STRUCTURAL MECHANICS

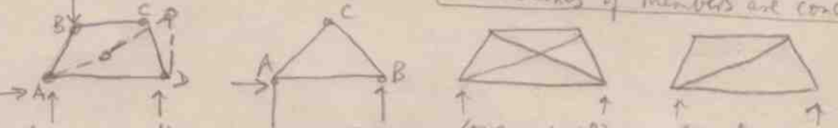
Trusses, Frames, Machines, Beams, Cables.

TRUSSES

- usually fully constrained & stationary.
- straight members connected @ joints @ two ends only ⟹ 2-Force members (i.e. to avoid buckling)
- assume → slender members that can't support lateral load. Thus loads applied @ joints only. When conc load to be applied betwn joints or distr load to be apply (eg. bridges) then use floor system that uses stringers & floor beams to transmit load to joints.
- assume → wt of members act @ joints only (distributed half-half @ the 2 jts. the member connects)
- assume → pinned joints (no momt rigidity). connection actually done through gusset plates (c.f. Shames). Ball-socket jt for 3-D case. This approx is valid if centrelines of members are concurrent @ connection
- usually members have uniform ⊗ section

Rigid v/s Non-rigid Truss

over-rigid: removal of member doesnt destroy rigidity
just-rigid: -do- does destroy rigidity



SIMPLE TRUSS
(are just-rigid)

eg 

▶ simple plane truss : $\boxed{m = 2j - 3}$

To basic Δ add 2 members to distinct existing jts, & connect these 2 mem's @ a new jt (i.e., add 2 mem's & 1 jt.).
Note that the 3 jts involved (ie 2 new mem's involved) should not be collinear else it'll become over-rigid.

eg basic tetrahaedron

▶ simple space truss : $\boxed{m = 3j - 6}$

To basic Δ add 3 members to distinct existing jts & connect these 3 mem's @ a new jt. (ie, add 3 mem's & 1 jt.)
Note The 4 jts involved (ie, 3 new mem's involved) must not be coplanar, else it'll become over-rigid

▶ Can be shown that $m = 2j-3$ & $m = 3j-6$ are valid in general for just-rigid (not necessarily simple) plane & space truss, resp'ly (not done in this course)

▶ note that simple ⇒ just-rigid. However rigid ⇏ simple (could be compound)

▶ In general we consider only Trusses having Stat Det Support Reactions (ie, externally SD) & complete constrainment.

Solution

▶ Determine external reactions (Note : R.B. Truss). This can be done whether or not truss is internally SD. CHECK Calculations @ this stage.

(A) Method of Joints

▶ FBD of joints drawn. Unkws $= m+3$; Eqns $= 2j$ for planar ; $\because m = 2j-3$
$= m+6$; Eqns $= 3j$ for space ; $\because m = 3j-6$
> we see that simple truss with SD supports are S.D (internally)

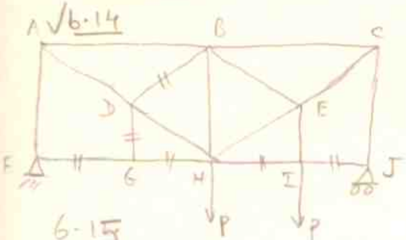
Note: eqns used to det. ext. reactions are not indep of these $2j$ eqs.

▶ The arrangement of pins, members in simple truss is : we can always find a pin [ball-socket] with only 2[3] unkws (eg last jt. formed, or support jt.'s after reactions are determined). Then just work backwards. This way solving many simultaneous eqns is avoided

(omit) ▶ Graphical method option – Maxwells Diagram.
Draw forces (concurrent) @ a jt. in tip-to-tail fashion in the order of encountering their lines of action when moving clock-or anticlock-wise around a jt.

P. 6-14 – 6.16 B&J Determine zero force members.

✓6.14



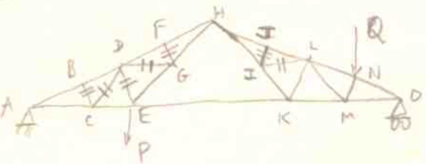
$G: F_{DG} = 0$

$D: F_{DB} = 0$

$\Sigma f_x = 0 \Rightarrow F_x = 0 \Rightarrow F_{FG} = F_{GH} = 0$

$J: F_{IJ} = 0 \Rightarrow F_{HI} = 0$

6-15



$B: F_{BC} = 0$

$C: F_{CD} = 0$

$F: F_{FG} = 0$

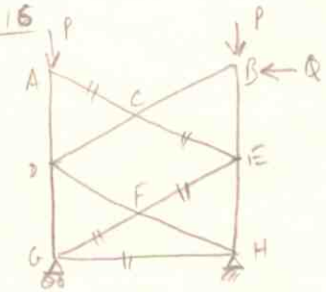
$G: F_{DG} = 0$

$D: F_{DE} = 0$

$J: F_{JI} = 0$

$I: F_{IL} = 0$

6.16



$A: F_{AC} = 0$

$C: F_{CE} = 0$

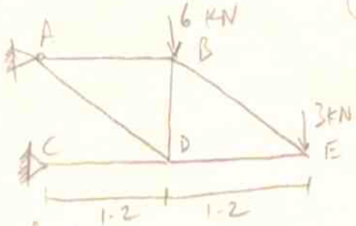
$E: F_{EF} = 0$

$F: F_{GF} = 0$

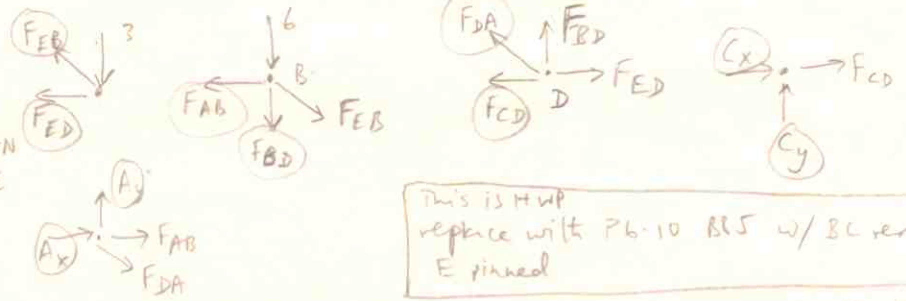
$G: F_{GH} = 0$

P. 6-6

use method of jts

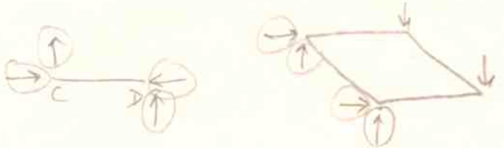


not simple truss

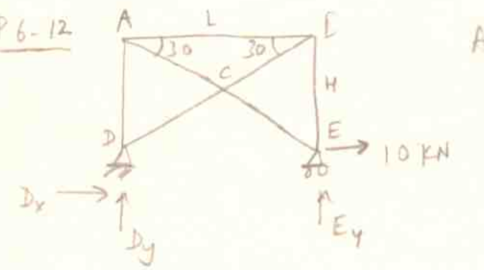


This is HWP
replace with P6-10 B&J w/ BC removed & E pinned

Note: Here $m \neq 2j-3$ but $m+r = 2j$. This is a truss that will collapse when removed from supports. So the alternate method is to isolate CD & ADEB as two bodies



✓P6-12



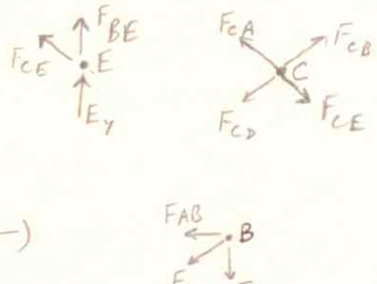
Analyze by method of jts.

Full Truss

$\Sigma M_D: E_y \times L = 0 \Rightarrow E_y = 0$

$\Sigma F_y: D_y + E_y = 0 \Rightarrow D_y = 0$

$\Sigma F_x: D_x + 10 = 0 \Rightarrow D_x = -10 = 10(\leftarrow)$



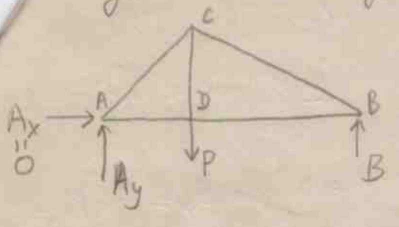
Jt. E $\Sigma F_x: -F_{CE}\cos 30 + 10 = 0$

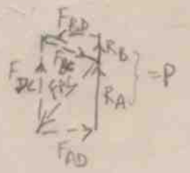
$\Sigma F_y: \cancel{E_y} + F_{BE} + F_{CE}\sin 30 = 0$ } solve F_{BE}, F_{CE}

Jt B $\Sigma F_x = \Sigma F_y = 0$
solve F_{AB}, F_{CB}

Jt C: solve for $F_{CA} = F_{CE}$, $F_{CD} = F_{CB}$; Jt A: solve for F_{AD}, check $\Sigma F_x = 0$; Jt D: check equil $\Sigma F_x = \Sigma F_y = 0$

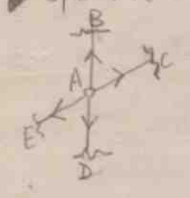
eg: Maxwells Diag





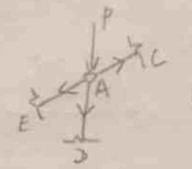
<u>Note</u>: Equations are better way than graphic sol.

- checks can be made along the way when considering support jts or intermediate jts.
- member in compr or tension. Write value followed by C or T on complete truss to help in book-keeping.
- Special cases:



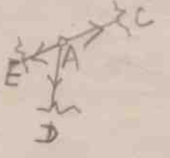
$F_{AE} = F_{AC}$

$F_{AB} = F_{BD}$



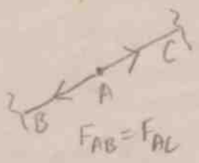
$F_{AD} = -P$

$F_{AC} = F_{AE}$

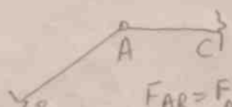


$F_{AE} = F_{AC}$

$F_{AD} = 0$

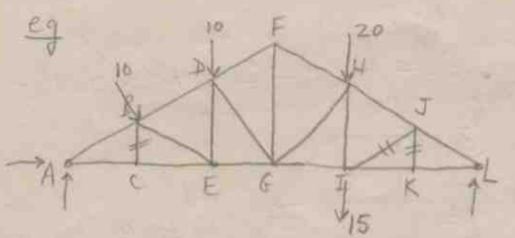


$F_{AB} = F_{AC}$



$F_{AB} = F_{AC} = 0$

<u>eg</u>



C: $F_{BC} = 0$; $F_{CA} = F_{CE}$

K: $F_{JK} = 0$; $F_{KI} = F_{KL}$

J: (resolve along IJ & $\perp$ar to it) $F_{JH} = F_{JL}$; $F_{IJ} = 0$

I: $F_{IH} = 15$ (member in tension) ; $F_{IG} = F_{IK}$

<u>Caution</u>: $\because$ 10 not along BE $\therefore$ $F_{BA} \neq F_{BD}$; $F_{BE} \neq -10$.
also $F_{DE} \neq 10$.

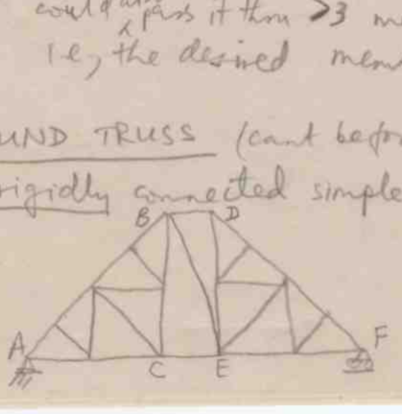
X In the beginning better to avoid quick analysis & draw FBD's of all jts whether or not they have the abovementioned special loading

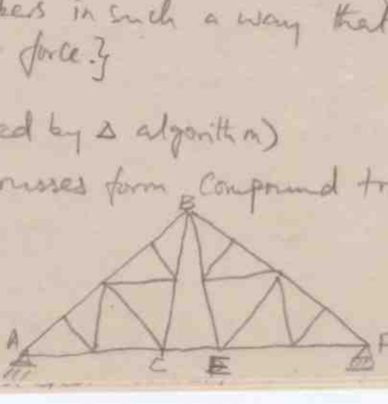
(B) <u>Method of Sections</u>

- useful if all member forces not reqd
- pass section thru members (generally 3) to isolate a part of truss as a RB: the member force desired now becomes an ext. force {In general pass section thru 3 members but sometimes could pass it through additional 0 force member(s). Sometimes could also pass it thru >3 members in such a way that equil eqns involve only one unknown i.e, the desired member force.}

<u>COMPOUND TRUSS</u> (cant be formed by Δ algorithm)

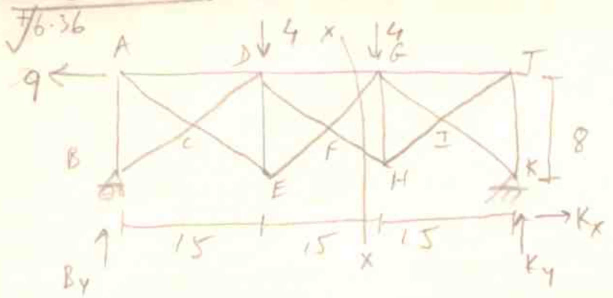
- <u>rigidly connected</u> simple trusses form compound truss.





remove either BD, BE, CE in either truss & it will rotate about A & F will translate → ie, become non rigid

P6-35 B&J
P6-36



Det force in AD, CD, CE (6-35) — HWP
— do — DG, FG, FH (6-36) — done here)

Full Truss

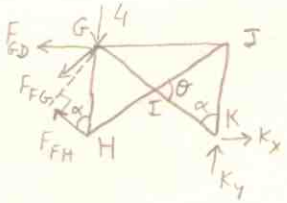
$\Sigma M_K: B_y(45) - 9(8) - 4(30) - 4(15) = 0 \Rightarrow B_y = 5.6(\uparrow)$

$\Sigma F_y: -4 - 4 + B_y + K_y = 0 \Rightarrow K_y = 2.4(\uparrow)$

$\Sigma F_x: K_x = 9(\rightarrow)$

$\theta = 2\tan^{-1}(4/7.5) = 56.15^\circ$

$\alpha = \frac{180 - \theta}{2} = 61.925^\circ$



$\Sigma M_F: F_{GD}(4) - 4(7.5) + K_x(4) + K_y(22.5) = 0 \Rightarrow F_{GD} = -15 = 15(C)$

$\Sigma M_G: -F_{FH} \times 8\sin\alpha + K_x(8) + K_y(15) = 0 \Rightarrow F_{FH} = 15.3 = 15.3(T)$

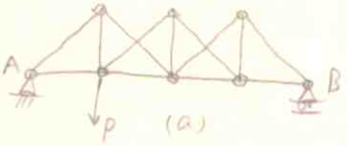
$\Sigma F_y: F_{FH}\cos\alpha - F_{FG}\cos\alpha + K_y - 4 = 0 \Rightarrow F_{FG} = 11.9 = 11.9(T)$

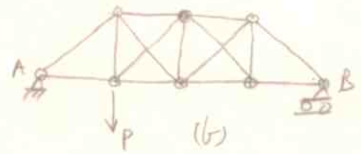
P6-46 B&J j = no.s of jts., m = no of members, u = no of unknowns, n = no. of eqs.

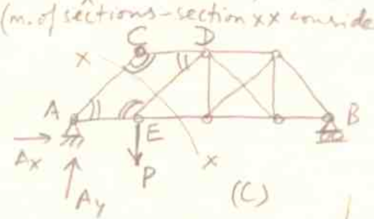
(a) Simple Truss, $m = 2j - 3$ $(m = 13, j = 8)$, $u = m + 3 = 16$, $n = 2j = 16$ → Completely constr, SD

(b) $m = 15$, $j = 8$, $u = m + 3 = 18$, $n = 2j = 16$ → Completely constr, SID ($\because u > n$)

(c) $m = 13$, $j = 8$, $u = m + 3 = 16$, $n = 2j = 16$ → Improperly constr (see below for reasoning)
However, consider jt. C → for given loading we get $F_{CA} = F_{CD} = 0$. Now $F_{CA} = 0 \Rightarrow A_y = 0 \Rightarrow \Sigma M_B \neq 0$ & $\Sigma M_A \neq 0$
If instead load was appd @ C then $F_{DE} = 0 \Rightarrow \Sigma M_A \neq 0$ (m. of sections - section xx considered). In general the truss will rotate about A & B,





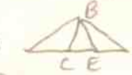


jt. E will translate (↕)
jt B will transl (→)
jt C will transl (↕)

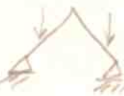
members CD, CA, AE will rotate wrt each other i.e, parallelogram ACDE will not stay rigid

- nonconcurrent, non-parallel, connecting links, to maintain rigidity(?) [B&J p.256].
- For pin-roller (or eqvt) supported, compound trusses (shown on previous pg), are SD (ext & int) & completely constrained (also rigid which is implied in 'compound')

  Here $\boxed{m = 2j - 3}$

- Add cross-member CD to existing comp. truss . Becomes overrigid (still compound truss); $m > 2j-3$; One of 4 members (labelled) is redundant. {Unkw = $m+3$} > {eqs = $2j$} & hence SID.
- Remove CE in  truss. Its non-rigid (hence not compound truss) & will collapse. {Unkw = $m+3$} < {eqs = $2j$} ⇒ all eqns not satisfied, ie., equil violated
- Remove CE in  & replace right roller with pin. Truss is rigid. {Unkw = $m+4$} = {eqs = $2j$}.

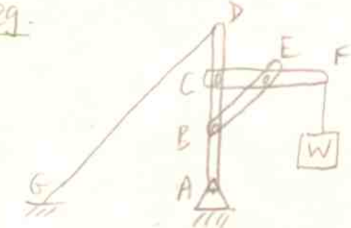
  Necessary condt for comp. truss to be SD, rigid, completely constr is $\boxed{m+r=2j}$ where r = nos of support reactions. This condt is not sufficient for structure that looses rigidity when detached from supports, e.g:

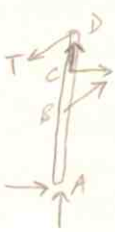


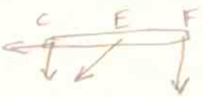
FRAMES

- have at least one multiforce member.
- fully constrained (usually).

eg.







Entire frame: ($\Sigma M, \Sigma F$) ⇒ find $\underline{A}$, T

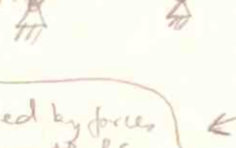
CEF: $(\Sigma M, \Sigma F_{x,y}) = 0$

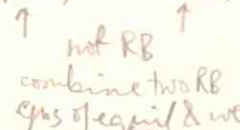
ABCD: -- do --

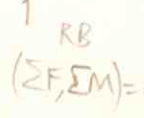
then check A_x, A_y

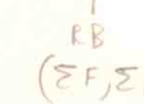
- Need to dis-member & analyse. Hence method of joints/sections not sufficient.
- ⇸ When pin connects ≥ 3 members, or 2 members & support, or load applied to pin, a decision must be made as to which member the pin is an integral part of. When multiforce members are involved must attach pin to one of these members. For pin connecting only 2 members the choice is arbitrary.
- Frame that loose rigidity when supports removed

SD & rigid

⇸ 







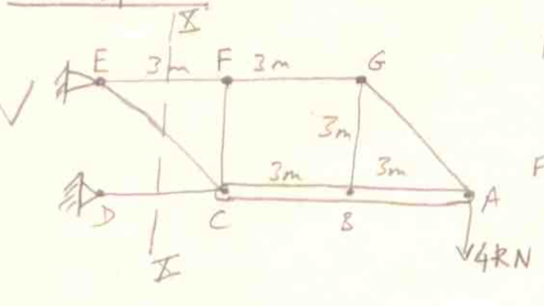
not RB | RB | RB

combine two RB eqns of equil & we see that $(\Sigma F, \Sigma M) = 0$ for ABC also ∵ int forces cancel out in eqn assembly

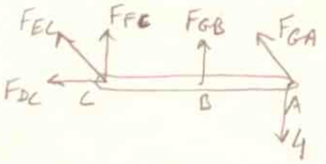
$(\Sigma F, \Sigma M) = 0$ | $(\Sigma F, \Sigma M) = 0$

$\Sigma F = \Sigma M = 0$ eqs/x satisfied by forces acting indep on AB, BC. They must be satisfied when two force systems considered together

MK 4/109



Find force in member BG.



From jt. F → $F_{FC} = 0$

FBD of ABC:

$\Sigma M_C: F_{GB}(3) - 4(6) + F_{GA}(\sqrt{18}) = 0 \rightarrow (1)$

$\Sigma M_E: F_{GB}(6) + F_{GA}(\sqrt{18}) - 4(9) - F_{DC}(3) = 0 \rightarrow$

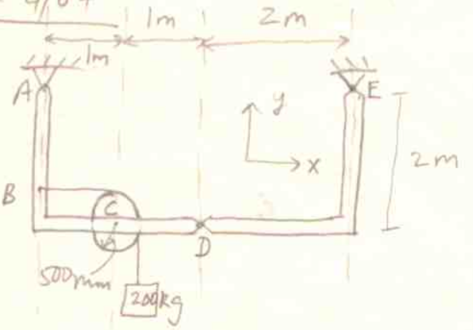
FBD to right of section X-X:

$\Sigma M_E: F_{DC}(3) - 4(9) = 0$

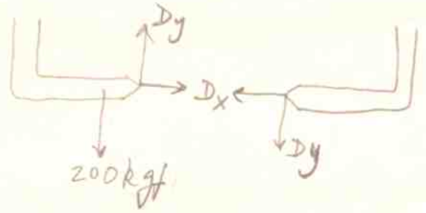
$\Rightarrow F_{DC} = -12 = 12(C)$

$F_{GB} = -8 = 8(C)$

MK 4/67



Find x, y comp's of force @ D which member AD exerts on D.



$\Sigma M_A: D_x(2) + D_y(2) - 200 * 9.81 * 1.5 = 0$

$\Sigma M_E: D_x(2) - D_y(2) = 0$

$\Rightarrow D_x = D_y = 735.75$ N (in dir. as shown)

However indep RB's must be considered for complete static determination since 4 reactions at (A) supports.

FBD of AB → 3 eqs $\underbrace{\text{4 unkw ; FBD of BC} \rightarrow \text{3 eqs}}_{\text{combined 6 eqs, 6 unkw} \Rightarrow \text{SD.}}$ 2 additional unkw

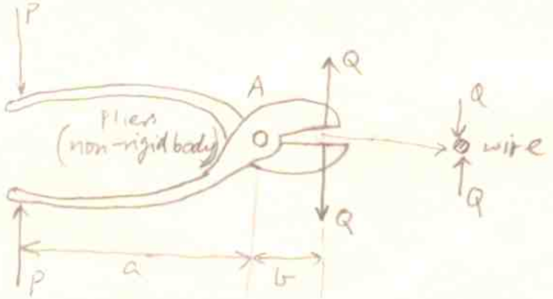
This is because ACB equil eqs are necessary not suff condits for equil. Thus AC + AB eqs reqd for equil being assured.

General Rule To determine whether structure is SD & rigid:

- isolate members, draw FBD, count unkws (reactions + member forces) = p
- count eqs = n
- If p > n then SID
- If p < n then non-rigid (all eqs not satisfied)
- If p = n & all p unkws can be determined & hence all n eqs satisfied then SD & rigid under general load
- If due to improper arrangement of supports & members all p unkws can't be determined & all n eqs can't be satisfied then SID & nonrigid

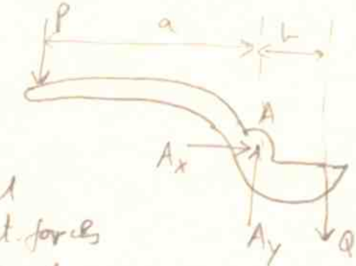
MACHINES

- transmit or modify forces (transform input forces into output forces)
- contain moving parts & at least one multiforce member.



Determine P given Q or vice-versa

∴ pair of pliers is non-RB we need to consider component parts as free body



(Given either P or Q)

$\Sigma M_A: Pa - Qb = 0$

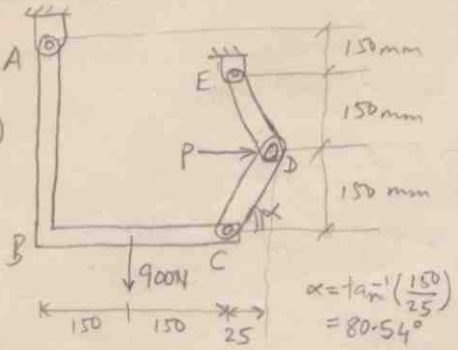
$\Sigma F_x: A_x = 0$

$\Sigma F_y: -P - Q + A_y = 0$

- in general for machines will need to use several FBD's & solve simultaneous eqs involving int. forces
- FBD's chosen to reveal input forces & output reactions

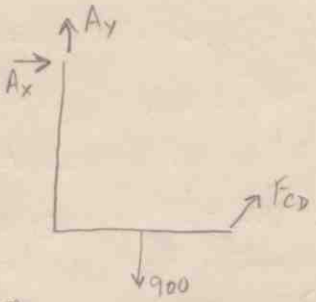
6.117 & 6.118

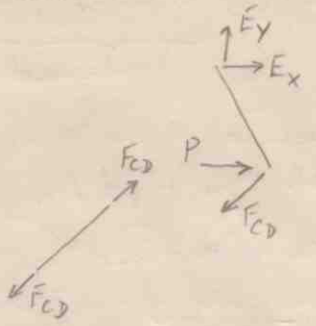
(6.117)



Find P reqd to be applied to toggle CDE to maintain bracket in position shown

(ABC)





Note: ∵ ED is 2-Force member, resultant of (E_x & E_y), and (P & F_{CD}) shown in FBD will be directed along ED.

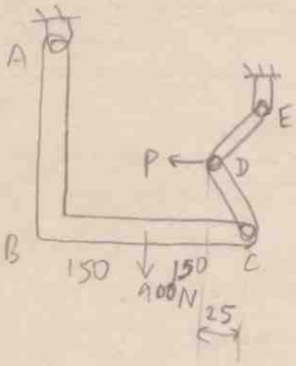
$\alpha = \tan^{-1}\left(\frac{150}{25}\right) = 80.54°$

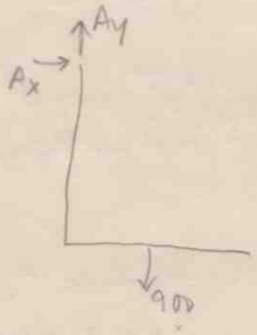
ABC

$\Sigma M_A: -900 * 150 + F_{CD}\cos\alpha * 450 + F_{CD}\sin\alpha * 300 = 0 \Rightarrow F_{CD} = 364.97\ N\ (T)$

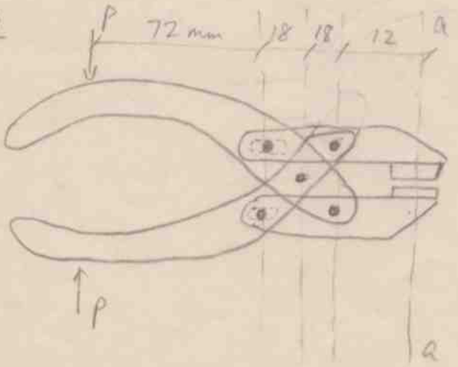
$\Sigma M_E: P * 150 - F_{CD}\cos\alpha * 150 - F_{CD}\sin\alpha * 25 = 0 \Rightarrow P = 120\ N\ (\rightarrow)$

(6.116)





6.114



Given

Clamping jaws remain //. Require 300 N output forces reqd along a-a. Pins A, D slide freely

Find: P reqd