***3.139** A flagpole is guyed by three cables. If the tensions in the cables have the same magnitude P, replace the forces exerted on the pole with an equivalent wrench and determine (a) the resultant force $\mathbf{R}$, (b) the pitch of the wrench, (c) the point where the axis of the wrench intersects the xz plane.

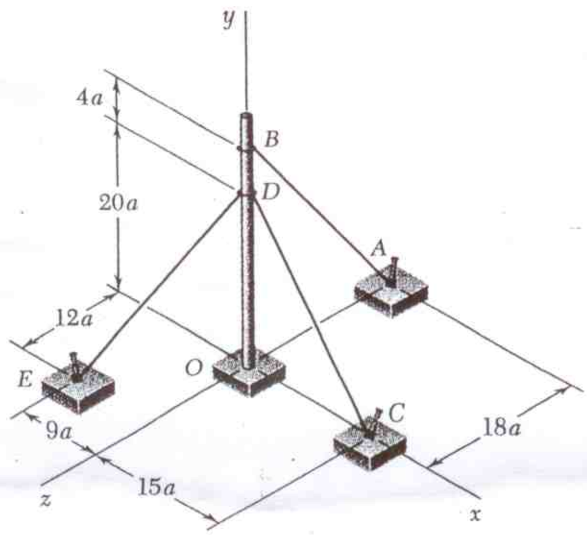


Fig. P3.139

$$\underline{R} = \underline{T}_{DE} + \underline{T}_{DC} + \underline{T}_{BA}$$ (replace $P \to T$ for uniqueness in notation).

$$= T\left\{ \frac{(-9\underline{i} - 20\underline{j} + 12\underline{k})}{25} + \frac{(15\underline{i} - 20\underline{j})}{25} + \frac{(-24\underline{j} - 18\underline{k})}{30} \right\}$$

$$= T\left\{ \frac{7.2\underline{i} - 72\underline{j} - 3.6\underline{k}}{30} \right\} = T(0.24\underline{i} - 2.4\underline{j} - 0.12\underline{k})$$

Does not matter about which point we do the reduction for the couple. Choose point as D (for convenience ∵ two couples, due to T_{DE}, T_{DC}, vanish).

$$\underline{M}_D^R = \text{due to } \underline{T}_{BA} \text{ alone}$$

$$= 4a\underline{j} \times \underline{T}_{BA} = 4aT\left(\frac{-18}{30}\right)\underline{i} = -2.4\,aT\,\underline{i}$$

$$p = \frac{\underline{R} \cdot \underline{M}_D^R}{R^2} = \frac{-2.4aT^2 \times 0.24}{5.832T^2} = -0.09877a$$

$$p\underline{R} + \underline{r} \times \underline{R} = \underline{M}_D^R, \quad \underline{r} = (x\underline{i} - 20a\underline{j} + z\underline{k})$$

$$-0.0988aT(0.24\underline{i} - 2.4\underline{j} - 0.12\underline{k}) + T(x\underline{i} - 20a\underline{j} + z\underline{k}) \times (0.24\underline{i} - 2.4\underline{j} - 0.12\underline{k}) = -2.4\,aT\,\underline{i}$$

2

$\underline{i}:\ (-0.0988\,aT)(0.24) + Tz(2.4) + 2.4aT = -2.4aT$

$\Rightarrow z = -1.9901\,a$

$\underline{k}:\ (-0.0988\,aT)(-0.12) + Tx(-2.4) + 4.8aT = 0$

$\Rightarrow x = 2.0049\,a$

$\underline{j}:\ (-0.0988\,aT)(-2.4) + T(0.12x + 0.24z) = 0$

identically satisfied..

***3.137 and *3.138** Two bolts at A and B are screwed into a block by applying the forces and couples shown. Replace the two wrenches with a single equivalent wrench and determine (a) the resultant **R**, (b) the pitch of the single equivalent wrench, (c) the point where the axis of the wrench intersects the xz plane.

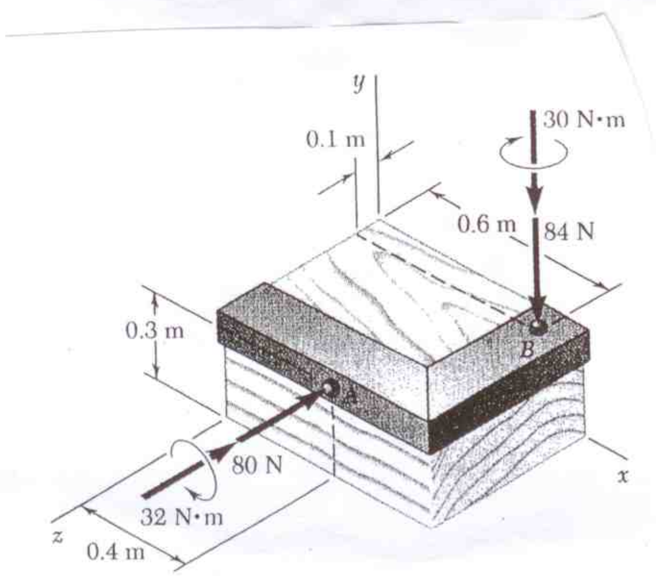


Fig. P3.137

$\underline{R} = (-80\underline{k} - 84\underline{j})\,N$

$\underline{M}_O^R = -32\underline{k} - 30\underline{j} + (0.4\underline{i} + 0.3\underline{j} + a\underline{k}) \times (-80\underline{k})$

$\quad + (0.6\underline{i} + 0.1\underline{k}) \times (-84\underline{j})$

$\quad = -15.6\underline{i} + 2\underline{j} - 82.4\underline{k}$

$$p = \frac{\underline{R}\cdot\underline{M}_O^R}{R^2} = \frac{(-80)(-82.4) + (-84)(2)}{80^2 + 84^2} = 0.477408$$

$p\underline{R} + (x\underline{i} + z\underline{k}) \times \underline{R} = \underline{M}_O^R$

③

$$0.477408(-80\underline{k}-84\underline{j}) + (x\underline{i}+z\underline{k})\times(-80\underline{k}-84\underline{j})$$
$$= -15.6\underline{i} + 2\underline{j} - 82.4\underline{k}$$

$\underline{i}$: $84z = -15.6 \Rightarrow z = -0.1857\,m$

$\underline{k}$: $-80 * 0.477408 - 84x = -82.4 \Rightarrow x = 0.5263\,m$

$\underline{j}$: equation will be identically satisfied, as usual.

***3.143** Replace the wrench shown with an equivalent system consisting of two forces perpendicular to the y axis and applied respectively at A and B.

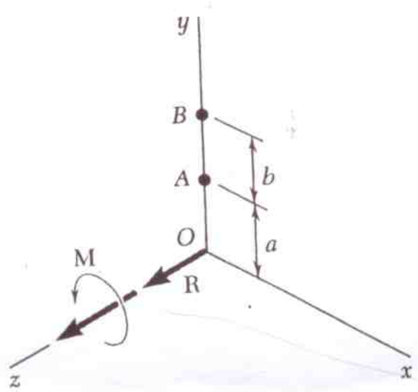


Fig. P3.143

$\underline{F}_A = (p\underline{i} + q\underline{k})$, $\underline{F}_B = (r\underline{i} + s\underline{k})$

Static equivalence:

$R\,\underline{k} = \underline{F}_A + \underline{F}_B = (p+r)\,\underline{i} + (q+s)\,\underline{k} \longrightarrow$ ①.

$\Rightarrow p = -r, \quad q + s = R. \longrightarrow$ (i, ii)

②

$\underline{M}_o^R = M\,\underline{k} = \underline{r}_A \times \underline{F}_A + \underline{r}_B \times \underline{F}_B = a\underline{j}\times(p\underline{i}+q\underline{k}) + (a+b)\underline{j}\times(r\underline{i}+s\underline{k})$

$= [aq + (a+b)s]\,\underline{i} - [ap + (a+b)r]\,\underline{k}$

$\Rightarrow aq + (a+b)s = 0, \quad ap + (a+b)r = -M \longrightarrow$ (iii, iv)

(i)(iv) $\rightarrow r = -M/b, \quad p = M/b$

(ii, iii) $\rightarrow s = -\frac{a}{b}R, \quad q = \left(\frac{b+a}{b}\right)R$

$$\left.\right\} \quad \underline{F}_A = \frac{M}{b}\underline{i} + R\left(1+\frac{a}{b}\right)\underline{k}, \qquad \underline{F}_B = -\frac{M}{b}\underline{i} - \frac{a}{b}R\,\underline{k}$$

Shames

4.64. Find the *simplest* resultant acting on the vertical wall *ABCD*. Give the coordinates of the center of pressure. The pressure varies such that $p = E/(y+1) + F$ psi, with y in feet, from 10 psi to 50 psi, as indicated in the diagram. E and F are constants.

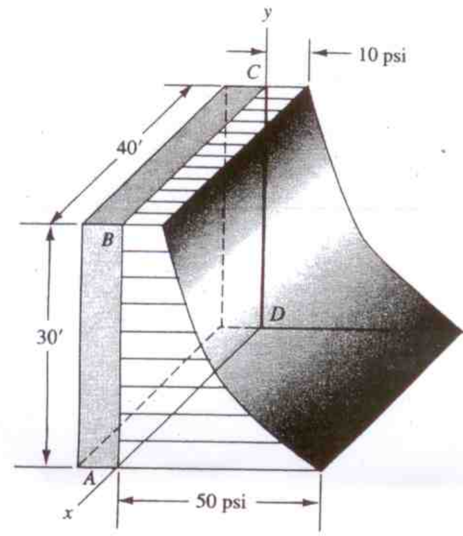


Figure P.4.64.

$$p = \left(\frac{E}{y+1} + F\right) \text{ psi}$$

$$50 = \frac{E}{1} + F, \quad 10 = \frac{E}{31} + F$$

$$\Rightarrow E = \frac{124}{3}, \quad F = \frac{26}{3}$$

$$p = 144\left(\frac{E}{y+1} + F\right) \text{ lb/ft}^2$$

$$R = \int dR = \int 40p\,dy = 144 * 40 \int_1^{31} \left(\frac{E}{x} + F\right) dx \quad \left(\text{put } x = y+1\right)$$

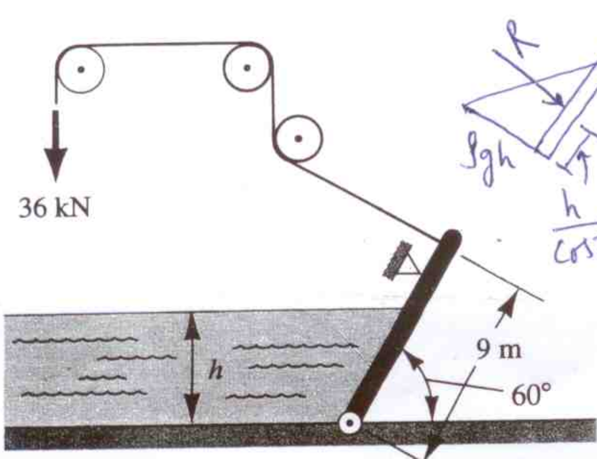
$$= 144 * 40 \left[E \ln 31 + F * 30\right] = 2.3151636 74 * 10^6$$

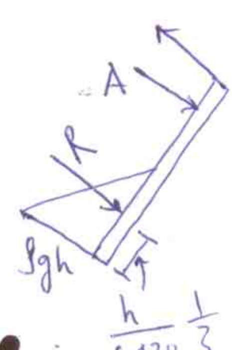
$$R\bar{y} = \int dM = \int_0^{30} 40p\,dy \cdot y = \int_0^{30} \left[40p(y+1)\,dy - 40p\,dy\right]$$

$$= 40 * 144 \left[\int_0^{30} \left(E + F(y+1)\right) dy\right] - R = 40 * 144 \left[E(30) + F\left(\frac{30^2}{2} + 30\right)\right] - R$$

$$\bar{y} = 28.7888 * 10^6 / R = 12.4349'$$

4.73. At what height h will the water cause the door to rotate clockwise? The door is 3 m wide. Neglect friction and the weight of the door.





$$\sum M_o = 0 \text{ with } \quad = 0$$

$$R = \frac{1}{2} \rho g h \frac{h}{\cos 30}(3), \quad \rho g = 9806$$

$$\bar{x} = \frac{1}{3} h/\cos 30$$

$$R\bar{x} = 36 * 10^3 * 9$$

$$h^3 = \frac{(36 * 10^3)(9)(3\cos 30)(2\cos 30)}{(9806)(3)}$$

$$h = 3.6732 \text{ m.}$$

Shames

(5)

4.71. Imagine a liquid which when stationary stratifies in such a way that the specific weight is proportional to the square root of the pressure. At the free surface, the specific weight is known and has the value γ_0. What is the pressure as a function of depth from the free surface? What is the resultant force on one face AB of a rectangular plate submerged in the liquid? The width of the plate is b.

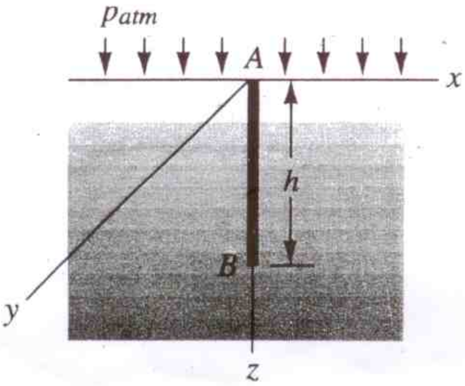
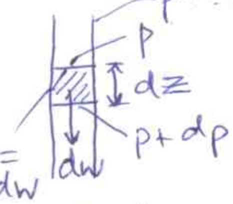


Figure P.4.71.

given: $\overset{\rho g}{\gamma} = k\sqrt{p}$.

prismatic column of unit cross-section area



$$\frac{dp}{dz} = \frac{dW}{dz} = k\sqrt{p}$$

$$\Rightarrow \frac{dp}{\sqrt{p}} = k\,dz$$

$$2\sqrt{p}\Big|_{p_{atm}}^{p} = kz$$

$$\sqrt{p} - \sqrt{p_{atm}} = \frac{kz}{2}$$

$$p = \frac{k^2 z^2}{4} + p_{atm} + kz\sqrt{p_{atm}}$$

$$\gamma\Big|_{z=0} = k\sqrt{p}\Big|_{z=0} = k\sqrt{p_{atm}} = \gamma_0 \text{ (given)}$$

$$\Rightarrow k = \gamma_0/\sqrt{p_{atm}}$$

$$p = \frac{\gamma_0^2 z^2}{4p_{atm}} + p_{atm} + \gamma_0 z$$

$$R = \int_0^h p\,dz\,b = b\left[\frac{\gamma_0 h^3}{12 p_{atm}} + p_{atm}h + \frac{\gamma_0 h^2}{2}\right]$$

$$R\bar{z} = \int_0^h p\,dz\,b\,z = b\left[\frac{\gamma_0^2 h^4}{16 p_{atm}} + p_{atm}\frac{h^2}{2} + \frac{\gamma_0 h^3}{3}\right]$$

$$\bar{z} = \left(\frac{\gamma_0^2 h^3}{16 p_{atm}} + \frac{p_{atm}h}{2} + \frac{\gamma_0 h^2}{3}\right) \Big/ \left(\frac{\gamma_0 h^2}{12 p_{atm}} + p_{atm} + \frac{\gamma_0 h}{2}\right)$$

4.111. A cylindrical tank of water is rotated at constant angular speed ω until the water ceases to change shape. The result is a free surface which, from fluid mechanics considerations, is that of a paraboloid. If the pressure varies directly as the depth below the free surface, what is the resultant force on a quadrant of the base of the cylinder? Take $\gamma = 62.4\ \text{lb/ft}^3$. [*Hint:* Use circular strip in quadrant having area $1/4(2\pi r\ dr)$.]

$$y = kr^2 = \frac{5}{36}r^2 \quad 6$$

$$p = \gamma h = \gamma(y+5) = \gamma\left(\frac{5}{36}r^2 + 5\right)$$

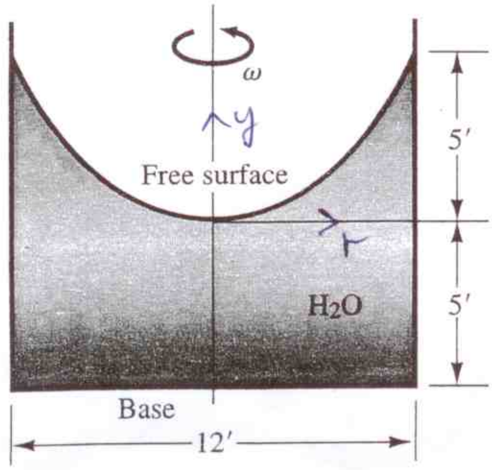


Figure P.4.111.

$$R = \int_0^6 p\,\frac{2\pi r\,dr}{4} = \frac{\pi}{2}\gamma\int_0^6\left(\frac{5}{36}r^3 + 5r\right)dr$$

$$= \frac{\pi}{2}\gamma\left[\frac{5}{36}\left(\frac{6^4}{4}\right) + 5\left(\frac{6^2}{2}\right)\right] = 13232.388$$