P.1 (a)

$$\underline{R} = (120+200)\underline{i} + (100-340)\underline{j} = 320\underline{i} - 240\underline{j}$$

$$\underline{M}_B^R = (-120*0.12 + 100*0.24)\underline{k} + (-16-32)\underline{k}$$
$$= -38.4\,\underline{k} \text{ N-m}$$

(b) Simplest resultant (coplanar forces)

$\underline{r} \times \underline{R} = \underline{M}_B^R$ (equivalent moment requirement)

$$(x\underline{i} + y\underline{j}) \times (320\underline{i} - 240\underline{j}) = -38.4\,\underline{k}$$

$$\Rightarrow -240x - 320y = -38.4 \quad (*)$$

{eqn of line of action of $\underline{R}$ for simplest resultant}

Intersect of (*) with AB → put $x=0 \Rightarrow y = 0.12$ m
" " " " BC → put $y=0 \Rightarrow x = 0.16$ m
" " " " CD → put $x = 0.24 \Rightarrow y = -0.06$

P2 $\underline{R} = \underline{T}_{AB} + \underline{T}_{AC} + \underline{T}_{AD} = -T_A\underline{j}$

T_{AD} (given)

$$\Rightarrow T_{AB}\frac{(-12\underline{j} - 8\underline{k})}{\sqrt{208}} + T_{AC}\frac{(-4\underline{i} - 12\underline{j} + 3\underline{k})}{\sqrt{169}} + 2520\frac{(6\underline{i} - 12\underline{j} + 4\underline{k})}{\sqrt{196}} = -T_A\underline{j}$$

$$\underline{i}: \quad T_{AC}\left(-\frac{4}{13}\right) + 2520\left(\frac{6}{14}\right) = 0 \Rightarrow T_{AC} = 3510$$

$$\underline{k}: \quad T_{AB}\left(-\frac{8}{\sqrt{208}}\right) + T_{AC}\left(\frac{3}{13}\right) + \frac{2520}{14}(4) = 0 \Rightarrow T_{AB} = 2758.25\text{ N.}$$

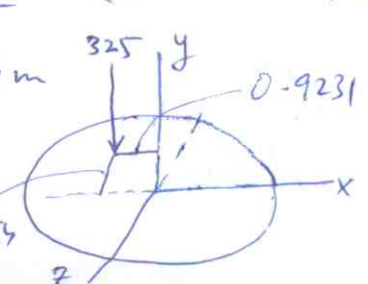
P.3. $\underline{R} = (-100-75-25-125)\underline{j} = -325\underline{j}$

$\underline{r} \times \underline{R} = \underline{M}_0^R$ where O is taken at origin.

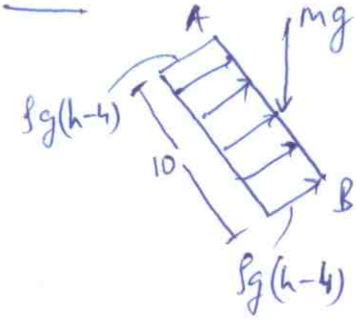
$\underline{r} = (x\underline{i} + z\underline{k})$

$$\Rightarrow -325x\,\underline{k} + 325z\,\underline{i} = (100*4 - 25*4)\underline{k} + (75*4 - 125*4)\underline{i}$$

$$\Rightarrow x = -\frac{300}{325} = -0.9231\text{ m}, \quad z = -\frac{200}{325} = -0.6154\text{ m}$$



P4



$$\underline{R} = -mg\,\underline{j} + \rho g(h-4)(10)(6)\left[\cos\theta\,\underline{i} + \sin\theta\,\underline{j}\right]$$

$$\theta = \frac{\pi}{2} - \tan^{-1}\left(\frac{8}{6}\right), \quad \cos\theta = 0.8, \ \sin\theta = 0.6$$

$$\underline{R} = \rho g(h-4)(48)\underline{i} + \{\rho g(h-4)(36) - mg\}\,\underline{j}$$

Impending opening of gate when line of action of $\underline{R}$ coincides with $\overrightarrow{AB}$.

$$\Rightarrow \frac{\rho g(h-4)(36) - mg}{\rho g(h-4)(48)} = -\frac{8}{6} \Rightarrow \rho g(h-4)[36*6 + 48*8] = 6mg$$

$$\Rightarrow h = 4 + \frac{6mg}{600}\frac{1}{\rho g} = 4 + \frac{50,000}{100*1000} = 4.5\text{ m}.$$

Check by equilibrium approach:

Put $\sum M_B = 0$ with reaction at A equal zero.

$$mg(3) - \rho g(h-4)(10)(6)\left(\frac{10}{2}\right) = 0 \Rightarrow \frac{3mg}{300\rho g} + 4 = h = 4.5\text{m}.$$

P.5. $$\underline{R} = \int d\underline{R} = -\underline{j}\int_1^5 10(1+z)\underbrace{\left(3.75 - \frac{3}{4}z\right)dz}_{\text{element of lowest order.}} = -\underline{j}\,10\int_1^5\left(3.75 + 3z - \frac{3}{4}z^2\right)dz$$

$$= -\underline{j}\,10\left(3.75*4 + 3*\frac{(5^2-1^2)}{2} - \frac{3}{4}*\frac{(5^3-1^3)}{3}\right) = -200\underline{j}$$

To find point of action of R find center of wind pressure distribution.

$$\bar{z}R = \int z\,dR = \int_1^5 z\,10(1+z)\underbrace{\left(3.75 - \frac{3}{4}z\right)dz}_{\text{element of lowest order}}$$

$$= 10\int\left(3.75z + 3z^2 - \frac{3}{4}z^3\right)dz$$

$$\bar{z} = \frac{10\left(3.75*\frac{(5^2-1^2)}{2} + 3*\frac{(5^3-1^3)}{3} - \frac{3}{4}*\frac{(5^4-1^4)}{4}\right)}{200} = 2.6\text{ m}$$

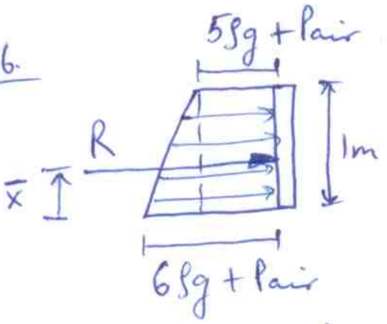
$$\bar{x}R = \int x\,dR = \int_1^5 -\frac{(3.75-\frac{3}{4}z)}{2} * 10(1+z)\underbrace{(3.75-\tfrac{3}{4}z)\,dz}_{\text{element of lowest order}}$$

$$= -5\int_1^5 (3.75-\tfrac{3}{4}z)(3.75+3z-\tfrac{3}{4}z^2)\,dz$$

$$= -5\int (14.0625+8.4375z-5.0625z^2+\tfrac{9}{16}z^3)\,dz$$

$$\bar{x} = \frac{-5\left(14.0625*4 + 8.4375*\frac{(5^2-1^2)}{2} - 5.0625*\frac{(5^3-1^3)}{3} + \frac{9}{16}*\frac{(5^4-1^4)}{4}\right)}{200} = -0.9\,m$$

P. 6.



$P_{air} = 0.1380\ N/mm^2 = 0.1380 * 10^6\ N/m^2$ (given)

$R = \frac{1}{2}(5\rho g + P_{air} + 6\rho g + P_{air})(1)(0.5)$

(use $\rho g = 9810\ N/m^3$)

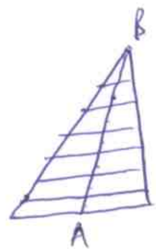
$R = (5.5\rho g + P_{air}) * 0.5 = 95977.5\,N$

$\bar{x}R = (5\rho g + P_{air})(1)(0.5)(\frac{1}{2}) + \frac{1}{2}(1)(\rho g)(0.5)(\frac{1}{3})$

$\bar{x} = 47580/95977.5$

$\bar{x} = 0.49574\,m$ from bottom of door.

Shortcut



For every horizontal strip the CG lies in the middle. So AB is locus of CG of horizontal strips. Thus the overal CG must lie on this locus. ∴ $\bar{z} = 2.6$,

$\bar{x} = \frac{1}{2}(3.75 - \frac{3}{4}\bar{z}) = 0.9$