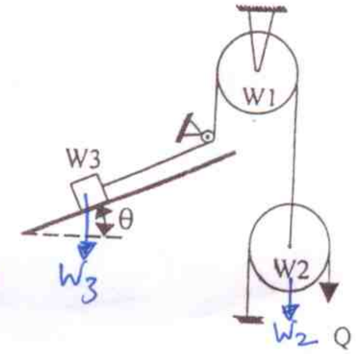
P49

P.49. Determine Q for equilibrium for the system shown. The pulleys are frictionless and have masses W_1 and W_2. The sliding body has mass W_3.



1-DOF system (s)

AFD. : only W_3, W_2, Q appear in AFD. All tensions are internal forces (equal & opp so they cancel out when finding δU for system).

Let δs = virtual displ. of W_2, downward (+).

$$\delta U = -W_3 \sin\theta\, \delta s + W_2\, \delta s + Q\,(2\delta s) = 0$$

$$Q = \frac{W_3 \sin\theta - W_2}{2}$$

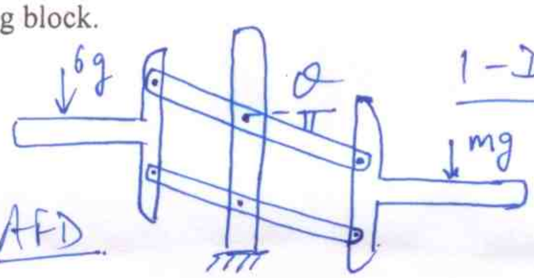
· Compare with Newtons law approach:

T = tension in rope = uniform ∵ frictionless pulleys, & massless rope.

Equilibrium ⇒ $T = W_3 \sin\theta \rightarrow$ (i) } give same result as above.
$T = W_2 + 2Q \rightarrow$ (ii)

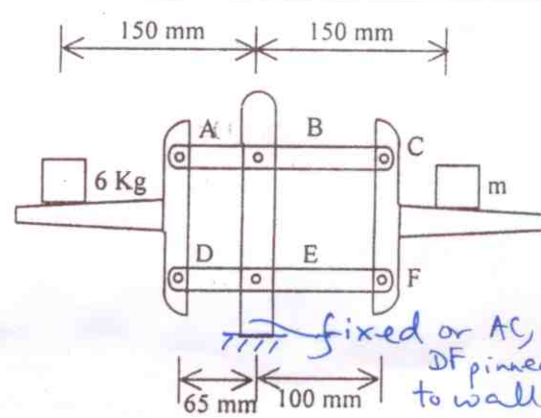
P.50

P.50. Determine the mass m that balances the 6 kg block.



1-DOF (θ)

AFD

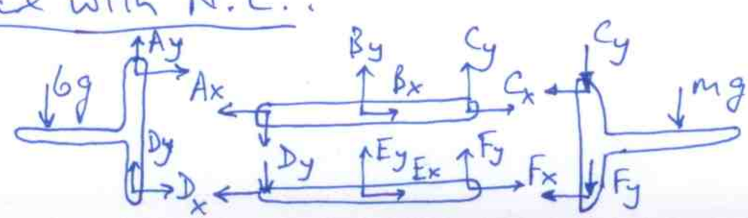


fixed or AC, DF pinned to wall.

Let δy = virtual displ of m.

$$\delta U = m\,\delta y - 6\,\delta y\left(\frac{65}{100}\right) = 0 \Rightarrow m = 6 * \frac{65}{100} = 3.9\ \text{kg}.$$

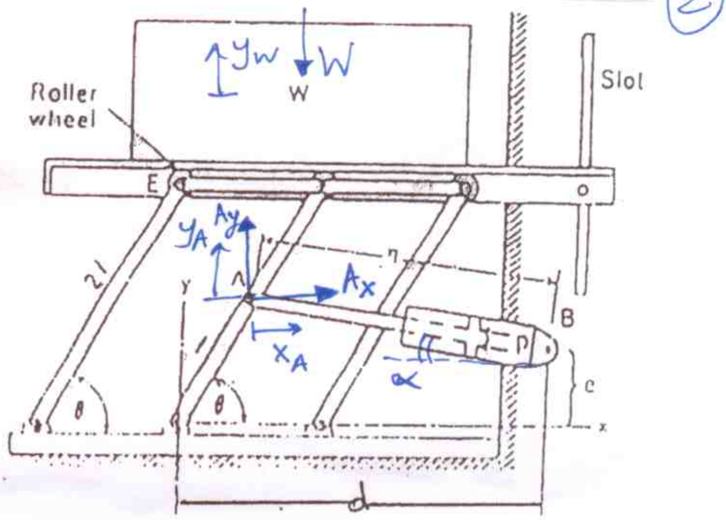
Compare with N.L.:



12 eqns, 13 unknowns (SID) (too cumbersome to eliminate & solve for m)

P.51

P.51. A hydraulic lift platform for loading trucks supports a weight W of 5000 N. Only one side of the system has been shown; the other side is identical. If the diameter of the piston in the cylinder (two) is 40 mm, what pressure p is needed to support W when $\theta = 60^\circ$. Assume $l = 240$ mm, $d = 600$ mm, and $e = 100$ mm. Neglect friction everywhere.



1-DOF (θ)

$$y_W = 2l\sin\theta \Rightarrow \delta y_W = 2l\cos\theta\,\delta\theta$$
$$y_A = l\sin\theta \Rightarrow \delta y_A = l\cos\theta\,\delta\theta$$
$$x_A = l\cos\theta \Rightarrow \delta x_A = -l\sin\theta\,\delta\theta$$

AFD contains only $W(\downarrow)$, $A_x(\leftrightarrow)$, $A_y(\uparrow)$, since piston is isolated from lift

$$\delta U = 0 = (-W)\delta y_W + A_x\,\delta x_A + A_y\,\delta y_A$$
$$= (-W)(2l\cos\theta\,\delta\theta) + \left(-p\,2\pi\frac{d^2}{4}\cos\alpha\right)(-l\sin\theta\,\delta\theta) + \left(p\,2\pi\frac{d^2}{4}\sin\alpha\right)(l\cos\theta\,\delta\theta) = 0 \longrightarrow ①$$

$d = 40$ mm.

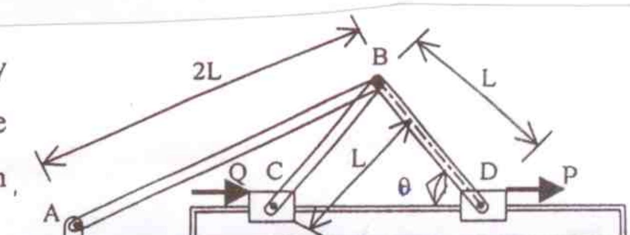
$$\alpha = \tan^{-1}\left\{\frac{l\sin\theta - e}{d - l\cos\theta}\right\} = 12.6629^\circ$$

$$① \Rightarrow p = \frac{2W\cos\theta}{2\pi\frac{d^2}{4}(\sin\alpha\cos\theta + \cos\alpha\sin\theta)} = 2.084 \text{ N/mm}^2$$

same answer as Tute #2 prob #6 whose solution by N.L. approach (SID structure) was more tedious.

P.52

P.52. The mechanism shown is acted upon by the force P; derive an expression for the magnitude of the force Q required to maintain equilibrium.



1-DOF (θ)

AFD — same as given fig. (all other forces are internal or do no virtual work, friction at sliders C, D is absent).

$$x_p^2 + L^2 - 2x_p L\cos\theta = (2L)^2$$

$$\Rightarrow 2x_p\,\delta x_p - 2L\,\delta x_p\cos\theta + 2L x_p \sin\theta\,\delta\theta = 0$$

$$\delta x_p = \frac{-x_p\sin\theta}{2(x_p - L\cos\theta)}\,2L\,\delta\theta$$

$$x_q^2 + L^2 - 2x_q L\underbrace{\cos(\pi-\theta)}_{=-\cos\theta} = 2L^2$$

$$\Rightarrow 2x_q\,\delta x_q + 2L\,\delta x_q\cos\theta - 2L x_q\sin\theta\,\delta\theta = 0$$

$$\delta x_q = \frac{x_q\sin\theta}{2(x_q + L\cos\theta)}\,2L\,\delta\theta$$

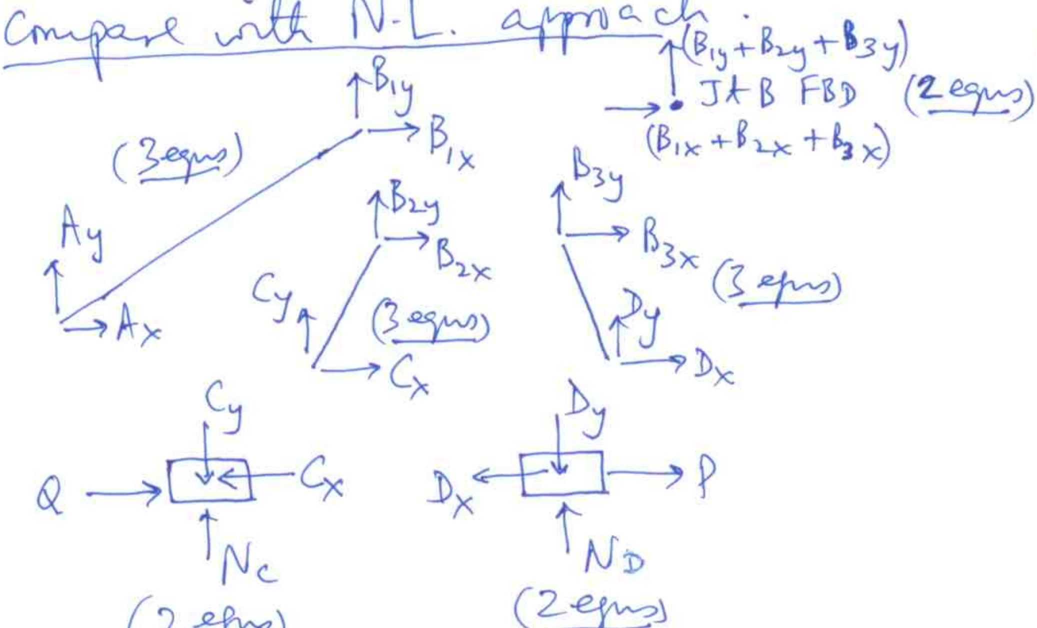
$$\delta U = P\,\delta x_p + Q\,\delta x_q = 0$$

$$\Rightarrow P\left(\frac{-x_p\cancel{\sin\theta}}{\cancel{2}(x_p - L\cos\theta)}\right)\cancel{2L\,\delta\theta} + Q\left(\frac{x_q\cancel{\sin\theta}}{\cancel{2}(x_q + L\cos\theta)}\right)\cancel{2L\,\delta\theta} = 0$$

$$Q = \frac{x_p}{x_q}\left(\cancel{\frac{x_q + L\cos\theta}{x_p - L\cos\theta}}\right)P = \left(\frac{x_p}{x_q}\right)P$$

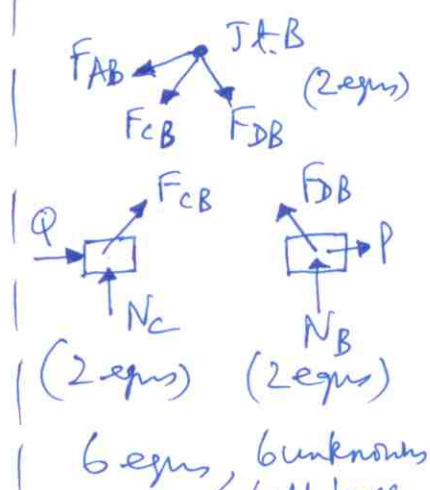
$$= \left(\frac{\sqrt{(2L)^2 - (L\sin\theta)^2} + L\cos\theta}{\sqrt{(2L)^2 - (L\sin\theta)^2} - L\cos\theta}\right)P = \left(\frac{\sqrt{4-\sin^2\theta} + \cos\theta}{\sqrt{4-\sin^2\theta} - \cos\theta}\right)P$$

Compare with N-L. approach:



15 eqns, 15 unknowns (ie all forces except P)

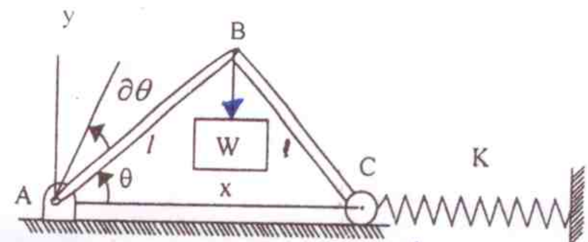
or doing as 2-Force members



6 eqns, 6 unknowns (all forces except P).

P.53.

P.53. If members AB and BC are 4 m long, find the angle θ corresponding to equilibrium for $W = 50$ kg if the spring constant k is 20 N/m. Neglect the weight of the members and friction everywhere. Take $\theta = 30^o$ for the configuration where the spring is unstretched.



AFD is figure itself

1 DOF (θ).

$$V = V_g + V_e$$
$$= Wgl\sin\theta + \frac{1}{2}k\left(2l\cos\theta - 2l\cos 30\right)^2$$

(used $\theta = 0^\circ$ as datum for V_g, and $\theta = 30^\circ$ as datum for V_e).

$$\delta U = -\delta V = -\frac{\partial V}{\partial\theta}\delta\theta \quad (\because \text{ no other active forces}) \Rightarrow \frac{\partial V}{\partial\theta} = 0$$

$$\frac{\partial V}{\partial\theta} = Wgl\cos\theta + k\left(2l\cos\theta - 2l\cos 30\right)\left(-2l\sin\theta\right) = 0 \rightarrow ①$$

$$\frac{\partial^2 V}{\partial\theta^2} = -Wgl\sin\theta + 4kl^2\left(\sin^2\theta - \cos^2\theta + \cos\theta\cos 30\right) \longrightarrow ②$$

Solve ① by Newton Raphson method $\left(\Delta\theta_i = -\left.\frac{\partial V/\partial\theta}{\partial^2 V/\partial\theta^2}\right|_{\theta=\theta_i}, \ \theta_{i+1} = \theta_i + \Delta\theta_i\right)$ or by trial & error method.

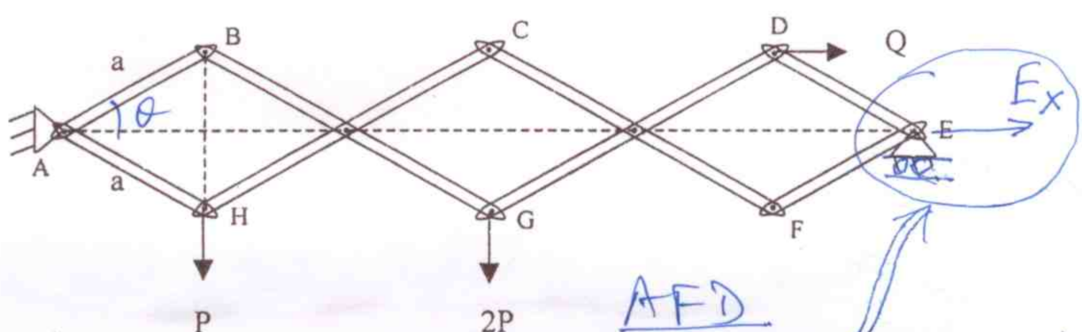
Result is $\underline{\theta = -70.726^\circ}$, $\left.\frac{\partial^2 V}{\partial\theta^2}\right|_{\theta=70.726} = 3219 > 0 \Rightarrow$ stable Equilibrium.

P.54. Find reaction at E by using method of virtual work.

P.54.

1st method (VW+NL)

1-DOF (θ)



AFD

(original pinned support replaced by roller + E_x)

$$y_H = y_G = -a\sin\theta$$
$$y_E = 0$$
$$x_D = 5a\cos\theta, \quad x_E = 6a\cos\theta$$

$$\delta U = (-P)\delta y_H + (-2P)\delta y_G + Q\,\delta x_D + E_x\,\delta x_E = 0$$

$$\Rightarrow P\cancel{a}\cos\theta\,\delta\theta + 2P\cancel{a}\cos\theta\,\delta\theta + Q\left(-5\cancel{a}\sin\theta\,\delta\theta\right) + E_x\left(-6\cancel{a}\sin\theta\,\delta\theta\right) = 0$$

$$\Rightarrow E_x = \frac{3P\cos\theta - 5Q\sin\theta}{6\sin\theta}$$

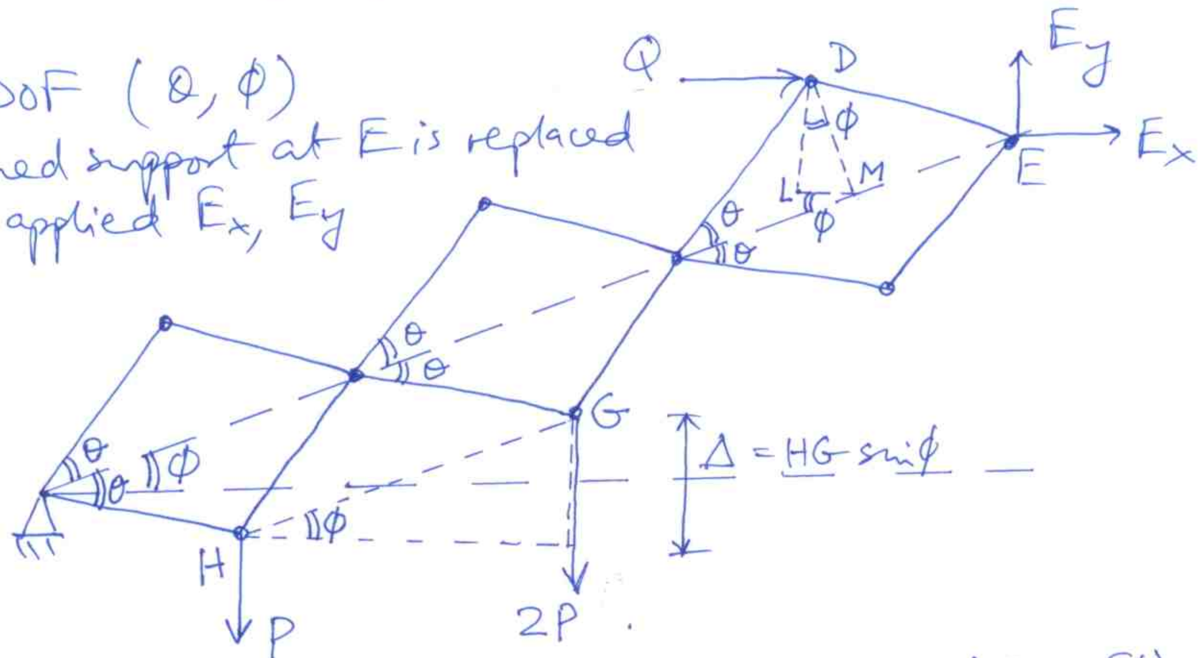
For E_y use N.L, i.e.,

$$\sum M_A = 0 = Pa\cos\theta + 2P(3a\cos\theta) + Qa\sin\theta - E_y(6a\cos\theta)$$

$$\Rightarrow E_y = \frac{7P\cos\theta + Q\sin\theta}{6\cos\theta}$$

Method 2 : Fully by Virtual work

2-DOF (θ, ϕ)
Pinned support at E is replaced by applied E_x, E_y



$$y_H = -a\sin(\theta-\phi) \Rightarrow \delta y_H = -a\cos(\theta-\phi)(\delta\theta - \delta\phi)$$

$$y_G = y_H + \Delta = -a\sin(\theta-\phi) + 2a\cos\theta\sin\phi$$

$$\Rightarrow \delta y_G = -a\cos(\theta-\phi)(\delta\theta-\delta\phi) - 2a\sin\theta\sin\phi\,\delta\theta + 2a\cos\theta\cos\phi\,\delta\phi$$

$$x_E = 6a\cos\theta\cos\phi \Rightarrow \delta x_E = -6a(\sin\theta\cos\phi\,\delta\theta + \cos\theta\sin\phi\,\delta\phi)$$

$$y_E = 6a\cos\theta\sin\phi \Rightarrow \delta y_E = -6a(\sin\theta\sin\phi\,\delta\theta - \cos\phi\cos\theta\,\delta\phi)$$

$$x_D = x_M - LM = 5a\cos\theta\cos\phi - a\sin\theta\sin\phi$$

$$\Rightarrow \delta x_D = -5a(\sin\theta\cos\phi\,\delta\theta + \cos\theta\sin\phi\,\delta\phi) - a(\cos\theta\sin\phi\,\delta\theta + \sin\theta\cos\phi\,\delta\phi)$$

$$\delta U = (-P)\delta y_H + (-2P)\delta y_G + Q\,\delta x_D + E_x\,\delta x_E + E_y\,\delta y_E$$

$$\delta U = \underbrace{\Big(Pa\cos(\theta-\phi) + 2Pa\cos(\theta-\phi) + 4Pa\sin\theta\sin\phi - 6aE_x\sin\theta\cos\phi - 6aE_y\sin\theta\sin\phi - 5aQ\sin\theta\cos\phi - aQ\cos\theta\sin\phi \Big)}_{\mathrm{I}}\delta\theta$$

$$+ \underbrace{\Big(-Pa\cos(\theta-\phi) - 2Pa\cos(\theta-\phi) - 4Pa\cos\theta\cos\phi - 6aE_x\cos\theta\sin\phi + 6aE_y\cos\theta\cos\phi - 5aQ\cos\theta\sin\phi - aQ\sin\theta\cos\phi \Big)}_{\mathrm{II}}\delta\phi = 0$$

$\because$ $\delta\theta$, $\delta\phi$ are independent, get,

$(\mathrm{I}) = 0 \rightarrow ①$ } Two equations in 2 unknowns E_x, E_y.

$(\mathrm{II}) = 0 \rightarrow ②$

To solve, put $\phi = 0$, $\theta = \theta$ (ie given equilibrium configuration).

$$\Rightarrow 3P\cancel{a}\cos\theta - 6\cancel{a}E_x\sin\theta - 5\cancel{a}Q\sin\theta = 0 \rightarrow ①$$

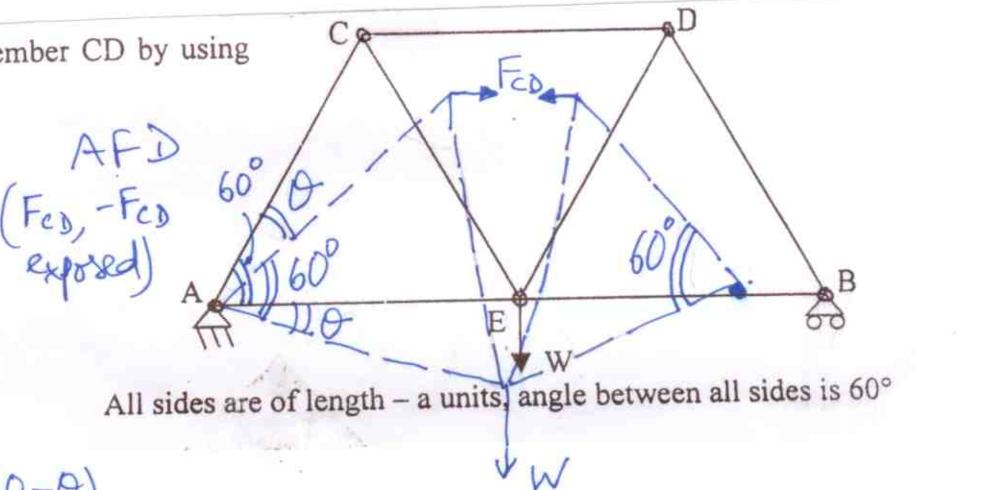
$$-3P\cancel{a}\cos\theta - 4P\cancel{a}\cos\theta + 6\cancel{a}E_y\cos\theta - \cancel{a}Q\sin\theta = 0 \rightarrow ②$$

$$① \rightarrow E_x = \frac{3P\cos\theta - 5Q\sin\theta}{6\sin\theta}$$

$$② \rightarrow E_y = \frac{7P\cos\theta + Q\sin\theta}{6\cos\theta}$$

(Same result as method I)

P55 P.55. Determine force in member CD by using method of virtual work.



All sides are of length – a units, angle between all sides is 60°

$x_C = a\cos(60-\theta)$

$y_E = -a\sin\theta$

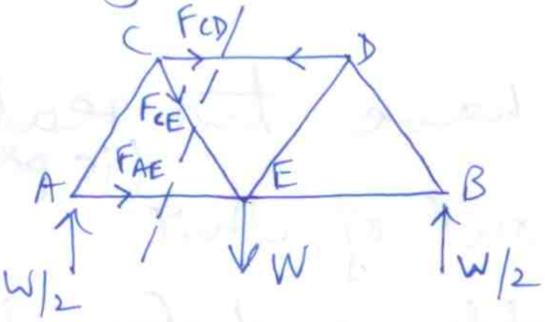
$x_D = x_B - a\cos(60-\theta)$
$= 2a\cos\theta - a\cos(60-\theta)$

$$\delta U = 0 = F_{CD}\,\delta x_C + (-F_{CD})\,\delta x_D + (-W)\,\delta y_E$$

$$= F_{CD}\Big(a\sin(60-\theta)\,\delta\theta - \{-2a\sin\theta\,\delta\theta - a\sin(60-\theta)\,\delta\theta\}\Big) + Wa\cos\theta\,\delta\theta$$

put $\theta = 0° \longrightarrow F_{CD}(\cancel{a}\sin 60 + \cancel{a}\sin 60) = -W\cancel{a} \Rightarrow F_{CD} = -\dfrac{W}{2\sin 60}$

Check by NL: use method of sections

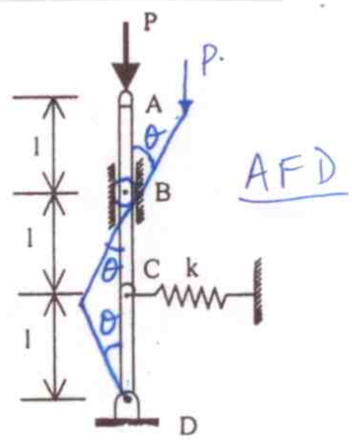


$$\sum M_E = 0 = \frac{W}{2}a + F_{CD}\, a \sin 60$$

$$\Rightarrow F_{CD} = \frac{-W}{2\sin 60^\circ}$$

P.56

P.56. Two bars *ABC* and *CD* are attached to a single spring of constant *k* which is unstretched when the bars are vertical. Determine the range of values of *P* for which the equilibrium of the system is stable in the position shown $\rightarrow$ ie $\theta = 0^\circ$ undeformed



Note: Length of spring not given. So assume it is very long and essentially remains horizontal (otherwise data is insufficient).

Since P is constant, we can include its work done in the potential (similar to gravitation potential).

$$V = P(3l\cos\theta) + \frac{1}{2}k\underbrace{(l\sin\theta)^2}$$

ie, tacitly assumed that spring stays essentially horizontal $\because$ it is very long.

$$\frac{\partial V}{\partial \theta} = -3Pl\sin\theta + kl^2\sin\theta\cos\theta = 0$$

$$\Rightarrow \underbrace{\sin\theta = 0}_{\text{equilibrium position of our interest}} \quad \text{or} \quad \cos\theta = \frac{3Pl}{kl^2}, \quad \begin{cases} \text{ie } \theta_{1e} = 0^\circ \\ \theta_{2e} = \cos^{-1}\left(\frac{3P}{kl}\right) \end{cases}$$

$$\left.\frac{\partial^2 V}{\partial \theta^2}\right|_{\theta=0^\circ} = \left[-3Pl\cos\theta + kl^2(\cos^2\theta - \sin^2\theta)\right]_{\theta=0^\circ} = -3Pl + kl^2$$

Stable $\Rightarrow \left.\frac{\partial^2 V}{\partial \theta^2}\right|_{\theta=0^\circ} > 0$, ie $P < \frac{kl}{3}$ for stable

Note: If $P < \frac{kl}{3}$ then second equilibrium position is real (ie possible) $\because \cos\theta = \frac{3P}{kl} < 1$.

$$\text{Now } \left.\frac{\partial^2 V}{\partial \theta^2}\right|_{\theta=\theta_{2e}} = -\frac{9P^2l^2}{kl^2} + \left(\frac{9P^2l^2}{kl^2} * 2 - kl^2\right) = \frac{9P^2}{k} - kl^2 < 0 \text{ if } 3P < kl.$$

so θ_{2e} is unstable.

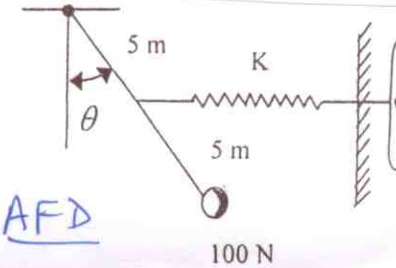
(P.56) (Contd)

So If $P < \frac{kl}{3}$ we have two real (ie possible) equilibrium positions out of which only $\theta_{1e} = 0°$ is stable and θ_{2e} is unstable.

& If $P > \frac{kl}{3}$ we have only one real (ie possible) equilibrium position, ie $\theta_{1e} = 0°$ which is unstable, ie system will buckle.

(P.57) (8)

P.57. The spring is unstretched when $\theta = 30^o$. At any position of the pendulum, the spring remains horizontal. If the spring constant is 50 N/m, at what position will the system be in equilibrium? 1-DOF (θ)



AFD

$$V = -100 * 10\cos\theta + \frac{1}{2}k(5\sin\theta - 5\sin 30)^2$$

$$\frac{\partial V}{\partial \theta} = 1000\sin\theta + 50*(5\sin\theta - 5\sin 30)*5\cos\theta = 0 \text{ for equilibrium} \rightarrow (1)$$

$$\frac{\partial^2 V}{\partial \theta^2} = 1000\cos\theta + 1250(\cos^2\theta - \sin^2\theta + \sin\theta\sin 30) \rightarrow (2)$$

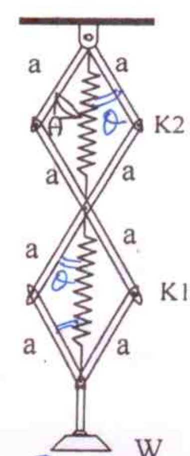
NOTE: This is not required if you are not interested in finding the nature of the equilibrium solution, i.e., whether stable or unstable. However, it is useful if you are solving (1) by Newton Raphson (see P.53), so stability part comes as a by-product.

Solution of (1) is $\theta = 15.842°$ (by Newton Raphson).

$$\left.\frac{\partial^2 V}{\partial \theta^2}\right|_{\theta=15.842} = 2196 > 0 \Rightarrow \text{stable equilibrium.}$$

(P.58.)

P.58. If the springs are unstretched when $\theta = \theta_o$, find the angle θ when the weight W is places on the system. Use the method of the minimum potential energy. 1-DOF (θ)



AFD

$$V = \frac{1}{2}(k_1 + k_2)(2a\cos\theta - 2a\cos\theta_0)^2 - W(4a\cos\theta)$$

$$\frac{\partial V}{\partial \theta} = (k_1 + k_2)4a^2(\cos\theta - \cos\theta_0)(-\sin\theta) + 4Wa\sin\theta = 0 \rightarrow (1)$$

$$\Rightarrow \underset{(\text{ie } \sin\theta = 0)}{\theta = 0^0} \text{ or } \theta: \cos\theta = \cos\theta_0 + \frac{W}{(k_1 + k_2)a} \longrightarrow \text{equilibrium solutions.}$$

Check Stability of equilibrium:

$$\frac{\partial^2 V}{\partial \theta^2} = 4a^2(k_1 + k_2)\left[(-\cos\theta)(\cos\theta - \cos\theta_0) + \sin^2\theta\right] + 4Wa\cos\theta \rightarrow (2)$$

For $\theta = 0° \longrightarrow \frac{\partial^2 V}{\partial \theta^2} = 4a^2(k_1 + k_2)\left[-1 + \cos\theta_0\right] + 4Wa$

(P.58) contd. On the other hand if we assume $\cos\theta_0 + \frac{W}{a(k_1+k_2)} > 1$

then θ_{2e} is not real (ie not possible) and $\theta_{1e} = 0°$ is stable.

Summary: (1) For $\left|\cos\theta_0 + \frac{W}{a(k_1+k_2)}\right| < 1$ both equilibrium positions θ_{1e} & θ_{2e} are real (ie possible) but only θ_{2e} is stable.

(2) For $\cos\theta_0 + \frac{W}{a(k_1+k_2)} > 1$, only θ_{1e} is possible (ie real) and it is stable

Note: The case when $\cos\theta_0 + \frac{W}{a(k_1+k_2)} < -1$ is not physically possible since $\frac{W}{a(k_1+k_2)} > 0$.

So (1) & (2) are the only possible cases for which θ_{2e} and θ_{1e} are the stable solutions, respectively.

$$\Rightarrow \frac{\partial^2 V/\partial\theta^2}{4a^2(k_1+k_2)} = -1 + \cos\theta_0 + \frac{W}{a(k_1+k_2)}$$ (9)

So if we assume (or its given) that $\left|\cos\theta_0 + \frac{W}{a(k_1+k_2)}\right| < 1$

then $\left.\frac{\partial^2 V}{\partial\theta^2}\right|_{\theta=0^\circ} < 0$, ie, unstable equilibrium.

For θ: $\cos\theta = \cos\theta_0 + \frac{W}{(k_1+k_2)a} \rightarrow$ (3) from (2), & (3)

$$\left.\frac{\partial^2 V/\partial\theta^2}{4a^2(k_1+k_2)}\right|_{\theta = \text{satisfies (3)}} = (\cancel{\cos\theta - \cos\theta_0})\cos\theta - \cancel{\cos^2\theta} + \cancel{\cos\theta\cos\theta_0} + \sin^2\theta$$

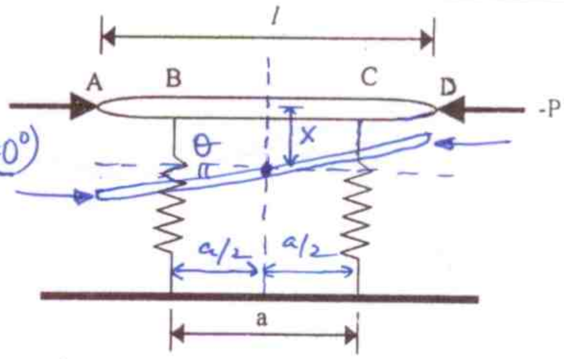
$= \sin^2\theta \quad > 0$, so second equil solution is unconditionally stable.

So with above assumption, answer is,

θ : $\cos\theta = \cos\theta_0 + \frac{W}{(k_1+k_2)a}$ ◀ (contd on p.8a)

P.59.

P.59. The horizontal bar *AD* is attached to two springs of constant *k* and is in equilibrium in the position shown. (ie $\theta = 0^\circ$) Determine the range of values of the magnitude *P* of the two equal and opposite horizontal forces *P* and –*P* for which the equilibrium position is stable (a) if *AB* = *CD*, (b) if *AB* = 2 (*CD*).



2-DOF (x, θ).

As in P.56, assume springs are very long & essentially remain vertical always.

Case (a) $AB = CD$.

$$V = \frac{1}{2}k\left(x + \frac{a}{2}\sin\theta\right)^2 + \frac{1}{2}k\left(x - \frac{a}{2}\sin\theta\right)^2 - 2P\frac{l}{2}(1-\cos\theta)$$

$$= kx^2 + k\frac{a^2}{4}\sin^2\theta - Pl(1-\cos\theta)$$

Equilibrium $\Leftrightarrow$ $\frac{\partial V}{\partial\theta} = 0$, $\frac{\partial V}{\partial x} = 0$

$$\frac{\partial V}{\partial\theta} = 0 = \frac{k}{2}a^2\sin\theta\cos\theta - Pl\sin\theta \rightarrow (1)$$

$$\frac{\partial V}{\partial x} = 0 = 2kx = 0 \rightarrow ②$$

Equilibrium solutions (of ①, ②) are: $x=0$ (expected since all masses are neglected)

and $\underbrace{\theta = 0}_{\text{of our interest.}}$ or $\theta = \cos^{-1}\left(\frac{2Pl}{ka^2}\right)$

Stability:

$$\frac{\partial^2 V}{\partial \theta^2} = \frac{k}{2}a^2(\cos^2\theta - \sin^2\theta) - Pl\cos\theta$$

$$\frac{\partial^2 V}{\partial x^2} = 2k, \quad \frac{\partial^2 V}{\partial \theta \partial x} = 0$$

For stability, $\left.\frac{\partial^2 V}{\partial \theta^2}\right|_{\substack{\theta=0\\x=0}} > 0$, $\left.\frac{\partial^2 V}{\partial x^2}\right|_{\substack{\theta=0\\x=0}} > 0$, $\left[\left(\frac{\partial^2 V}{\partial \theta^2}\right)\left(\frac{\partial^2 V}{\partial x^2}\right) - \left(\frac{\partial^2 V}{\partial x \partial \theta}\right)\right]_{\substack{x=0\\\theta=0}} > 0$

satisfied if $\frac{ka^2}{2} - Pl > 0$

i.e. $P < \frac{ka^2}{2l}$ for stable equil.

satisfied always $\because k > 0$

satisfied if previous two conditions are satisfied $\because \frac{\partial^2 V}{\partial x \partial \theta} = 0$

Case b.: $AB = 2CD \Rightarrow 3CD = l - a \Rightarrow CD = \frac{l-a}{3}$

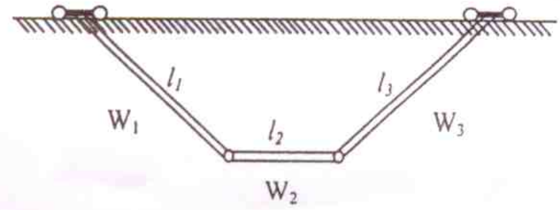
$$V = kx^2 + \frac{ka^2}{4}\sin^2\theta - P\left[\left(\frac{a}{2} + \frac{2}{3}(l-a)\right) + \left(\frac{a}{2} + \frac{l-a}{3}\right)\right](1-\cos\theta)$$

$$= kx^2 + \frac{ka^2}{4}\sin^2\theta - Pl(1-\cos\theta)$$

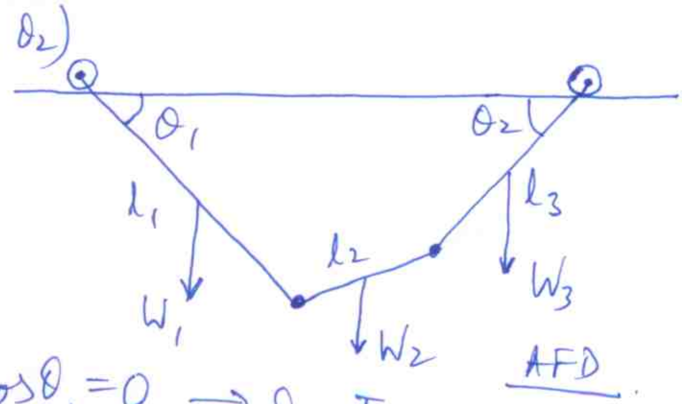
i.e., V remains same, so solution remains same as case (a) — Interesting !!

NOTE: We could have treated this as 1-DOF (i.e, θ only) since you see that the d.o.f.'s de-couple, i.e, eqn ① involves θ only and eqn ② involves x only. So if you take only θ as d.o.f., i.e, $x=0$, you still get the same result.

P.60. Determine the angle of inclination of each linkage in the figure shown. The rollers move without friction on the support.



2-DOF. (θ_1, θ_2)



$$V = -W_1 \frac{l_1}{2}\sin\theta_1 - W_3 \frac{l_3}{2}\sin\theta_2 - W_2\left(\frac{l_1\sin\theta_1 + l_3\sin\theta_2}{2}\right)$$

Equilibrium:

$$\frac{\partial V}{\partial \theta_1} = -W_1 \frac{l_1}{2}\cos\theta_1 - W_2 \frac{l_1}{2}\cos\theta_1 = 0 \Rightarrow \theta_1 = \frac{\pi}{2}$$

$$\frac{\partial V}{\partial \theta_2} = -W_3 \frac{l_3}{2}\cos\theta_2 - W_2 \frac{l_3}{2}\cos\theta_2 = 0 \Rightarrow \theta_2 = \frac{\pi}{2}$$

Stability:

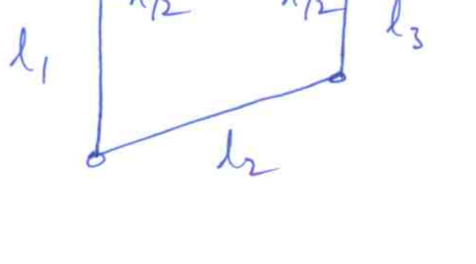
$$\left.\frac{\partial^2 V}{\partial \theta_1^2}\right|_{\substack{\theta_1=\pi/2\\ \theta_2=\pi/2}} = \left[W_1 \frac{l_1}{2}\sin\theta_1 + W_2 \frac{l_1}{2}\sin\theta_1\right]_{\substack{\theta_1=\theta_2\\ =\pi/2}} = (W_1+W_2)\frac{l_1}{2} > 0$$

$$\left.\frac{\partial^2 V}{\partial \theta_2^2}\right|_{\substack{\theta_1=\theta_2=\\ \pi/2}} = \left.(W_3+W_2)\frac{l_3}{2}\sin\theta_2\right|_{\theta_1=\theta_2=\pi/2} = \frac{(W_3+W_2)}{2} l_3 > 0$$

$$\frac{\partial^2 V}{\partial \theta_1 \partial \theta_2} = 0$$

So all 3 stability conditions indicate Stable equilibrium

Equilibrium configuration is,



→ ie position of minimum potential energy — could have guessed it at the beginning (!!!) but you should verify it in this way.