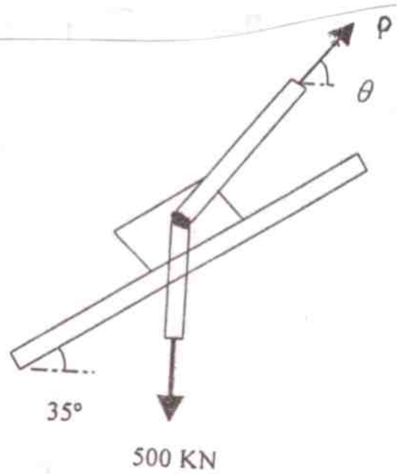
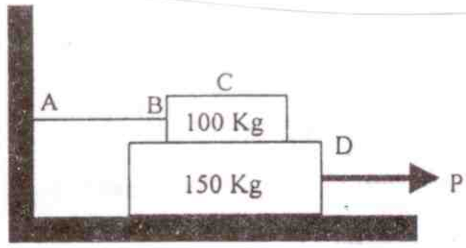
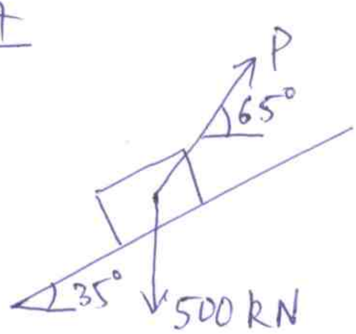
P.37. The coefficient of friction between the block and the rail are are $\mu_s = 0.30$ and $\mu_k = 0.25$. Knowing that $\theta = 65°$, determine the smallest value of P required (a) to start the block up the rail, (b) to keep it from moving down.



P.38. The coefficients of friction are $\mu_s = 0.30$ and $\mu_k = 0.25$ between all surfaces of contact. Determine the smallest force P required to start block D moving if (a) block C is restrained by cable AB as shown, (b) cable AB is removed.



P.37

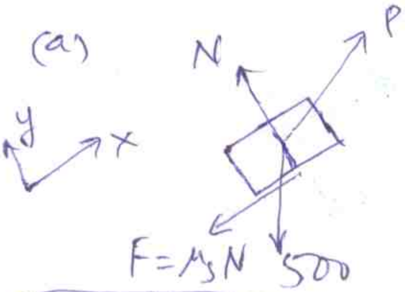


Given: $\mu_s = 0.3$, $\mu_k = 0.25$

Find: (a) smallest value of P to start moving up.

(b) smallest value of P to keep it from moving down.

(a)



Unknowns: P, N
eqns: no moment eqn

ΣF_x: $P\cos 30 - 500\sin 35 - \overset{\mu_s}{0.3}N = 0$

ΣF_y: $N - 500\cos 35 + P\sin 30 = 0$

$$P = \frac{500(\sin 35 + 0.3\cos 35)}{\cos 30 + 0.3\sin 30} = 403.2\text{ kN}.$$

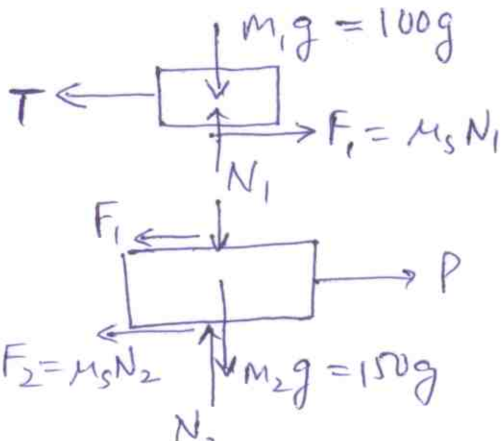
(b) Impending downward slip, so friction reverses direction, i.e, $F = -\mu_s N$ in above. (or $\mu_s = -0.3$).

$$\Rightarrow P = \frac{500(\sin 35 - 0.3\cos 35)}{\cos 30 - 0.3\sin 30} = 228.9\text{ kN}.$$

If $P<0 \Rightarrow$ without P (ie $P=0$) the downward slide is not impending. Alternatively you can first check that for $P=0$ downward slide will occur $\because$ $N = 500\cos 35$ and

$(500\sin 35 - \mu_s N) > 0$.

P.38. (a)



$\mu_s = 0.3$

$\mu_k = 0.25$

No mont eqn
Unkns: T, N_1, N_2, P

$P = F_1 + F_2 = \mu_s(N_1 + N_2)$

$N_1 = m_1 g$

$N_2 = N_1 + m_2 g = (m_1 + m_2)g$

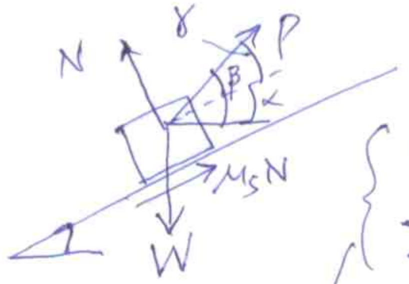
$P = \mu_s(2m_1 + m_2)g$.

$= 1030.05\text{ N}$

(b) $T = 0 \Rightarrow F_1 = 0 \Rightarrow P = F_2 = \mu_s N_2 = \mu_s(m_1+m_2)g$

$\Rightarrow P = 735.75\text{ N}$.

P.37 Explanations for <u>part (b)</u>.



$\gamma = \beta - \alpha$

$$\begin{cases} \Sigma F_x : P\cos\gamma - W\sin\alpha + \mu_s N = 0. \\ \Sigma F_y : N - W\cos\alpha + P\sin\gamma = 0. \end{cases}$$

$$\Rightarrow P = \frac{W(\sin\alpha - \mu_s\cos\alpha)}{(\cos\gamma - \mu_s\sin\gamma)}$$ = load at which downward slip impends.

(i) If $P > 0$ then this value of P is the answer.

(ii) If $P < 0$: When does this happen?

When $\tan\alpha > \mu_s$ & $\tan\gamma > \mu_s^{-1}$ $\rightarrow$ case (a)

or $\tan\alpha < \mu_s$ & $\tan\gamma < \mu_s^{-1}$ $\rightarrow$ case (b).

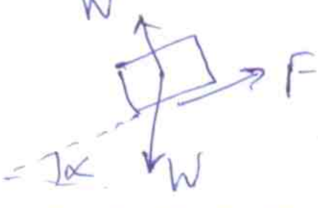
Now $$\tan\beta = \frac{\tan\alpha + \tan\gamma}{1 - \tan\alpha\tan\gamma}$$

For case (a), $(\tan\alpha\tan\gamma) > 1 \Rightarrow \beta > \frac{\pi}{2}$.

for case (b), $(\tan\alpha\tan\gamma) < 1 \Rightarrow \beta < \frac{\pi}{2}$

Assume $\beta \ngtr \frac{\pi}{2}$ is given $\rightarrow$ so ~~our~~ case of interest is case (b).

For case (b), $\underbrace{\tan\alpha < \mu_s \Rightarrow \text{for } P = 0 \text{ it is in equil.}}_{\text{This is well known result. (JEE !!)}}$



$\Longrightarrow$ shows block in equil.

ie $W\sin\alpha = F$, $N = W\cos\alpha$

ie $\tan\alpha = \frac{F}{N}$

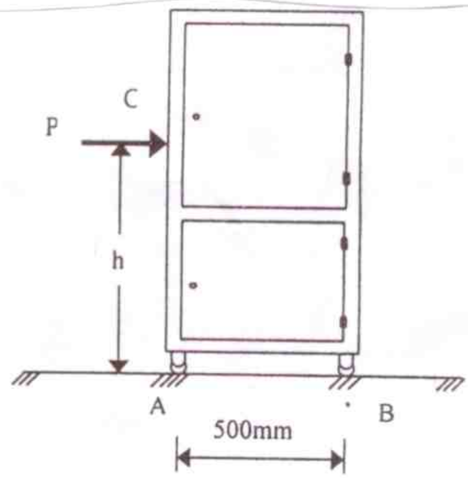
ie $\tan\alpha \leq \mu_s$.

eg of case (b) is
$\alpha = 15^\circ$, $\gamma = 45$, $\beta = 60^\circ$,
<u>gives</u> P downward.

<u>Note</u>: Without above discussion also we can infer this $\because$ in Part (a) P is upward & in part (b) if P is downward it means that for P between these values there is no slip.

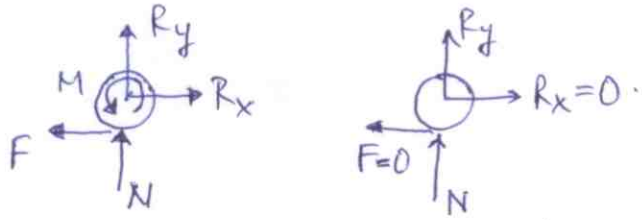
P.39.

P.39. A 60 kg cabinet is mounted on casters, which can be locked to prevent their rotation. The coefficient of static friction is 0.35. If h = 600 mm, determine the magnitude of force P required to move the cabinet to the right (a) if all casters are locked, (b) if the casters at B are locked and the casters at A are free to rotate, (c) if the casters at A are locked and the casters at B are free to rotate.

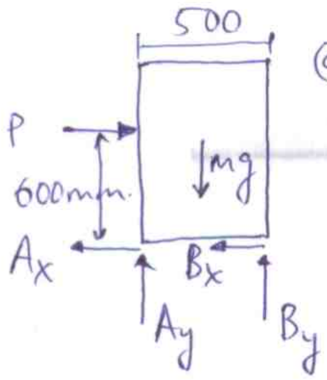


P.40. A 60 kg cabinet is mounted on casters, which can be locked to prevent their rotation. The coefficient of static friction between the floor and each caster is 0.35. Assuming that the casters at both A and B are locked, determine (a) the force P required to move the cabinet to right, (b) the largest allowable value of h if the cabinet is not to tip over.

P.39



Locked caster — Unlocked caster.



(a) Both casters locked.

$$P = A_x + B_x = \mu_s (A_y + B_y) = \mu_s mg$$
$$= 0.35 * 60g = 206.01$$

(b) Caster A free, B locked.

So A_x not present.

$$P = B_x = \mu_s B_y = \mu_s \left[\frac{P(600) + mg(250)}{500}\right]$$
$$\Rightarrow P = \left[\mu_s mg/2\right] / \left[1 - \mu_s \frac{(6)}{(5)}\right] = 177.6 \text{ N.}$$

Momt eqn available
Unknws: P, A_y, B_y

check: use $P = 177.6$, get $B_y = 507.42$
$\Rightarrow B_y < mg (= 588.6\text{N})$
$\Rightarrow A_y > 0$, so OK.

(C) Caster A locked, B free.

So B_x not present.

$$P = A_x = \mu_s A_y = \mu_s \left[\frac{-P(600) + mg(250)}{500}\right]$$
$$\Rightarrow P = 72.53 \text{ N}$$

or directly $\because P > 0$, $A_y > 0$

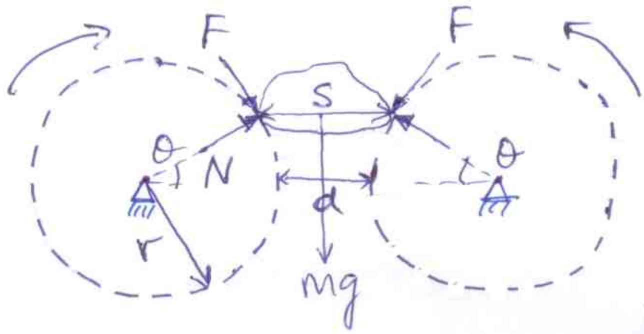
check: $\Rightarrow A_y = 207.26 > 0$, so OK.

P.40 (a) same as 39(a), ie $P = 206.01$

(b) Tips over when $A_y = 0 = \dfrac{-206.01(h) + mg(250)}{500}$

$\Rightarrow h = 250\,mg/206.01 = 714.2\text{mm}.$

P.41



P.41. Two large cylinder, each of radius $r = 500$ *mm*, rotates in the opposite directions and form the main elements of a crusher for stone aggregate. The distance d is set equal to maximum desired size of crushed aggregate. If $d = 20$ *mm* and $\mu_s = 0.30$, determine the size s of the largest stones which will be pulled by crusher by friction alone.

$r = 500$ mm, $d = 20$ mm, $\mu_s = 0.3$

$\sum F_x = 0$ identically satisfied.

$\sum F_y = 0 \Rightarrow mg - 2N\sin\theta + 2F\cos\theta = 0$

$F = N\tan\theta - \dfrac{mg}{2\cos\theta} < N\tan\theta$

neglect mg, $\Rightarrow F = N\tan\theta$ = friction required to pull stones without aid of mg.

when $F = \mu_s N$ slip impends

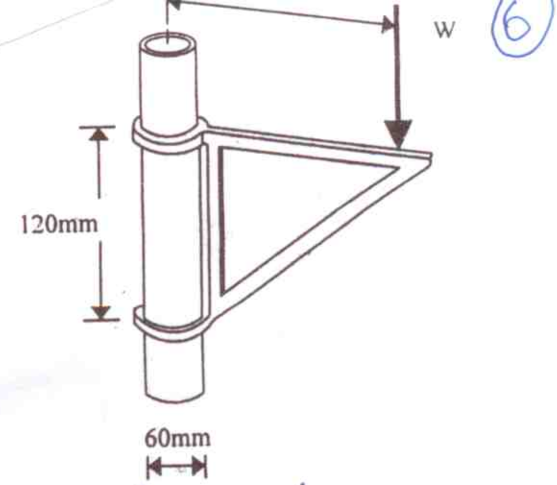
$\Rightarrow F_{max} = N\tan\theta_{max} = \mu_s N \Rightarrow \theta_{max} = \tan^{-1}(\mu_s) = 16.6992°$

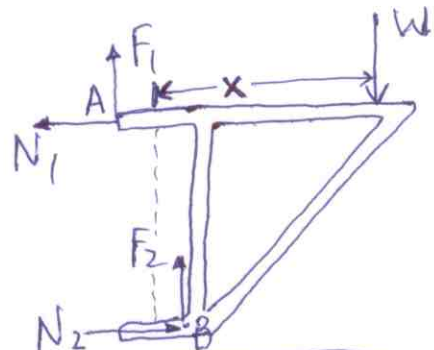
$S = 2r + d - 2r\cos\theta_{max} = 62.17\text{mm}$ = max size of stone that can be pulled in by friction alone (ie unaided by its weight).

What about moment equation ?? It is identically satisfied if CG lies on axis bisecting the line joining centers of cylinders. Otherwise it will not be satisfied, in general, when mass is not neglected and CG lies off bisecting axis).

P42

P.42. The movable bracket shown may be placed on the 60 mm diameter pipe. If the coefficient of static friction between the pipe and bracket is 0.25, determine the minimum distance x at which the load W can be supported. Neglect the weight of the bracket.





unknowns : N_1, N_2, x

Assume impending slip,

$F_1 = \mu_s N_1, \quad F_2 = \mu_s N_2$

$\sum F_x : N_1 = N_2 \longrightarrow (1)$

$\sum F_y : W = F_1 + F_2 \longrightarrow (2)$

$\sum M_A : W(x+30) = F_2(60) + N_2(120) \quad (3)$

~~So if slip not impending, i.e, $F_1 < \mu_s N_1$, $F_2 < \mu_s N_2$, then unknowns are N_1, N_2, F_1, F_2 for given x, and problem is SID.~~

Slip impending $\Rightarrow F_1 = \underbrace{\mu_s N_1}_{(F_1)_{max}}, \quad F_2 = \underbrace{\mu_s N_2}_{(F_2)_{max}}$

$(1), (2) \Rightarrow W = \mu_s (N_1 + N_2) = 2\mu_s N_2 \Rightarrow N_2 = 2W$

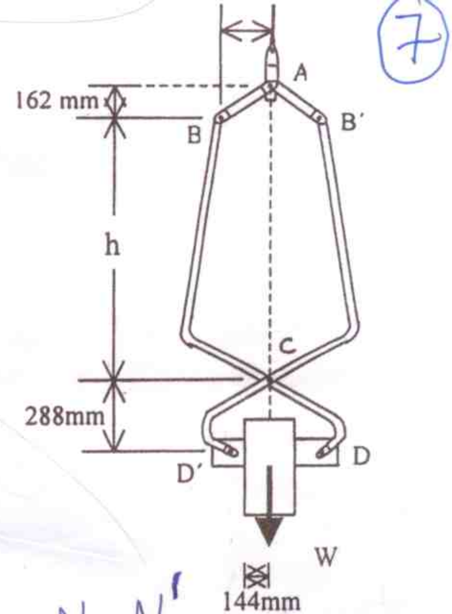
$(3) \Rightarrow W(x+30) = 0.25 * 2W * 60 + 2W * 120$

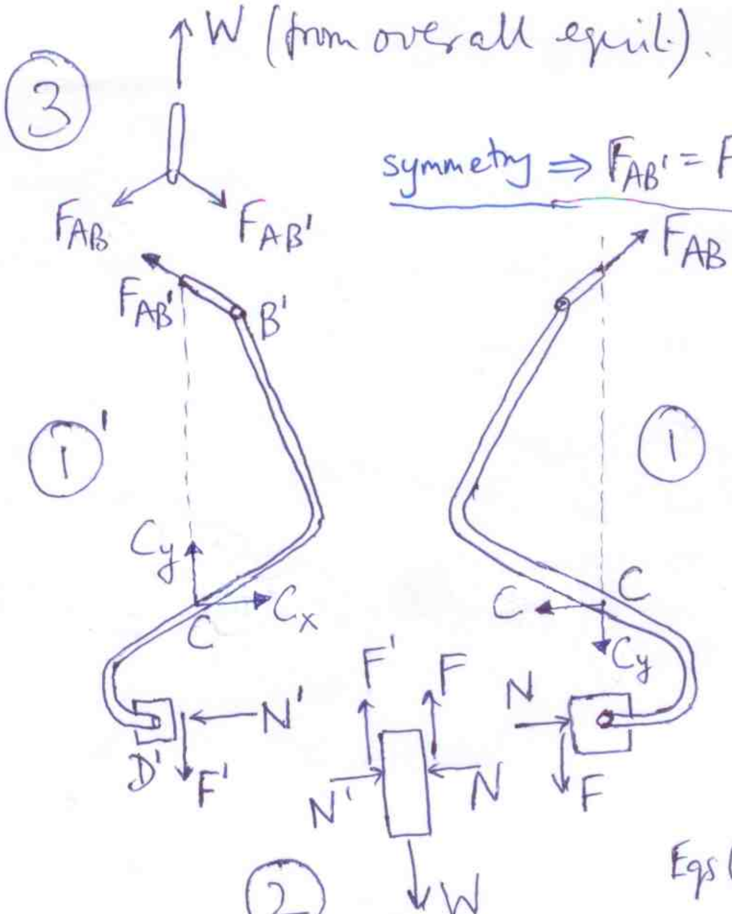
$\Rightarrow x_{min} = 240 \text{ mm}$

Why do we call it min 'x'. Physically it is evident. Mathematically, if $x \downarrow$ from x_{min}, eqn (3) implies $F_2 \downarrow$ or $N_2 \downarrow$ or both. If $F_2 \downarrow$, eqn (2) implies $F_1 \uparrow$, ie F_1 exceeds limiting value $(F_1)_{max}$. If $N_2 \downarrow$ then F_2 which is $(F_2)_{max}$ based on old N_2 has already exceeded its limiting value. Essentially eqns (2), (3) mathematically show why x obtained by assuming impending slip is x_{min} & not x_{max}.

P43.

P.43. The friction tongs shown are used to lift a 350 kg casting. Knowing that h = 864 mm, determine the smallest allowable value of the coefficient of static friction between the casting and blocks D and D'.





symmetry $\Rightarrow F_{AB'} = F_{AB}$, $F = F'$, $N = N'$

FBD ②: $2F' = W \longrightarrow$ ①

FBD ③: $2F_{AB'}\left(\frac{162}{270}\right) = W$ ②

FBD ①': ΣM_C:

$$F_{AB'}\left(\frac{216}{270}\right)(h+162) - N'(288) + F'(144) = 0 \longrightarrow ③$$

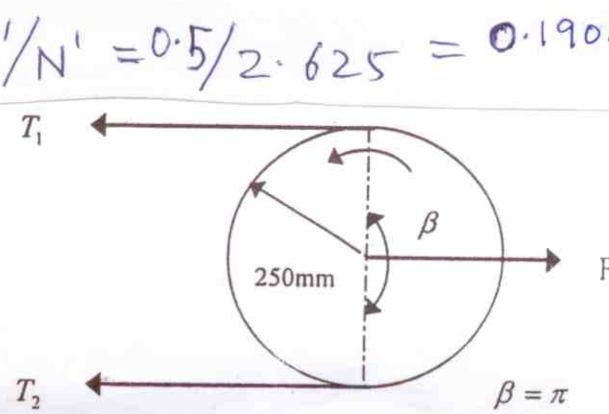
$$\text{Eqs ①,②,③} \longrightarrow \frac{W}{2}\left(\frac{270}{162}\right)\left(\frac{216}{270}\right)(1026) - N'(288) + \frac{W}{2}(144) = 0$$

$$\Rightarrow N' = 2.625\,W$$

For slip impending, $\mu_s = F'/N' = 0.5/2.625 = 0.1904$

P44

P.44. A pulley requires 200 Nm torque to get it rotating. The angle of wrap is π radians, and μ_s is known to be 0.25. What is the minimum horizontal force F required to create enough tension in the belt so that it can rotate the pulley?



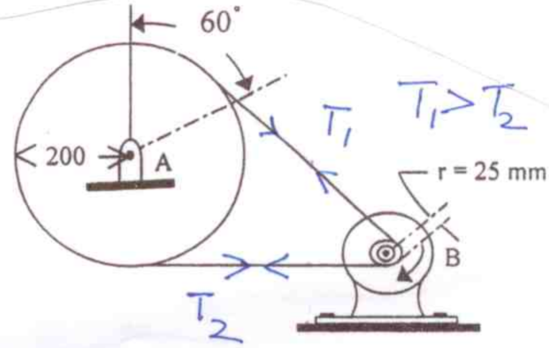
$T_1/T_2 = e^{0.25\pi}$, $(T_1 - T_2)(0.25) = 200$ $\Rightarrow T_1 = 800/(1 - e^{-0.25\pi})$

r

$\Rightarrow T_1 = 1470.42$, $T_2 = 670.42$ $\Rightarrow F = T_1 + T_2 = 2140.84$ N

P45

P.45. A flat belt connects pulley A, which drives a machine tool, to pulley B, that it connected to shaft of an electric motor. The coefficients of friction are $\mu_s = 0.25$ and $\mu_k = 0.20$ between both pulleys and the belt. If the maximum allowable tension in the belt is 3 kN, determine the largest torque that can be exerted by the belt on pulley A.



$T_1 = 3\,kN$ (given).

$$\beta_A = 240^\circ = \frac{240}{180}\pi, \quad \beta_B = 120^\circ = \frac{120}{180}\pi$$

If slip impends at A, then already slipping at B, since $\frac{T_1}{T_2} = e^{\mu_s \beta_A}$ and $e^{\mu_s\beta_A} > e^{\mu_s\beta_B}$. So this is not (physically possible.

If slip impends at B, then not impending at A.

So critical case is when slip impends at B.

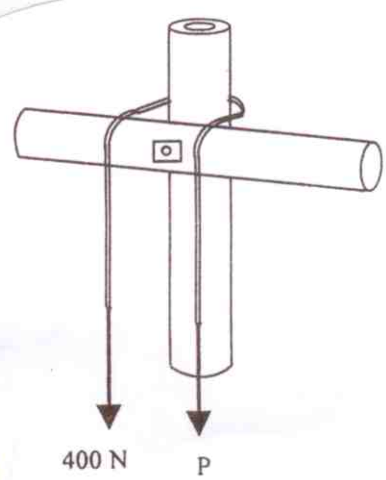
$$\Rightarrow \frac{T_1}{T_2} = e^{\mu_s \beta_B} \Rightarrow T_2 = T_1\, e^{-0.25 * \frac{2}{3}\pi}$$

$$M = (T_1 - T_2)\, r = (T_1 - T_2)\, 200 = T_1\left(1 - e^{-0.25 * \frac{2}{3}\pi}\right)$$

$= 244.57$ N.m (largest torque transmitted by B to A).

P.46.

P.46. If the coefficient of static friction is 0.25 between the rope and the horizontal pipe and 0.20 between the rope and the vertical pipe, determine the range of values of P for which equilibrium is maintained.



Case A: Impending slip downward at left end!

$$\frac{400}{T_1} = e^{0.2\,\pi/2}, \quad \frac{T_1}{T_2} = e^{0.25\pi}, \quad \frac{T_2}{P} = e^{0.2 \cdot \pi/2}$$

$$\Rightarrow P = T_2\, e^{-0.2\frac{\pi}{2}} = T_1\, e^{-0.25\pi}\, e^{-0.2\frac{\pi}{2}} = 400\, e^{-0.2\frac{\pi}{2}}\, e^{-0.25\pi} * e^{-0.2\frac{\pi}{2}}$$

$$\Rightarrow P = 400\, e^{-0.45\pi} = 97.295\,N$$

Case B: Impending slip upward at left end.

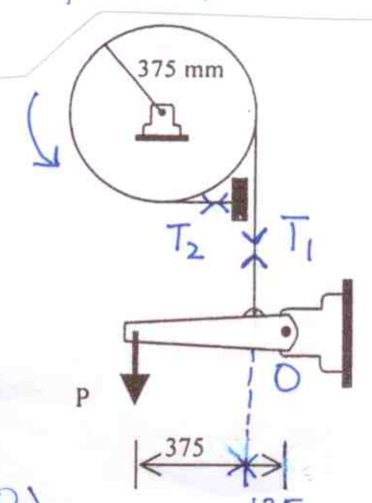
$$\Rightarrow P = 400\, e^{0.45\pi} \quad (\because \text{all prior relations get reciprocated})$$

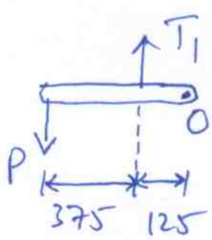
$$= 1644.48\,N.$$

$$\Rightarrow 97.295 \le P \le 1644.48\,N \text{ for equilibrium.}$$

P.47

P.47. A band brake is used to control the speed of a flywheel as shown. The coefficient of friction are $\mu_s = 0.30$ and $\mu_k = 0.25$. What couple should be applied to the flywheel to keep it rotating counterclockwise at a constant speed when $P = 50\ N$?





$$\Sigma M_0 = 0 \Rightarrow T_1 = P\left(\frac{500}{125}\right)$$

$$T_1 = 4P = 200\,N.$$

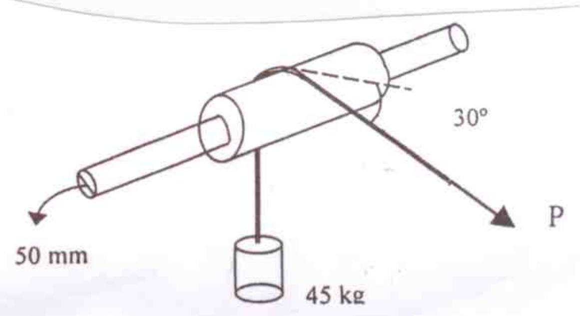
$$T_2 = T_1 e^{-1.5\pi \mu_k}$$

$$M = (T_1 - T_2) * \frac{375}{1000} = 200\left(1 - e^{-1.5\pi \cdot 0.25}\right) * \frac{375}{1000}$$

$$= 51.91\ N.m.$$

P.48.

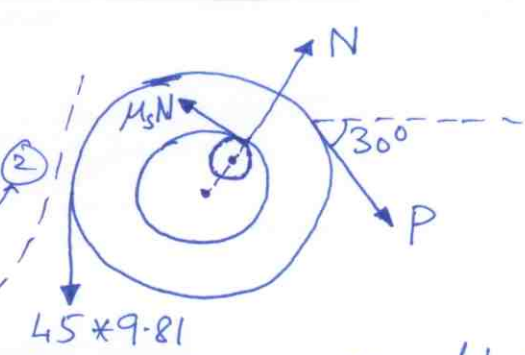
P.48. A bushing of 75 mm outside diameter fits loosely on a horizontal 50 mm diameter shaft. If a force P forming an angle of 30° with the horizontal and having a magnitude $P = 800$ N is required to start raising the 45 kg load, determine the coefficient of static friction between the shaft and the bushing. Assume that the rope does not slip on the bushing.



$$P\cos 30 + N\sin\theta - \mu_s N\cos\theta = 0 \rightarrow (1)$$

$$P\sin\theta + 45 * 9.81 - N\cos\theta - \mu_s N\cos\theta = 0 \rightarrow (2)$$

$$(P - 45 * 9.81) * 75 - \mu_s N(50) = 0 \rightarrow (3)$$



FBD at impending raising of load,

$$①^2 + ②^2 \rightarrow \underbrace{P^2 + (45*9.81)^2 + 2P\sin 30 * 45 * 9.81}_{} = N^2 + \mu_s^2 N^2 \qquad (10)$$

$$③ \rightarrow \quad = \left[\frac{(P - 45*9.81)*75}{50\,\mu_s}\right]^2 (1 + \mu_s^2)$$

$$\Rightarrow \frac{1+\mu_s^2}{\mu_s^2} = 4.10722 \quad \Rightarrow \quad \mu_s = 0.5673.$$

NOTE: To begin raising load we have condition of impending slip, hence μ_s involved. To continue to raise load at a uniform rate, μ_k gets involved and hence P reqd to continue raising is somewhat less that P reqd to commence raising, $\because \mu_k < \mu_s$.