FRICTION

- force distr on contact surface ; tgt to contact surface ; opposes slipping or impending slipping
- friction due to surf irregularities and partly due to molecular attraction

Coulomb Friction

↳ (contact (force) between dry surfaces)

Laws : (1) f is indep of contact area's magnitude. (2) sliding friction force is indep of rel. vel of surfaces in contact when rel. vel is low. (3) $F_{Total} = \mu N$ (μ is μ_s or μ_d) for impending or actual motion.

Simple Surface Contact Problems.

e.g. plane area contact or small area of contact (i.e. $A = \Delta A$)
- plane surf. of contact
- impending / actual motion in same dir for all surf. area elements (i.e., no impending / actual rot. betwn contact bodies)
- μ is uniform over contact surf.

Complex Surf Contact Probs. { when these condts are violated, we consider area elements dA (for which they still hold good) & integrate over contact area.

TYPES OF PROBLEMS

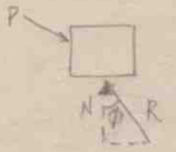
(1) Given : motion impending in certain (given) direction, & μ_s – Find : mag & dir of one of the applied forces
Method : take dir of F (i.e. $F_m = \mu_s N$) opposite to motion dir. & proceed [motion dir. is impending or actual motion dir].

(2) Given : motion impending & given all applied forces – Find : μ_s. Method : find F, N from FBD & equate F to $\mu_s N$
Here we can take any dir for F and the sign will tell whether assumed dir is correct or not

#

omit →

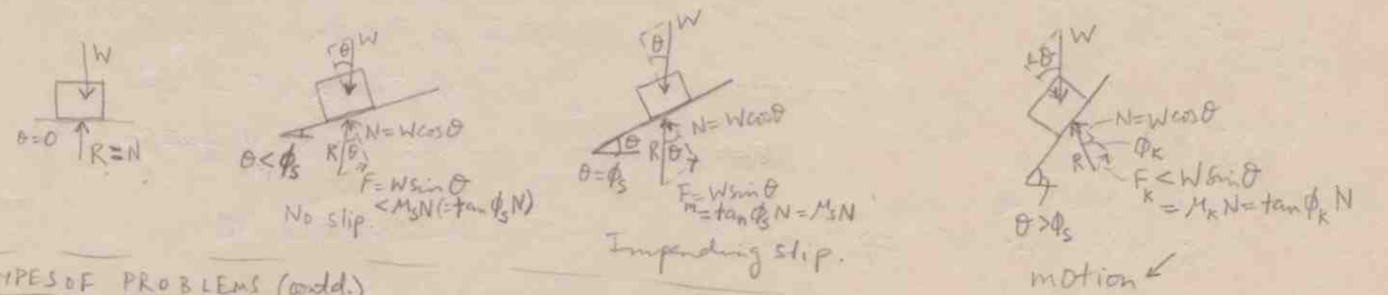
Angles of Friction :



$\tan\phi_s = \mu_s$, $\tan\phi_k = \mu_k$

no friction : $\phi = 0$; impending motion : $\phi = \phi_s$ = angle of static friction

no motion : $\phi < \phi_s$; motion : $\phi = \phi_k$, $\phi_k < \phi_s$ → " " kinetic "



TYPES OF PROBLEMS (contd.)

(3) Given : applied forces acting & μ_s – Find : whether equilibrium is maintained

1st Method : Assume impending motion in various possible directions, solve for ext forces required for such impending motion(s). Then compare the ext forces obtained (for the various directions of impending motion) with actual applied loads & decide whether equil maintained or not

2nd method : use FBD to solve for F. Then compare it with F_m (= $\mu_s N$). If $F < F_m$ then no slip, else slip & $F = \mu_k N$.

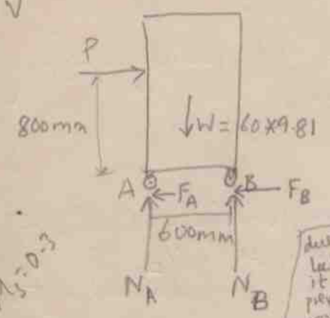
NOTE : For finite contact surface, we do not know line of action of resultant N since we dont know distribution of N over contact surf. Thus we cant take moments (on Σ Forces). However, for point contact cases we can do ΣM ∵ line of action of N & F pass thru contact pt.

NOTE : For rel motion between 2 surfaces, friction direction on surf A is opp to that of impending / actual motion of A as seen from B & vice-versa.
So use rel. motion dir. to get Friction dir

(dotted lines show rel. motion dir for A, B)

B&J P. 8.18 $\simeq$ 8.19 of 3rd SI ed.

√



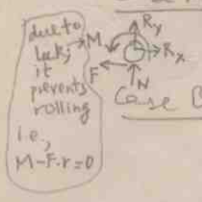
ΣF_y: $N_A + N_B = W$

ΣF_x: $P - (F_A + F_B) = 0$

ΣM_A: $P(800) + W(300) - N_B(600) = 0$

Find P reqd to move cabinet to right. (Type I problem)

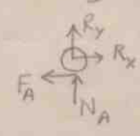
Case A: Both wheels locked



$F_A + F_B = \mu_s(N_A + N_B) = \mu_s W \Rightarrow P = \mu_s W = 176.6\,N$ ◀

Case B: Wheel B locked, A free to rotate

$F_B = \mu_s N_B$



for small wheel $(I_A = 0)$ ΣM: $F_A \cdot r = \cancel{I_A}^{0} \alpha \Rightarrow F_A = 0$

$$P = \frac{W(300)}{\frac{600}{\mu_s} - 800} = 147.2\,N$$ ◀

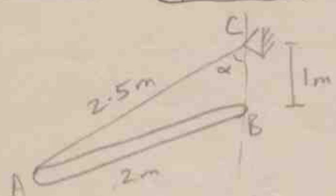
Case C: wheel A locked, B free to rotate

$F_A = \mu_s N_A$, $F_B = 0$ (by same reasoning as case A)

$$\therefore P = \frac{-W(300) + W(600)}{800 + \frac{600}{\mu_s}} = 63.1\,N$$ ◀

B&J P. 8.38 (Type 3 probl).

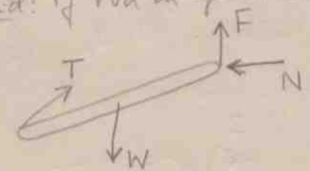
√



$\mu_s = 0.35$, $\mu_k = 0.25$, $W = 10\,kg * g$

Find: if rod in equil and mag & dir of F.

$$\alpha = \cos^{-1}\left[\frac{2.5^2 + 1^2 - 2^2}{2(2.5)(1)}\right] = 49.46°$$



$$\left\{\begin{array}{l} \Sigma F_x: T\sin\alpha - N = 0 \\ \Sigma F_y: T\cos\alpha + F - W = 0 \\ \Sigma M_C: N(1) - W(\tfrac{1}{2} * 2.5 * \sin\alpha) = 0 \end{array}\right.$$

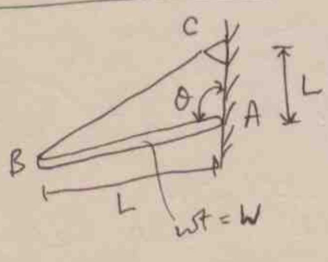
$\therefore N = 93.19$ Newt

$T = 122.625$ Newt

$F = 18.4$ Newt ◀

$F_{max} = \mu_s N = 32.6165 \Rightarrow F_{max} > F$ so equil maintained ◀

B&J 3rd ed 8.32

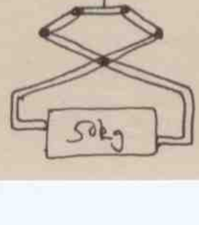


$\mu_s = 0.4$, $\mu_k = 0.3$

Find (a) value of θ for impending motion

(b) corresponding value of tension.

B&J 3rd ed 8.38.

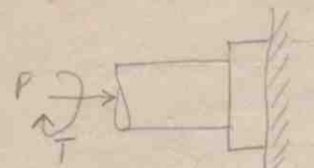


Find least μ_s between blocks and tong so as to be able to lift block. All geometry known.

omplex Contact Surface Friction Problems

(I) Disk Friction – Thrust Bearing

— (used in disk clutches, axial support for rotating shafts & axles)



$dN = \dfrac{P}{\pi(R_o^2 - R_i^2)} \, r\,d\theta\,dr$

$dF = \mu_k dN$

$(dM)_F = \dfrac{P\mu_k}{\pi(R_o^2-R_i^2)} \, r\,d\theta\,dr * r$

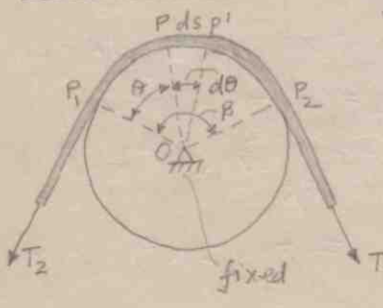
$\Rightarrow M_F = \dfrac{P\mu_k}{\pi(R_o^2-R_i^2)} \displaystyle\int_0^{2\pi}\int_{R_i}^{R_o} r^2\,dr\,d\theta$

$M_F = \dfrac{2}{3}\dfrac{P\mu_k}{(R_o^2-R_i^2)}(R_o^3 - R_i^3)$

M_F is moment due to frictional resistance. Equilibrium of shaft requires that torque $T (= M_F)$ should be applied to overcome M_F.

When shaft is solid ($R_i = 0$), $M_F = \frac{2}{3}P\mu_k R_o$. This is equivalent to the case where shaft has a point contact with bearing @ dist. of $\frac{2}{3}$ its radius from its axis.

(II) Belt Friction



- Assume stationary (fixed) drum; flexible (flat, massless) belt; impending motion (between belt & drum) is clockwise relative to drum, i.e $T_1 > T_2$
- β is ∠ of wrap in radians; β can be $> 2\pi$.

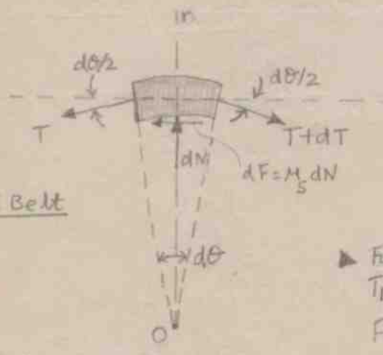
FBD of infinitesimal segment of belt:

$\Sigma F_t : -T\cos\frac{d\theta}{2} + (T+dT)\cos\frac{d\theta}{2} - \mu_s dN = 0 \Rightarrow dT\cos\frac{d\theta}{2} = \mu_s dN \Rightarrow dT = \mu_s dN$ — ①

$\Sigma F_n : -T\sin\frac{d\theta}{2} - (T+dT)\sin\frac{d\theta}{2} + dN = 0 \Rightarrow -2T\sin\frac{d\theta}{2} - dT\sin\frac{d\theta}{2} + dN = 0 \Rightarrow T\,d\theta = dN$ — ②

From ①, ②:

$\frac{dT}{T} = \mu_s\,d\theta \Rightarrow \displaystyle\int_{T_2}^{T_1} dT/T = \int_0^{\beta}\mu_s\,d\theta \Rightarrow \boxed{\ln(T_1/T_2) = \mu_s\beta \Rightarrow T_1/T_2 = e^{\mu_s\beta}}$ → ③



FBD of Belt

- ▶ If we neglect centrifugal effects the same rel. can be used for: Rotating drum with impending belt-drum slippage. Rotating / stationary drum with belt-drum slippage by replacing $\mu_s \to \mu_d$
- ▶ $T_1 > T_2$, i.e, T_1 [T_2] is tension in that part of the rope/belt which pulls [resists]
- ▶ For impending / actual slippage the ratio of T_1/T_2 is indep of F (it only depends on μ_s, β)

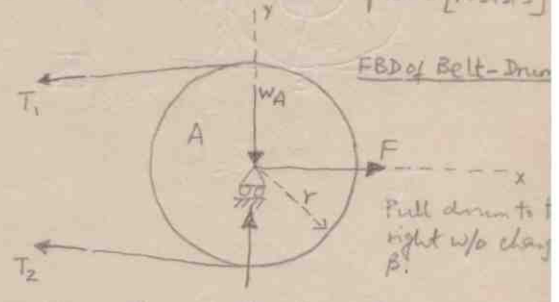
From FBD of belt-drum:

torque developed by belt on drum $= (T_1 - T_2)r$ → ④

$(T_1)_x + (T_2)_x = F_{(known)}$ → ⑤

So we can find T_1, T_2 from ③,⑤ & then the torque dev. by belt on drum from ④

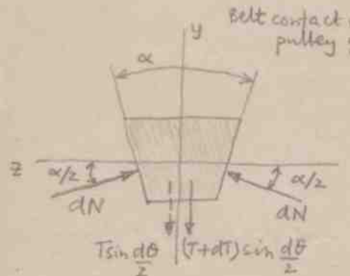
This torque depends on F, ∵ $(T_1 - T_2)$ depends on F even though their ratio does not.

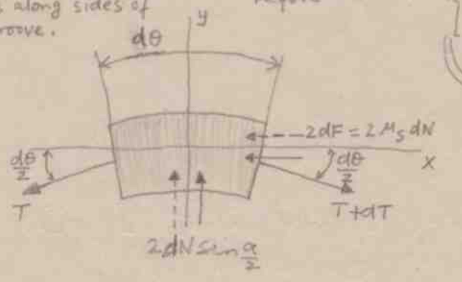


Belt Drives

Assumptions as before

Belt contact is along sides of pulley groove.





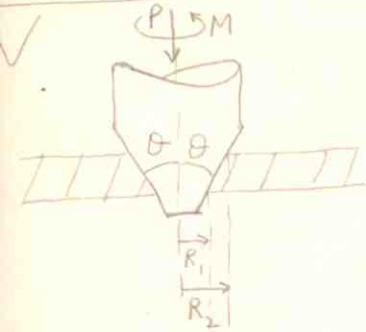
$\Sigma F_y : 2dN\sin\frac{\alpha}{2} - 2T\sin\frac{d\theta}{2} - dT\sin\frac{d\theta}{2} = 0 \Rightarrow dN\sin\frac{\alpha}{2} - T\frac{d\theta}{2} = 0$

$\Sigma F_x : dT\cos\frac{d\theta}{2} - 2\mu_s dN = 0 \Rightarrow dT - 2\mu_s dN = 0$

$\Rightarrow \frac{dT}{T} = \frac{\mu_s}{\sin\alpha/2}\,d\theta \Rightarrow \boxed{T_2/T_1 = e^{[\mu_s\beta/(\sin\alpha/2)]}}$ → ⑥

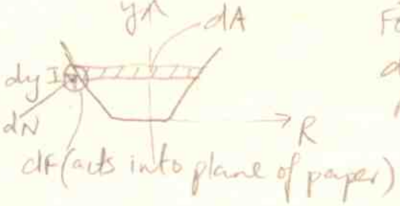
NOTE: (3, 6) only valid for impending/actual slippage. When no slippage & no impending slippage, use first principles.

8-100 BRJ = 8.111 of 3rd SI ed.



Assume: uniform pressure distr on contact surfaces

Find M to overcome frictional resistance for conical ~~pivot~~ bearing shown



Equivalent system:

For all strips (dA) we see that inclination of dN, dF with y axis is the same. Moreover ∵ only friction provides resisting moment about y axis & its moment arm is the same at all pts on a strip, we can replace friction by a single dF for the strip. (Also note that due to symmetry x comp of normal reaction at diametrically opp pts on strip cancel each other).

If pressure (normal force per unit area) is uniform over contact surface, then vertical comp of normal force per unit area is also uniform over contact area (∵ θ is const over contact area)!

$$\therefore dN\sin\theta = \frac{P \cdot 2\pi R\,dy}{(\text{contact area})} = \frac{2\pi P\,R\cot\theta\,dR}{\pi(R_2^2-R_1^2)\cot\theta}$$

contact area $= \int dA = \int 2\pi R\,dy$

$y = \cot\theta R - \cot\theta R_1$

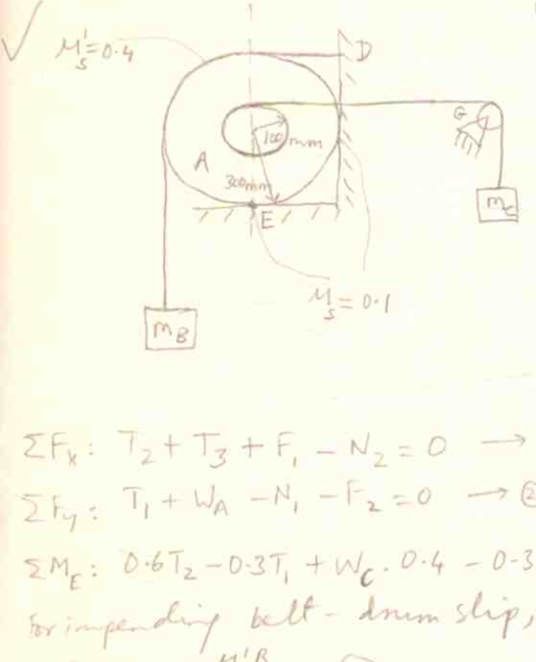
$$A = \int_{R_1}^{R_2} 2\pi R\cot\theta\,dR = \pi(R_2^2-R_1^2)\cot\theta$$

$dF = \mu\,dN$; $dM = dF \cdot R$

$$\text{So } M = \int dF \cdot R = \frac{2P\mu}{\sin\theta(R_2^2-R_1^2)}\int_{R_1}^{R_2} R^2\,dR = \frac{2}{3}\frac{P\mu}{\sin\theta}\frac{R_2^3-R_1^3}{R_2^2-R_1^2}$$

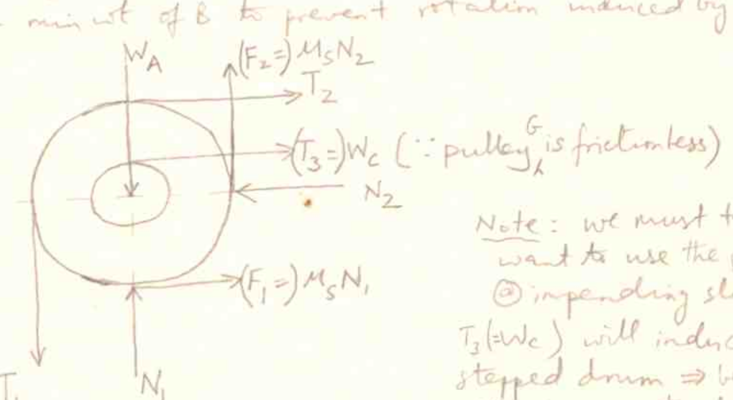
(use $\mu = \mu_s[\mu_d]$ if M to initiate [maintain steady] motion is reqd).

7.68 Shames = 7.73 of 4th ed.



Given: $W_C = 500N$, $W_A = 100N$, G is frictionless pulley

Find: min wt of B to prevent rotation induced by C.



Note: we must take $T_1 > T_2$ if we want to use the form $(T_1/T_2) = e^{\mu'\beta}$ @ impending slip. This is because $T_3 (= W_C)$ will induce CW rot of stepped drum ⇒ belt BD about to slip CCW wrt drum.

ΣF_x: $T_2 + T_3 + F_1 - N_2 = 0 \rightarrow$ 1

ΣF_y: $T_1 + W_A - N_1 - F_2 = 0 \rightarrow$ 2

ΣM_E: $0.6T_2 - 0.3T_1 + W_C \cdot 0.4 - 0.3(N_2 + F_2) = 0 \rightarrow$ 3

For impending belt-drum slip,

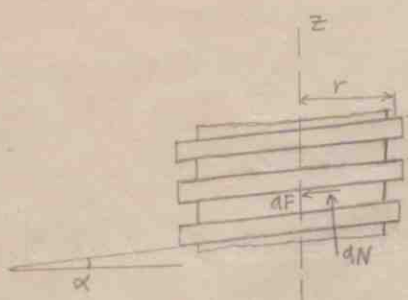
$T_1/T_2 = e^{\mu_s'\beta} \rightarrow$ 4

Solve 1–4 for T_1, T_2, N_1, N_2 (eliminate N_1 from 1, 2, subst expr of N_2 in 3 & in the resulting eqn subst for T_2 from 4 & get T_1)

& get $T_1 = W_B = 178.5N$

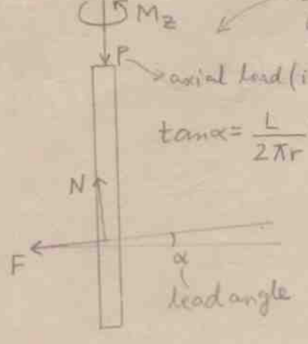
L11, 12, 13

III Square-Threaded Screw



r: mean radius from centerline to thread
p: pitch – dist along screw axis between adjacent threads
L: lead – dist the nut advances (along axis) per revolution
Note: $L = np$ for n-threaded screw (multi-threaded screws have several (n) independent threads)

Note: No forces transmitted from screw to nut along walls of screw threads. (i.e, no contact along walls)
Assume: due to narrow width of threads, that F, N distribution along thread is confined @ a dist. r from axis.



Limited Static Equivalence of distributed F, N with conc. F, N :-
- all $\underset{\sim}{dF}$'s, $\underset{\sim}{dN}$'s have same dir. cosine wrt z-axis. Hence for ΣF_z we can replace F, N distribution by single conc. forces F, N anywhere along thread as shown
- ∵ all $\underset{\sim}{dF}$'s, $\underset{\sim}{dN}$'s act @ dist r from z-axis, i.e., they have same moment arm about z-axis (in addition to common inclination with z-axis), we can use conc. F, N forces when considering ΣM_z.
- cant use this eqvt system for $\Sigma F_x, \Sigma F_y$ – i.e, we'd have to consider the actual F, N distr.

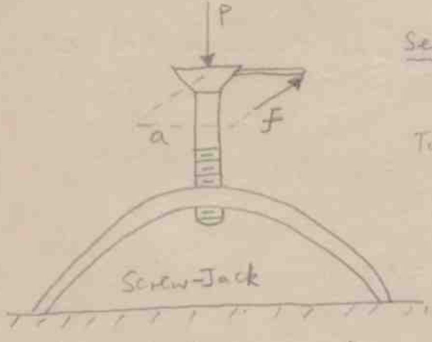
P → axial load (i.e, due to screw wt, wt of loads that screw-jack supports)

$$\tan\alpha = \frac{L}{2\pi r} = \frac{np}{2\pi r}$$

Equilibrium @ impending motion:

$\Sigma F_z: \; -P + N\cos\alpha - \mu N\sin\alpha = 0$; $\Sigma M_z: \; -\mu N\cos\alpha \cdot r - N\sin\alpha \cdot r + M_z = 0$

Eliminating N → $\boxed{M_z = \dfrac{Pr(\mu\cos\alpha + \sin\alpha)}{\cos\alpha - \mu\sin\alpha}}$ → for steady rotation of screw (i.e, to maintain steady rot) use μ_d. For impending motion use μ_s (i.e. to start motion)



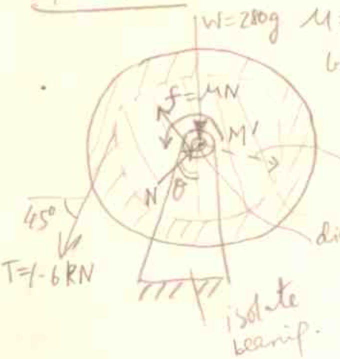
Self Locking device: If f removed after raising load P, and load remains in position (i.e, screw does not unwind)
To check if self locking or not → put $M_z = 0$, assume impending unwinding (i.e, reverse direction of F) and solve for $(\mu_s)_{min}$.

$$\frac{Pr(-[\mu_s]_{min}\cos\alpha + \sin\alpha)}{\cos\alpha + [\mu_s]_{min}\sin\alpha} = 0 \Rightarrow [\mu_s]_{min} = \tan\alpha$$

If $\mu_s < [\mu_s]_{min}$ → self unwinding → we require f (i.e, M_z) to maintain position of load.
If $\mu_s > [\mu_s]_{min}$ → self locking → we require some reverse torque M_z' to lower load – this is computed by assuming impending slippage in reverse dir. (i.e, reverse F by reversing sign of μ_s terms) & compute M_z' in the same manner.

Note: When M_z is applied by means of f, we dont need to bother about $\underline{f}$ as a force, i.e, only its contribution to M_z is important (i.e, in the eqvt force-couple system ($\underset{\sim}{f}$, M_z) acting at the screw axis (z) we can forget about f). This is because the horizontal (x, y) components of $\underline{dF}$, $\underline{dN}$, when added, will resist $\underline{f}$. This is further evident from the fact that the ΣM_z, ΣF_z eqns are not affected by $\underline{f}$. Thus the effect of $\underline{f}$ is only to redistribute the friction/normal forces (distributions) in such a way that their total z-component & total magnitude (N) is unchanged & their total horz-comp resists $\underline{f}$.

6/74 (MK 2nd ed) old → 111^ar to 6/80 of MK 3rd ed



W=280g μ = 0.3 for bearing

bearing dia = 62.5 mm

Find M' to turn telephone cable reel slowly ccw.

$\Sigma F_x: \ -T\cos 45 - \mu N \cos\theta + N\sin\theta = 0 \ \longrightarrow (1)$

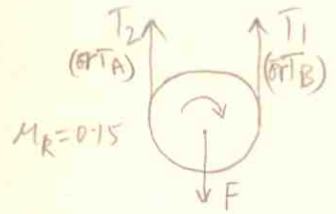
$\Sigma F_y: \ -W - T\sin 45 + N\cos\theta + \mu N \sin\theta = 0 \ \longrightarrow (2)$

$\Sigma M_{center}: \ -T(0.55) + \mu N\left(\frac{0.0625}{2}\right) - M' = 0 \ \longrightarrow (3)$

$(1)^2 + (2)^2: \ N^2(1+\mu^2) = (1.6\times10^3\cos45)^2 + (280\times9.81 + 1.6\times10^3\times\sin45)^2 \Rightarrow N = 3869.45\ N$

from (3), $M' = -843.72$ N.m $= 843.72$ N.m

8.118 (BJ) Given: T_1, T_2 are each 75 N when flywheel @ rest. Find T_1, T_2 when it rot c.w @ const spd



$F = (T_1 + T_2)$ = const (doesn't change whether flywheel @ rest or rot.)

$= 75 + 75 = 150$ N

When motion occurs so does slip.

$\therefore \ T_1/T_2 = e^{0.15\pi}$

Also $T_1 + T_2 = F = 150 \ \Rightarrow \ T_2\left(1 + e^{0.15\pi}\right) = 150$

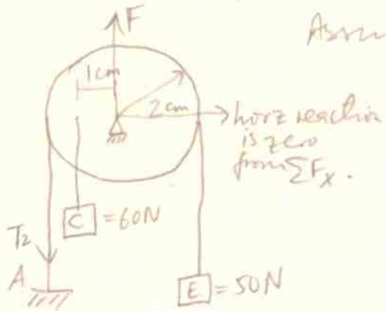
$\Rightarrow T_2 = 57.65$ N

$T_1 = 92.35$

7.56 Shames

Find min coeff of friction between rope-&-drum to maintain equil

Assume slip impending.



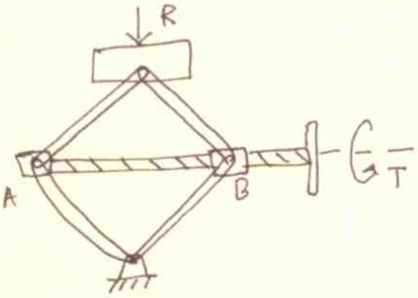
$\ln(T_1/T_2) = \mu_s \beta_{=\pi}$

$\Sigma M_{center}: \ (T_2 - 50)(2) + 60\times1 = 0$

$T_2 = \frac{40}{2} = 20$ N

$\therefore (\mu_s)_{min} = 0.2916$

Shames 4th ed, p.326, Prob 7.98.

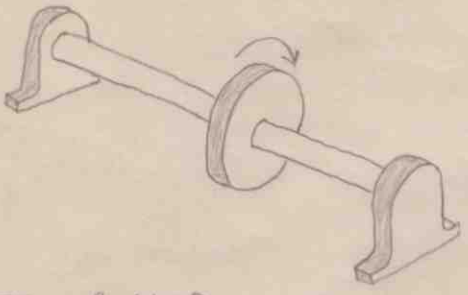


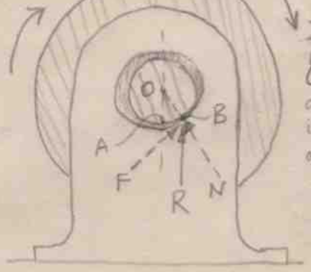
Given: scissors jack lifting car. 'A' is merely a bearing, 'B' is a threaded collar (nut). Single threaded screw with pitch p = 3mm, mean dia = 2r = 20mm, $\mu_s = 0.3$. Find T for $\theta = 45°$ and $60°$.

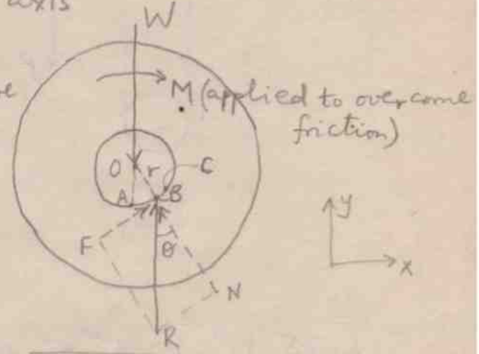
IV) Journal Bearings – Axle Friction (provide lateral support to rot. shafts/axles)

- For lubricated bearing, friction is viscous type & depends on axle-bearing clearance, speed of rot, viscosity of fluid – won't be studied here
- Assume direct contact of axle & bearing along single straight line (i.e, dry friction for case of unlubricated/partially lubricated bearing)

B is point of contact, i.e, intersection of line of contact with plane $\perp^{ar}$ to shaft axis







$\Sigma F_y:\ R - W = 0$

$\Sigma M_o:\ \boxed{M - Rr\sin\theta = 0}$ – θ is θ_B or θ_C

Note: $F = \mu N \Rightarrow \tan\theta = \frac{F}{N} = \mu \Rightarrow \boxed{\theta = \tan^{-1}(\mu)}$ – θ is θ_B or θ_C

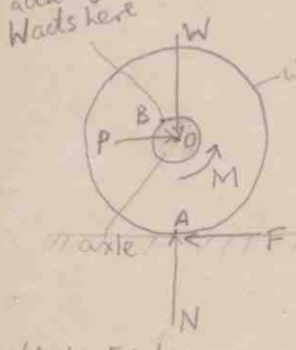
- when axle @ rest contact is at point A. When it is set in motion it climbs in the bearings until slippage occurs $(F = \mu_s N)$ at C. After slippage it slides back slightly, finally settling @ B $(F = \mu_d N)$
- Thus To find $\theta_B[\theta_C]$ use $\mu_d[\mu_s]$. To find M to be applied to maintain rotation [to cause slip] use $\theta_B[\theta_C]$, i.e, $\mu_d[\mu_s]$. Generally we are interested in θ_B & M reqd to maintain rotation.
- In the FBD if we wish to draw R thru pt. A then we should add a ccw couple ($= Rr\sin\theta$) representing the frictional resistance of the bearing. This is obvious (i.e, eqvt force-couple system, @ A, of R acting @ B)

Wheel Friction – Rolling Resistance

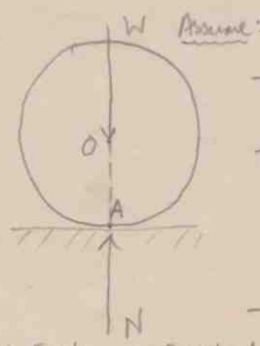
- Assume wheel rolls w/o slipping & wheel is rigid
- Resistance due to (1) Axle & rim friction (2) ground and/or wheel deforms so contact takes place over certain area instead of at-a-point.

(1) Axle + Rim friction

Consider FBD of wheel-axle mounted on bearing. The bearing is attached to (i.e, it supports the) load that is to be moved – e.g., railroad car.



w/Axle Friction



w/o Axle Friction – i.e Free Wheel

Assume: Wheel – axle (& car) moves to right @ constant speed

- W is reaction from bearing (& hence equals wt of the car if we assume no transverse motion).
- However W does not act thru centre O but instead @ a pt. to the left (see fig). Thus if we draw W thru O we must add the couple M (W * horz dist between B & O) that represents frictional resistance moment from bearing.
- For equil we need M balanced by couple due to P & F (P = F for equil), where F is friction @ rim from track & P is force that

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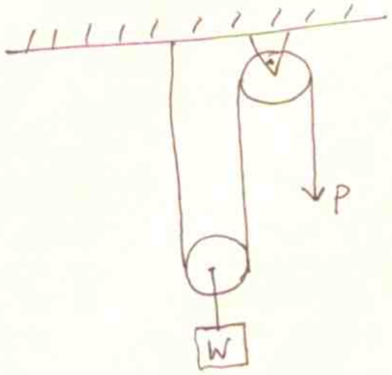

Given: $W = 153\,kg$, pulley dia $= 60\,mm$, axle dia $= 10\,mm$

$\mu_s = 0.2$.

Find: Tension in each portion for slowly raising the load.

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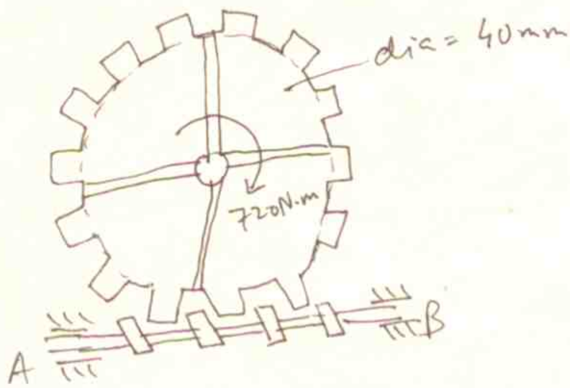


Given: Square threaded worm gear.

$r = 30\,mm$, $L = 7.5\,mm$.

$\mu_s = 0.12$

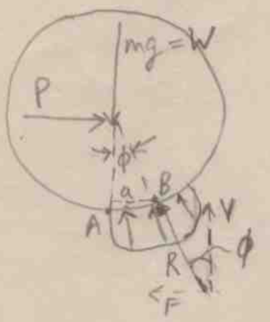
Find: Torque to be applied to shaft AB to rotate large gear CCW.

should be applied to wheel to keep it rolling @ uniform speed.

- For frictionless track → $F=0 \Rightarrow P=0 \Rightarrow M=0$, i.e, wheel slides on track w/o turning in bearing
- For frictionless bearing, i.e, no axle friction → $M=0 \Rightarrow P=F=0$. This is the case of a wheel that's not held in bearings & rolls freely @ const. speed.. No friction force will act on wheel regardless of μ between wheel & ground. Thus this freely rolling wheel will roll indefinitely.

(2) Rolling Resistance

- However, experimental evidence indicates that free wheel cannot roll indefinitely (i.e, eventually it stops). This is due to rolling resistance. For resistance we must have resisting force (F) ← (i.e, opp to motion dir.). Furthermore since F should not produce a C.W. moment (as this implies α is c.w which means ω increases) the resultant contact force R should pass thru wheel centre (note: F is x-comp of R).
  Thus P is reqd to overcome F & maintain steady motion



$$\Sigma M_B: Wa - Pr\cos\phi = 0 \Rightarrow \frac{P}{W} = \frac{a}{r\cos\phi} = \tan\phi$$

for small ϕ (which is true based on observations that contact area is small)

$\boxed{P/W = a/r}$ a → coeff of rolling resistance
→ not an actual coeff but has dim of length

Note: (i) The distribution of contact force over a finite surface is an exptal observation
(ii) P is reqd horz force to overcome x-comp of R in order to maintain steady motion
(iii) For steady motion, moment equil requires that R pass thru wheel centre (i.e, 3 non-parallel forces must be concurrent).
(iv) • Coulomb suggested that for fixed materials & r, the ratio $P/W = \text{const} \Rightarrow a$ is const for fixed materials & r.
• Also suggested that P/W varies inversely as r for fixed materials
Thus if we keep W const, $P/W \propto \frac{1}{r}$, i.e, for W fixed, $P\downarrow$ as $r\uparrow$
Also we conclude that a is fixed for all sizes (r) & loads (W) for roller of fixed material.

both these are strongly contested by investigators, especially the latter one.