Department Civil Engineering
Indian Institute of Technology Bombay

CE 102 Engineering Mechanics: MID-SEMESTER EXAM

Date: February 20, 2007 **Time: 9:30 – 11:30 hours** **Max. Marks: 30**

Note: Answer all questions. Assume suitable data, if required, and state the same clearly.
Draw Free Body Diagram(s) clearly.

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1. The mechanism shown in **Fig. 1** is acted upon by the force *P*; derive an expression for the magnitude of the force *Q* required to maintain the equilibrium. Collars *C* and *D* move on the horizontal rod without friction. **(7.5)**

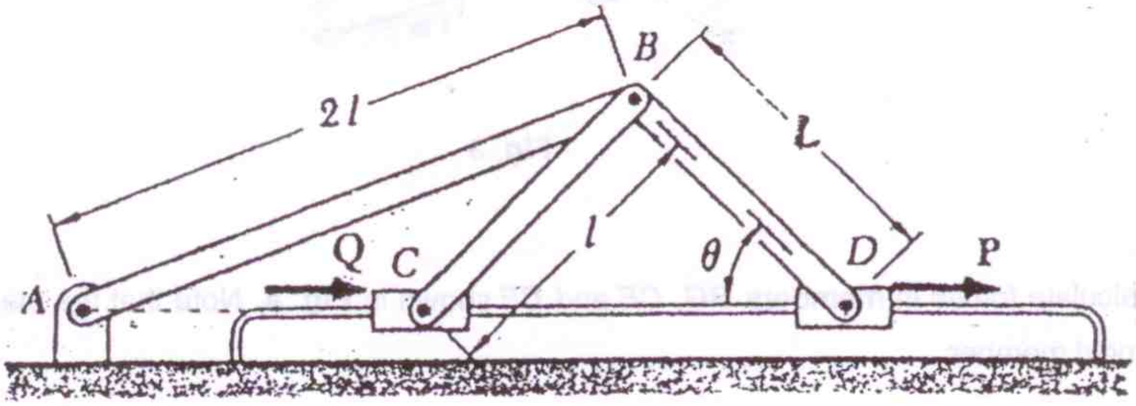

Fig. 1

2. Couple *M* is applied to pin *B* as shown in **Fig. 2**. Determine the reactions at the supports and the axial force in all the members (indicate tensile or compressive) when pin *B* is: (i) welded to *AB*, (ii) welded to *BC*. **(7.5)**

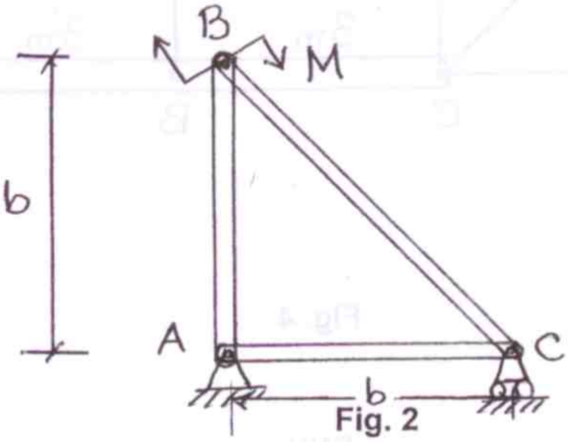

Fig. 2

3. The welded tubular frame shown in **Fig. 3** is secured to the horizontal *x-y* plane by a ball-and-socket joint at ***A*** and supported by a loose-fitting ring at ***B***. Under the action of ***2 kN*** load, determine the tension in the cable ***CD*** and the reactions at the supports ***A*** and ***B***. **(7.5)**

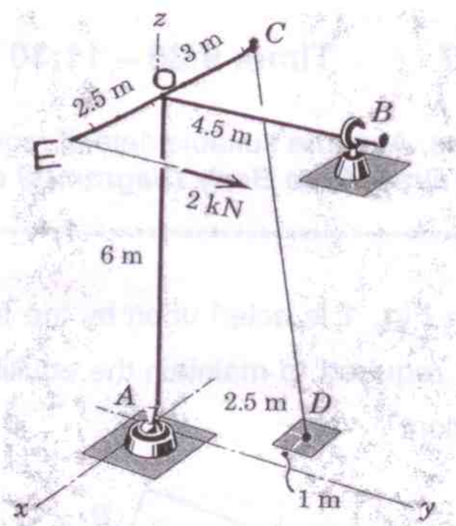

Fig. 3

4. Calculate forces in members ***BG***, ***CE*** and ***CF*** shown in **Fig. 4**. Note that the member ***ABC*** is a rigid member. **(7.5)**

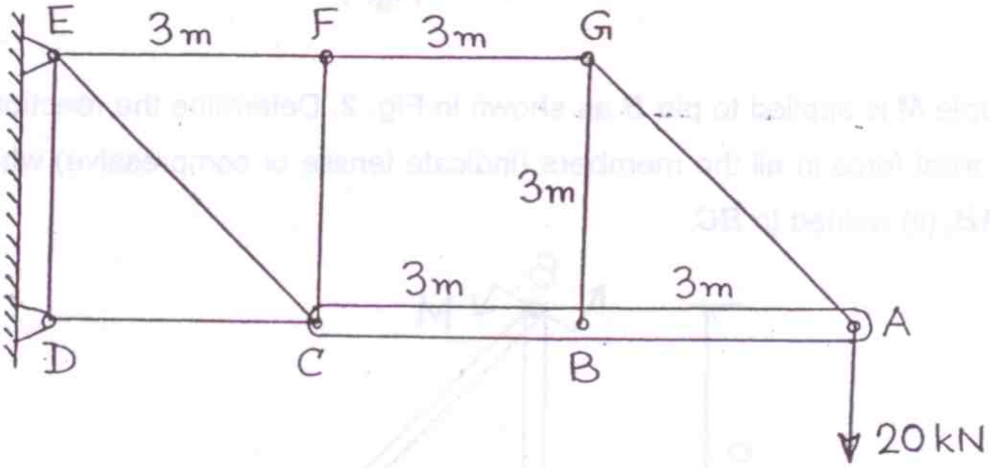

Fig. 4

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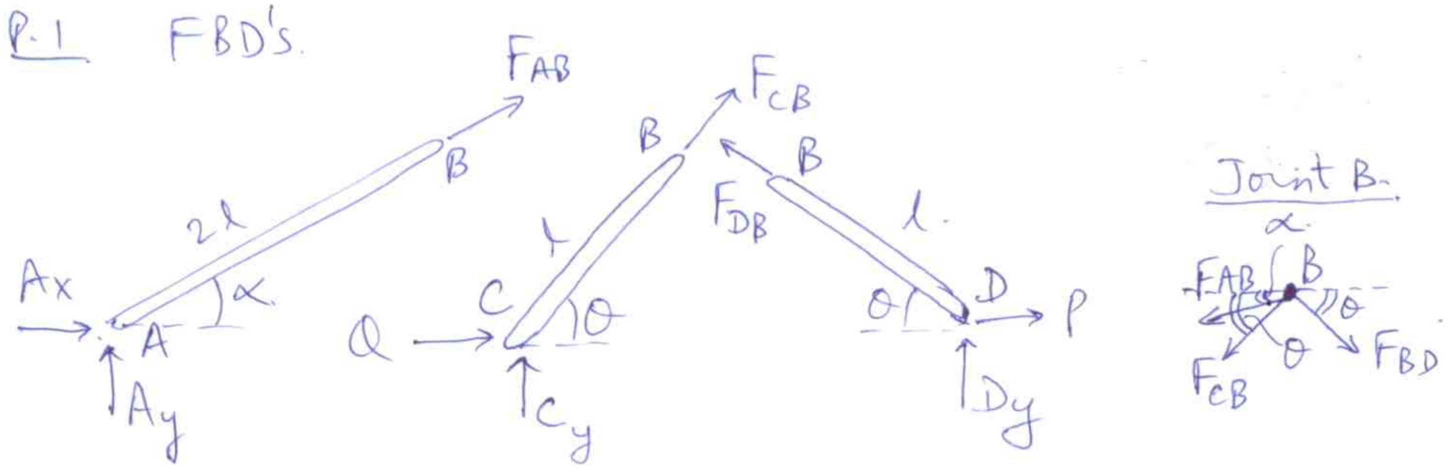
P.1 FBD's.



4 FBD's $\longrightarrow$ 8 equations (force balance eqns) $\longrightarrow$ 8 unknowns $(A_x, A_y, F_{AB}, F_{CB}, F_{DB}, Q, C_y, D_y)$

$\Longrightarrow$ Statically Determinate.

FBD of DB : $\Sigma F_x = 0 \Rightarrow F_{BD}\cos\theta = P \longrightarrow (1)$

FBD of Joint B : $\Sigma F_x = 0 \Rightarrow F_{BD}\cos\theta - F_{BA}\cos\alpha - F_{CB}\cos\theta = 0 \rightarrow (2)$

$\Sigma F_y = 0 \Rightarrow F_{BD}\sin\theta + F_{BA}\sin\alpha + F_{CB}\sin\theta = 0 \rightarrow (3)$

FBD of BC : $\Sigma F_x = 0 \Rightarrow Q + F_{CB}\cos\theta = 0 \longrightarrow (4)$

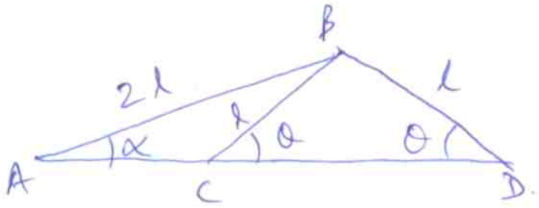
(1)–(4) are 4 eqns in 4 unknowns.

Solution of (1)–(4) :

$$(2) * \sin\alpha + (3) * \cos\alpha = F_{BD}\sin(\alpha+\theta) - F_{CB}\sin(\alpha-\theta) = 0 \longrightarrow (5)$$

$$(1)\ \&\ (5) \longrightarrow F_{CB} = F_{BD}\frac{\sin(\alpha+\theta)}{\sin(\alpha-\theta)} = \frac{P}{\cos\theta}\frac{\sin(\alpha+\theta)}{\sin(\alpha-\theta)} \longrightarrow (6)$$

$$(4)\ \&\ (6) \longrightarrow Q = -F_{CB}\cos\theta = -P\frac{\sin(\alpha+\theta)}{\sin(\alpha-\theta)} = \frac{P\sin(\alpha+\theta)}{\sin(\theta-\alpha)}$$

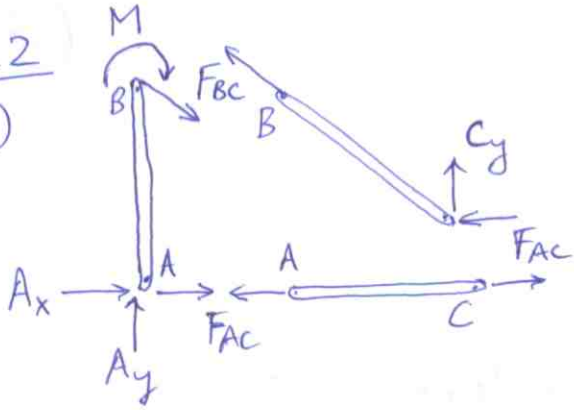


$$\frac{\sin\alpha}{l} = \frac{\sin\theta}{2l} \Rightarrow \alpha = \sin^{-1}\left(\frac{1}{2}\sin\theta\right)$$

$\theta > \alpha$. So Q is +ve as shown in FBD.

P.2

(i)



5 equations, 5 unknowns.

AB: $\sum F_x$: $A_x + F_{AC} + F_{BC}/\sqrt{2} = 0 \rightarrow (1)$

$\sum F_y$: $A_y - F_{BC}/\sqrt{2} = 0 \rightarrow (2)$

$\sum M_A$: $M + \frac{F_{BC}}{\sqrt{2}} \cdot b = 0 \rightarrow (3)$

BC: $-F_{BC}/\sqrt{2} - F_{AC} = 0 \rightarrow (4)$

$C_y + F_{BC}/\sqrt{2} = 0 \rightarrow (5)$

Solution:

$F_{BC} = -\frac{\sqrt{2}M}{b}$ (C)

$A_y = -M/b$

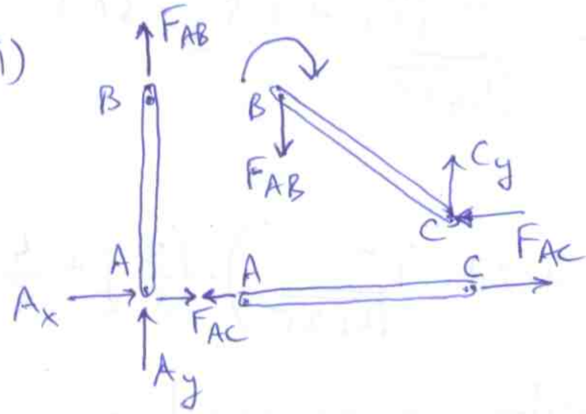
$F_{AC} = M/b$ (T)

$C_y = M/b$

$A_x = 0$

$H_{AB} = -A_y = \frac{M}{b}$ (T)

(ii)



5 eqns, 5 unknowns.

BC: $\sum M_c$: $-M + F_{AB} \cdot b = 0 \rightarrow (6)$

$\sum F_x$: $-F_{AC} = 0 \rightarrow (7)$

$\sum F_y$: $-F_{AB} + C_y = 0 \rightarrow (8)$

AB: $\sum F_y$: $F_{AB} + A_y = 0 \rightarrow (9)$

$\sum F_x$: $A_x + F_{AC} = 0 \rightarrow (10)$

Solution:

$F_{AB} = \frac{M}{b}$

$F_{AC} = 0$

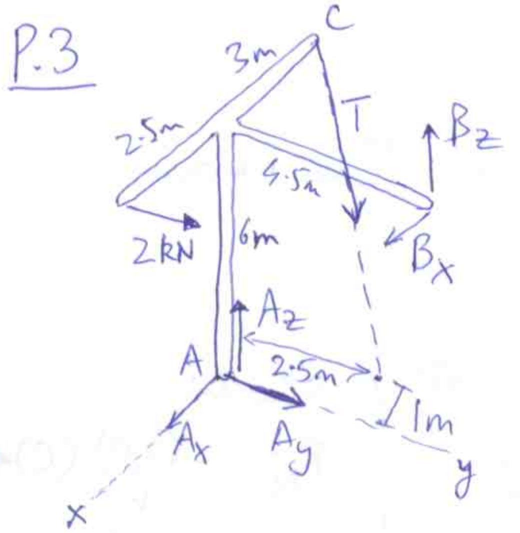
$C_y = \frac{M}{b}$

$A_y = -\frac{M}{b}$

$A_x = 0$

$H_{BC} = -\frac{F_{AB}}{\sqrt{2}} = \frac{-M}{b\sqrt{2}}$ (C)

P.3



Statically determinate 3-D problem. Unknowns are $A_x, A_y, A_z, B_x, B_z, T$ (6).

$$\underline{e}_{AB} = \frac{4.5\underline{j} + 6\underline{k}}{\sqrt{4.5^2+6^2}} = \frac{3}{5}\underline{j} + \frac{4}{5}\underline{k} \quad \text{(unit vector along AB).}$$

Take moments about line AB to get T.

$$\Sigma M_{AB} = 0 : \quad (2)(2.5)\underline{k}\cdot\underline{e}_{AB} + \underline{r}_{BC}\times\underline{T}\cdot\underline{e}_{AB} = 0$$

$$\underline{T} = \frac{(2\underline{i} + 2.5\underline{j} - 6\underline{k})T}{\sqrt{2^2+2.5^2+6^2}} = \frac{T(2\underline{i}+2.5\underline{j}-6\underline{k})}{\sqrt{46.25}}$$

$$\underline{r}_{BC} = -3\underline{i} - 4.5\underline{j}$$

$$\Rightarrow \left(5\underline{k} + (-7.5\underline{k} - 18\underline{j} + 9\underline{k} + 27\underline{i})\frac{T}{\sqrt{46.25}}\right)\cdot\left(\frac{3}{5}\underline{j} + \frac{4}{5}\underline{k}\right) = 0$$

$$\frac{T}{\sqrt{46.25}}\left(-\frac{54}{5} + \frac{6}{5}\right) = -4 \Rightarrow \boxed{T = 2.8336\,kN}$$

Now take moments about x & z axes passing thru A to get B_x, B_z

$$\Sigma M_{z\text{-axis}} = 0 : \quad -4.5B_x - \frac{2.5T}{\sqrt{46.25}}(3) + 2(2.5) = 0$$

$$\Rightarrow B_x = 0.4167\,kN$$

$$\Sigma M_{x\text{-axis}} = 0 : \quad B_z(4.5) - 2(6) - \frac{2.5T}{\sqrt{46.25}}(6) = 0$$

$$\Rightarrow B_z = 4.0555\,kN$$

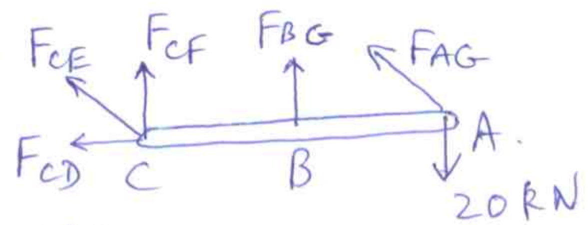
Sum forces in x, y, z directions to get A_x, A_y, A_z.

ΣF_x: $A_x + B_x + \frac{2T}{\sqrt{46.25}} = 0 \Rightarrow A_x = -1.25$ kN

ΣF_y: $A_y + 2 + \frac{2.5T}{\sqrt{46.25}} = 0 \Rightarrow A_y = -3.0416$ kN

ΣF_z: $A_z + B_z - \frac{6T}{\sqrt{46.25}} = 0 \Rightarrow A_z = -1.5555$ kN

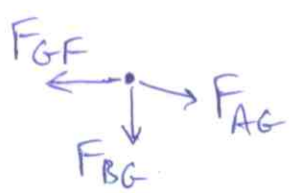
P.4



FBD ABC:

$\Sigma M_c = 0 \Rightarrow F_{BG}(3) - 20(6) + \frac{F_{AG}}{\sqrt{2}}(6) = 0 \rightarrow (1)$

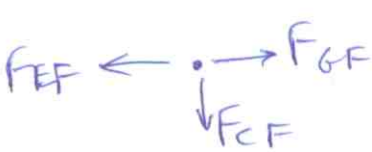
FBD Joint G:



$\Sigma F_y = 0 \Rightarrow \frac{F_{AG}}{\sqrt{2}} + F_{BG} = 0 \rightarrow (2)$

Solve (1), (2) $\longrightarrow$ $F_{BG} = -40$ kN (compressive).
$F_{AG} = 40\sqrt{2}$ kN (T).

FBD joint F:



$\Rightarrow F_{CF} = 0$

FBD ABC: $\Sigma F_y = 0 \Rightarrow \frac{F_{CE}}{\sqrt{2}} + \underbrace{\cancel{F_{CF}}}_{0} + \underbrace{\boxed{\cancel{F_{BG} + \frac{F_{AG}}{\sqrt{2}}}}}_{=0} - 20$ 0.

$\Rightarrow F_{CE} = 20\sqrt{2}$ kN (T)