FRICTION – Extra Problems

P.1 Each of the four wheels of the vehicle has a mass of 20 kg. and is mounted on a 80-mm. dia. journal (shaft). The total mass of the vehicle is 480 kg. including wheels, and is distributed equally on all four wheels. If a force
P = 80 N. is required to keep the vehicle rolling at a constant low speed on a horizontal surface, calculate the coefficient of friction which exists in the wheel bearings.

P.2 In the vise shown, the screw is single threaded in the upper member; it passes through the lower member and is held by a frictionless washer. The pitch of the screw is 3 mm., its mean radius is 12 mm., and μ_s = 0.15. Determine the magnitude P of the forces exerted by the jaws when a 60 N-m torque is applied to the screw.

P.3 The scissors-type jack has a double square thread which engages the threaded collars A and B. The thread has a mean dia. of 20 mm. and a lead of 10 mm. With a coefficient of friction of 0.25 for the threads, (a) calculate the torque M on the screw required to raise the load L = 12 kN from the position where h = l and (b) calculate the torque M' required to lower the load from the same position.

P.4 The two boards are clamped by using the hold-down clamp shown. What torque *M* must be applied to the screw in order to produce a 900 N compression between the boards. The 10 mm dia. single threaded screw has 11 square threads per 20 mm, and μ_s = 0.2 for the threads. Neglect friction in the small ball contact at *A* and also between the clamp and board at *C*, and assume that the contact force at *A* is directed along the screw axis. Also determine the torque M^* is required to loosen the clamp.

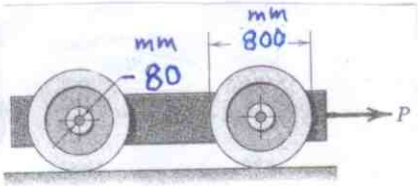


FIG. 1

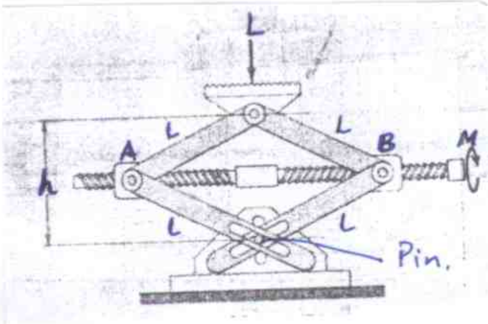


FIG. 3

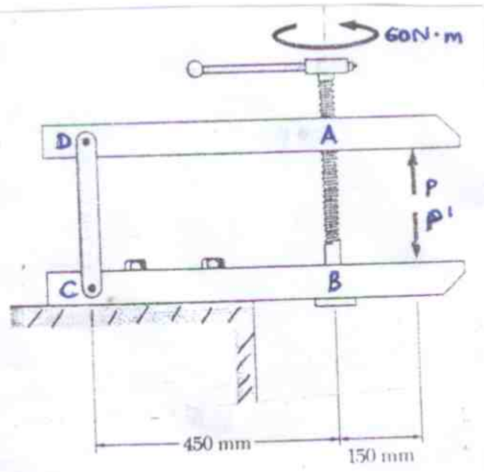


FIG. 2

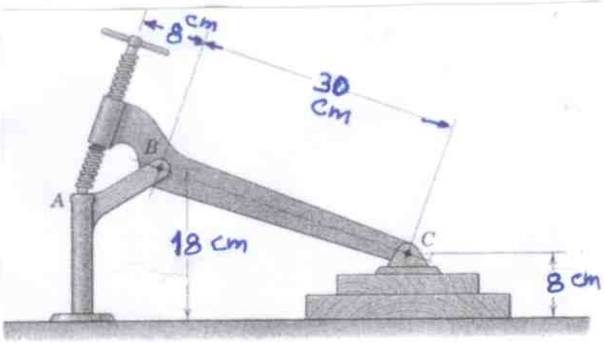
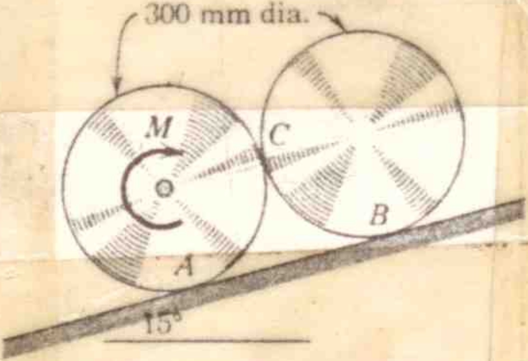
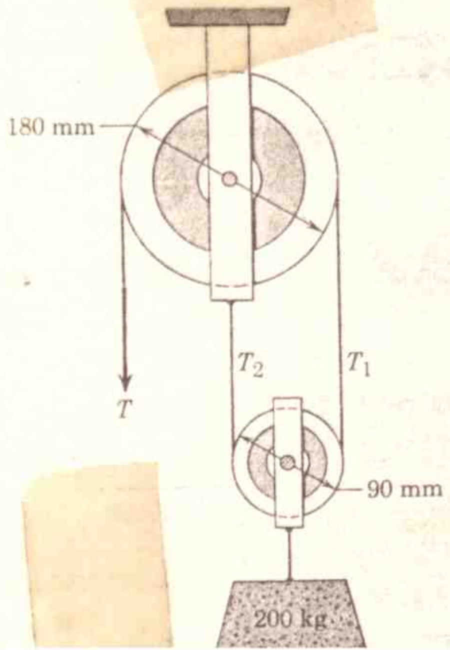


FIG. 4

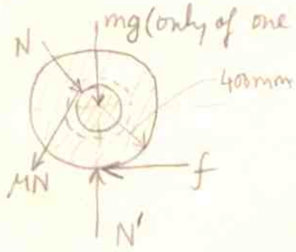
P.5 Find the couple M applied to the lower of the two 20-kg cylinders which will allow them to roll slowly down the incline. Take $\mu_s = 0.6$ and $\mu_k = 0.5$ for all contacting surfaces.

P.6 The dia. of the bearing for the upper pulley is 20 mm. and for the lower pulley it is 12 mm. For $\mu_s = 0.25$ for both bearings, calculate the tensions in the three cables if the block is being lowered slowly.

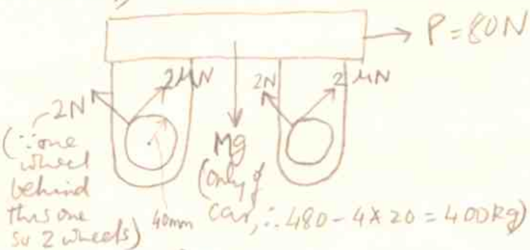

300 mm dia.
M
C
B
A
15°



180 mm
T_2
T_1
T
90 mm
200 kg



Single wheel



Car

Car: $\Sigma F_x : 4N(-\sin\theta + \mu\cos\theta) + 80 = 0 \to (1)$

$\Sigma F_y : 4N(\cos\theta + \mu\sin\theta) - 400g = 0 \to (2)$

$(1)^2 + (2)^2 : N^2(1+\mu^2) = \frac{(400g)^2 + 80^2}{16} \to (*)$

Single wheel

$\Sigma F_x : -f + N(\sin\theta - \mu\cos\theta) = 0 \to (3)$

$= \frac{80}{4}$ from (1)

$\Sigma M_{center} : f*0.4 - \mu N*0.04 = 0 \to (4)$

from (*), (4), $\mu = 0.208$

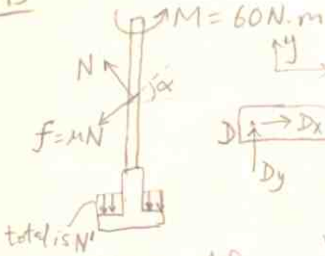
Note: if we take P/4 on wheel FBD & hence dont take P on car FBD

modified method { then: eqn (1) gets modified (i.e., the +80 term vanishes)
eqn (*) " " (i.e., the 80^2 " ")
eqn (3) remains the same ∵ we would have P/4 (=20) term & $N(\sin\theta - \mu\cos\theta) = 0$ due to modified eqn (1).

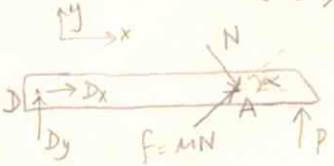
In the method detailed here we get $\frac{(\mu^2+1)}{\mu^2} = \frac{(400g)^2 + 80^2}{16} * \frac{(0.04)^2}{(20)^2(0.4)^2}$ whereas in the modified method the $(80)^2$ term vanishes from above eqn. This yields a marginal difference in results but the method detailed here is the correct one.

S.13

$\alpha = \tan^{-1}\left(\frac{3}{2\pi(12)}\right) = 2.279^\circ$



Screw AB



Vise arm DA

Screw: $\Sigma F_z : N(\cos\alpha - \mu \sin\alpha) - N' = 0 \rightarrow$ of no use

$\Sigma M_z : -M + rN(\sin\alpha + \mu\cos\alpha) = 0 \Rightarrow N = 26364\,N$

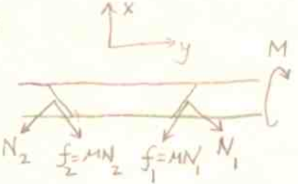
Vise arm DA: $\Sigma M_D : P(0.6) - N(\cos\alpha - \mu\sin\alpha)(0.45) = 0$

$P = 19.64$ kN.

Note: we are not assuming that pts D & A lie on the same horz line. However, the contribution in ΣM_D of the x-comp's of N & f are zero (i.e, cancel out). This is because the horz comp of f balances that of N when forces are summed along helical thread, so no horz force is transmitted from screw to vise arm DA. In this regard we recall that the conc. force representation of N & f had only limited static equivalence with the actual distributed forces along helical threads (i.e, conc N & f can only be used for ΣF_y & ΣM_y)

S.14

(a)



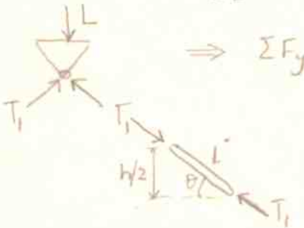
For M in dir. shown the right [left] half of screw has a tendency to move to the right [left] since it is left- [right-] handed. Since that is not possible (i.e, moving in opp dir.) as it is a single screw, the right [left] collar will move to the left [right] thus raising the load.

$\alpha = \tan^{-1}\left(\frac{10}{\pi(20)}\right) = 9.04°$

$\Sigma F_y : \cos\alpha(N_1 - N_2) + \mu \sin\alpha (N_2 - N_1) = 0 \Rightarrow N_1 = N_2 = N, \; f_1 = f_2 = f = \mu N \rightarrow ①$

$\Sigma M_y : -M + \frac{d}{2}\left[(N_1 + N_2)(\sin\alpha + \mu\cos\alpha)\right] \rightarrow ②$

Now, note that screw transmits only y-comp of f, N to collar (since the x & z comps balance out when Σ forces along helical thread). The conc f, N shown have limited equivalence with their actual distribution along thread i.e, conc f, N can only be used for $\Sigma M_y, \Sigma F_y$

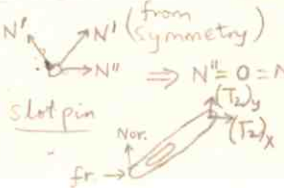


$\Rightarrow \Sigma F_y : 2T_1 \underbrace{\sin\theta}_{h/2l} - L = 0 \Rightarrow T_1 = \frac{Ll}{h} \rightarrow ③$

$\Sigma F_x : (T_2 - T_1)\sin\theta = 0$

$\Rightarrow T_1 = T_2 = T \rightarrow ④$

$\Sigma F_y : N(-\cos\alpha + \mu\sin\alpha) + (T_1 + T_2)\underbrace{\cos\theta}_{\sqrt{l^2 - h^2/4}/l} = 0 \rightarrow ⑤$



$\Rightarrow N'' = 0 = N'$, So slotted members are also 2-Force members.

(over)

from (3), (4), (5), for $h = l$,

$$N = \frac{2L\sqrt{3/4}}{(\cos\alpha - \mu\sin\alpha)} \rightarrow (*)$$

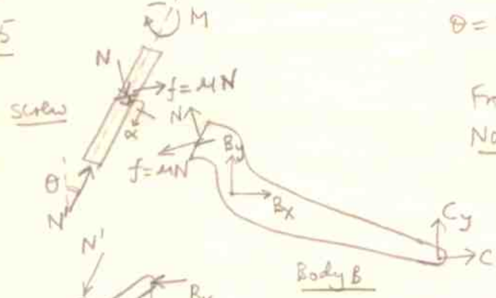
from (2) & (*),

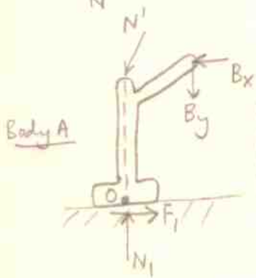
$$M = d/2\left[4L\sqrt{3/4}\left(\frac{\sin\alpha + \mu\cos\alpha}{\cos\alpha - \mu\sin\alpha}\right)\right] = 177.1 \text{ N-m}$$

(b) For lowering case, reverse signs on friction (μ) terms,

$$M' = d/2\left[4L\sqrt{3/4}\left(\frac{\sin\alpha - \mu\cos\alpha}{\cos\alpha + \mu\sin\alpha}\right)\right] = -36.34 \text{ N.m}$$

P.5





NOTE: Direction of f determined from direction of impending motion of screw.

$\theta = \sin^{-1}\left(\frac{10}{30}\right) = 19.47°$, $\alpha = \tan^{-1}\left(\frac{20/10}{\pi \cdot 10}\right) = 3.64°$

From FBD of clamp @ C we get C_y = compressive force = 900 N (given)

NOTE: Direction of N determined from fact that since $N' > 0$, upper face of screw threads contact lower face of sleeve thread

NOTE: • FBD of screw is only an eqvt FBD that is valid only for summing forces & moments about screw axis. Thus, the components of $\underline{N}$ & $\underline{f}$ (after integrating along thread) are non zero only along screw axis direction, and the components of $\underline{N}$ & $\underline{f}$ in the two directions $\perp^{ar}$ to screw axis sum to zero when integrated along thread.

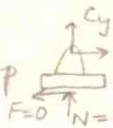
• So the FBD of Body B is drawn with the understanding that only the component of $\underset{\sim}{f}$, $\underline{N}$ along screw axis is non-zero, other comp's of $\underset{\sim}{f}$, $\underline{N}$ being zero in FBD of B.

• Assuming vertical rod in body A to be slender, we can conclude that N_1, F_1 can be taken as concentrated forces acting at pt O.

• Now if we consider all 3 FBD's we have 8 eqns (3 each for A & B, and 2 for screw) and 8 unknowns (N_1, F_1, B_x, B_y, N', C_x, N, M). Then assuming that friction forces developed @ O and clamp C are less than their limiting values @ impending slip, we can solve the problem in principle.

• However, dimensions given for A are insufficient & we have cannot take moment equilibrium of FBD A about pt. B. Thus we are forced to make the following assumption & completely neglect body A.

Assume: no friction between clamp C & board. ⇒ $C_x = 0$ (from FBD of clamp



)

For screw: $\sum M_{\text{axis of screw}}$: $-M + (\mu\cos\alpha + \sin\alpha)N \cancel{r}_{5/10} = 0 \rightarrow (1)$

$\sum F_{\text{axis of screw}}$: $N' + N(\mu\sin\alpha - \cos\alpha) = 0 \rightarrow (2)$

For body B : $\sum M_B$: $N(\mu\sin\alpha - \cos\alpha)(8) + C_y\sqrt{30^2 - 10^2} = 0 \rightarrow (3)$

(1) & (3) ⇒ N = 3229.5 N, M = 424.81 N.cm } for tightening ie, producing compression

To start loosening the clamp we need to unscrew the screw. Thus direction of f is reversed, ie, sign on μs terms reversed. Note that when we start loosening, the compressive force remains 900 N.

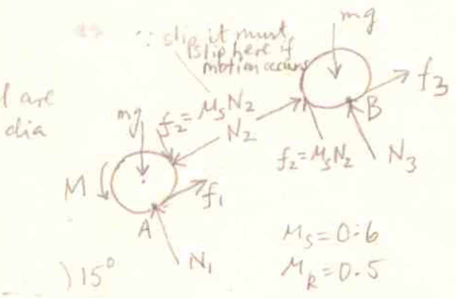
Thus we get N ∓ 3148 N, M = −214.26 N.cm

We can also see that when we start loosening, N' remains unchanged which is what we expect!

integration limits.

S.3

both cyl are 300 mm dia



Note: even though fig in book (MK) shows CW M we dont need to assume it as such. Infact we dont know a-priori if M_{reqd} is CW or CCW for rolling down, ∴ moment due to f_2 could be high enough (depending on wt, radii, & μ_k) to necessitate application of a CCW M.
Also we dont have to assume any particular dir for f_1, f_2. The signs in sdn will reveal dir.

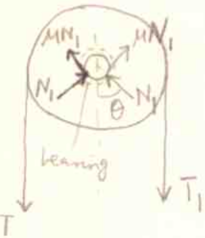
See discussions done in Spring 9 Sem in the 'Executive diary'

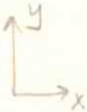
$$\Sigma M_B: \; -N_2 * 0.15 * (1+\mu_k) + mg \sin 15 * 0.15 = 0 \;\Rightarrow\; N_2 = 33.85 \text{ N}$$

$$\Sigma M_A: \; M + mg \sin 15 * 0.15 + N_2 * 0.15 * (1-\mu_k) = 0 \;\Rightarrow\; M = -10.16 \text{ N.m} = 10.16 \text{ N.m} \circlearrowright$$

S.11

upper pulley





Note : Both cases i.e, lowered & raised are done. For the latter case replace pencilled bearing forces with inked ones. θ, α defines angle between N_1 & vertical & N_2 & vertical, resp'ly.

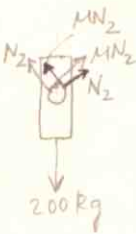
For the FBD of lower bearing block there will be unbalanced moments so we conclude that bearing block rotates slightly to balance moments (i.e, so that mg, N_2, μN_2 will meet @ a pt).

lower pulley

Lower Bearing Block

$$\left.\begin{aligned}\Sigma F_y &: N_2(\cos\alpha + \mu \sin\alpha) - 200 * 9.81 = 0 \\ \Sigma F_x &: N_2 \sin\alpha - \mu N_2 \cos\alpha = 0\end{aligned}\right\} \Rightarrow \alpha = 14.03^\circ,\ N_2 = 1903.42\ N$$

lower bearing block



Lower Pulley

$$\Sigma F_y : T_1 + T_2 - \underbrace{N_2(\cos\alpha + \mu \sin\alpha)}_{= mg = 200 * 9.81} = 0$$

$$\Sigma M_{center} : (T_2 - T_1) * \frac{90}{2} \pm \mu N_2 * \frac{12}{2} = 0$$

block being lowered (use − sign)

$T_2 = 1012.72\ N$, $T_1 = 949.28$

block being raised (use + sign)

$T_2 = 949.28\ N$, $T_1 = 1012.72\ N$

Upper Pulley

$$\Sigma F_x : -N_1 \sin\theta + \mu N_1 \cos\theta = 0$$

$$\Sigma F_y : -T - T_1 + N_1(\cos\theta + \mu \sin\theta) = 0$$

$$\Sigma M : (T - T_1) * \frac{180}{2} \mp \mu N_1 * \frac{20}{2} = 0$$

block being lowered (use + sign)

$N_1 = 1793.54\ N$, $T = 899.46\ N$

block being raised (use − sign)

$N_1 = 2019.38\ N$, $T = 1068.8$