M_{AC} = 0$: only reaction at D participates

$$\Rightarrow D_y(1.2) = 0 \Rightarrow D_y = 0 \longrightarrow (2)$$

$M_{BD} = 0$: only A_y and P participate.

$$\Rightarrow \left(\frac{0.8\underline{i} - 1.2\underline{k}}{\sqrt{2.08}} \right) \cdot \left[-0.8\underline{i} \times A_y\underline{j} + 0.8\underline{i} \times (-335\underline{j}) \right] = 0$$

$$\Rightarrow A_y = -335\ \text{kN}\ (\downarrow) \longrightarrow (3)$$

$$\Sigma F_y = 0 \Rightarrow \underset{T}{\cancel{T_{BG}}}\left(\frac{1.48}{1.8}\right) + \underset{T}{\cancel{T_{BH}}}\left(\frac{1.2}{1.8}\right) + \underset{(-335)}{\cancel{A_y}} - \underset{335}{\cancel{P}} + \underset{0}{\cancel{D_y}} = 0$$

$$\Rightarrow T\left(\frac{1.48+1.2}{1.8}\right) - 670 = 0$$ → this is proportional to ① i.e. not an independent equation.

Summary: either $\underline{M_{AD} = 0}$, or $\underline{M_{AC} = 0 \;\&\; M_{BD} = 0 \;\&\; \Sigma F_y = 0}$

gives the result.

Longer (straight fwd) way:

Do $\Sigma \underset{\sim}{M}_A = 0$, $\Sigma \underset{\sim}{F} = 0$, get 6 scalar equations in 7 unknowns. Do algebraic combination to get T. You will be able to get D_y, A_y also. Other 4 reactions (A_x, A_z, D_x, D_z) cannot be solved by statics alone ∵ it is SID problem.

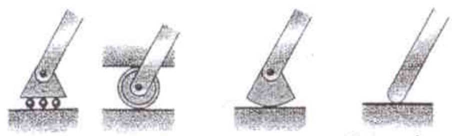
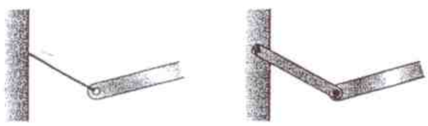
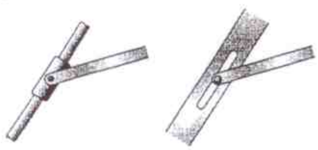
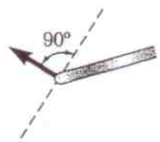
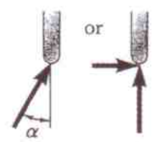
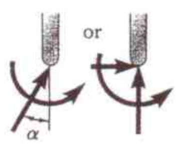
Support or Connection	Reaction	Number of Unknowns
 Rollers, Rocker, Frictionless surface	 Force with known line of action	1
 Short cable, Short link	 Force with known line of action	1
 Collar on frictionless rod, Frictionless pin in slot	 90° Force with known line of action	1
 Frictionless pin or hinge, Rough surface	 or α Force of unknown direction	2
 Fixed support	 or α Force and couple	3

Fig. 4.1 Reactions at supports and connections.

When the sense of an unknown force or couple is not readily apparent, no attempt should be made to determine it. Instead, the sense of the force or couple should be arbitrarily assumed; the sign of the answer obtained will indicate whether the assumption is correct or not.

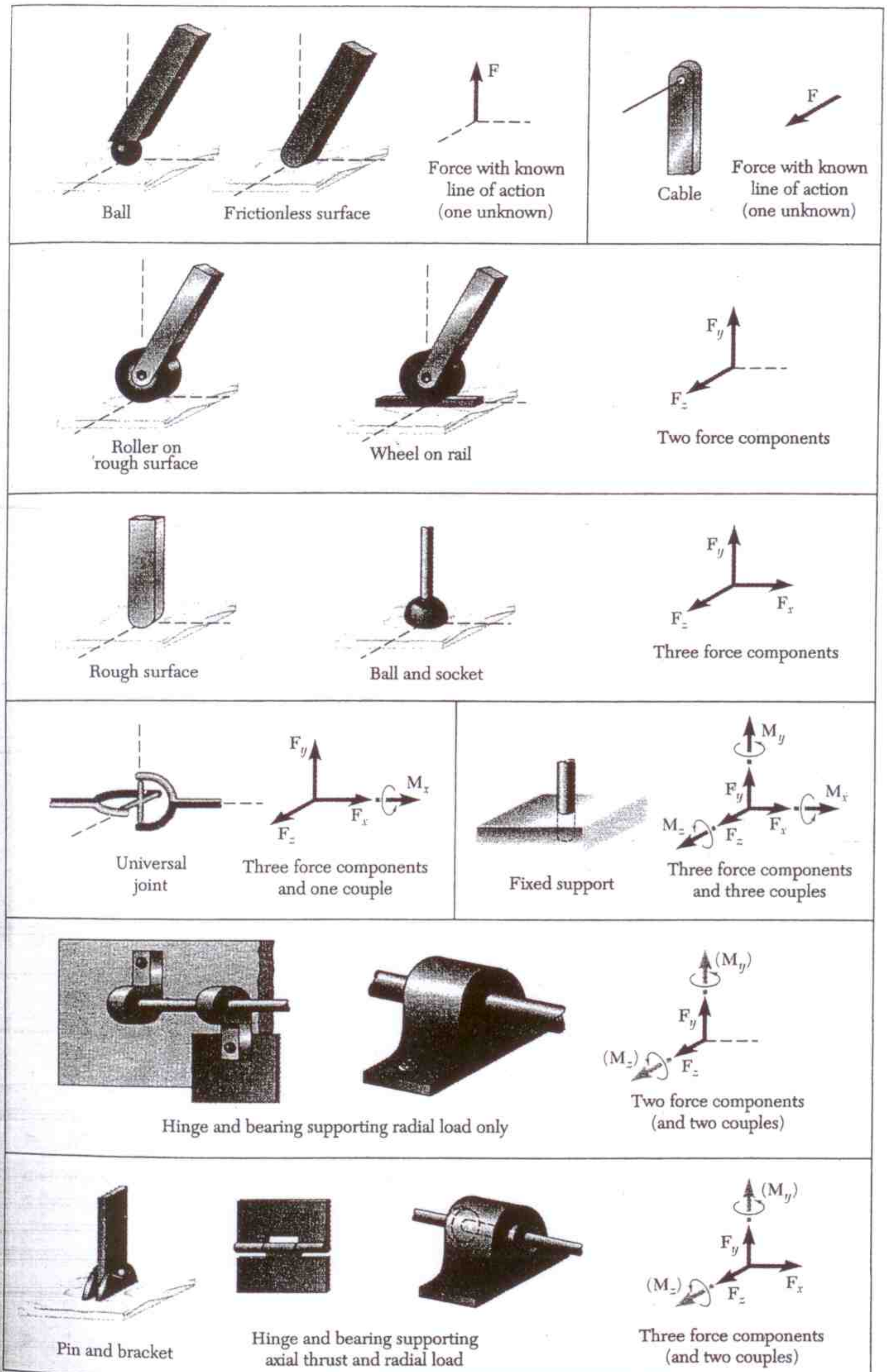


Fig. 4.10 Reactions at supports and connections.