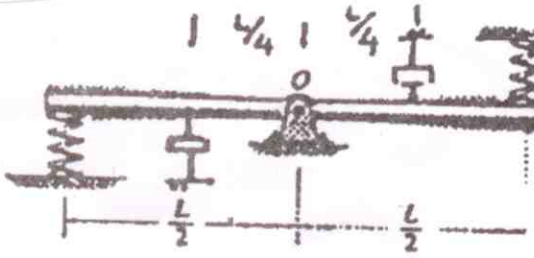
P.1

1. The uniform rod has a mass m and is supported by the pin O. If the rod is given a small displacement and released, develop the equation of motion. Then determine the undamped period of vibration and the damping ratio of the system. The springs are unstretched when the rod is in the position shown and both have the same spring constant k. Both the dashpots have the same dashpot constant c.



Given that static position is shown (∵ springs unstretched & rod is balanced at center).

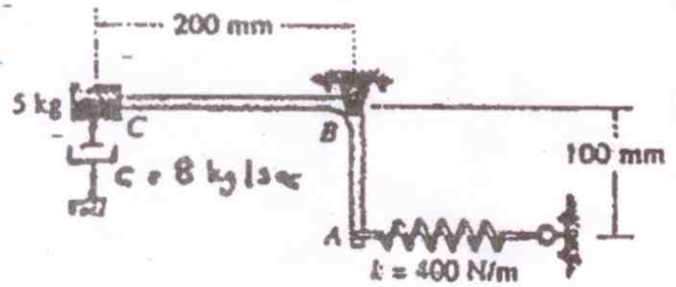
$$I_O\ddot{\theta} = \sum M_O \Rightarrow \frac{ml^2}{12}\ddot{\theta} = -2k\left(\frac{l}{2}\theta\right)\frac{l}{2} - 2c\left(\frac{l}{2}\dot{\theta}\right)\frac{l}{2}$$

$$\frac{m\cancel{l^2}}{12}\ddot{\theta} + \frac{c\cancel{l^2}}{2}\dot{\theta} + \frac{k\cancel{l^2}}{2}\theta = 0$$

$$\omega_n = \sqrt{\frac{kl^2/2}{ml^2/12}} = \sqrt{\frac{6k}{m}}, \quad T_n = 2\pi\sqrt{\frac{m}{6k}}$$

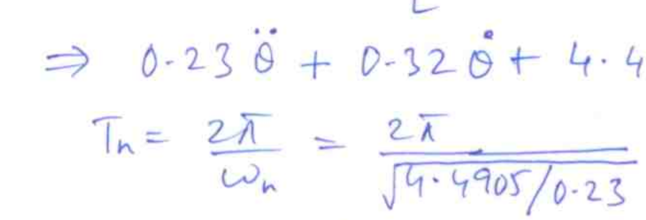
$$\zeta = \frac{c/2}{2\frac{m}{12}\sqrt{\frac{6k}{m}}} = \frac{3c}{\sqrt{6km}}$$

P.2.



2. The bent rod shown in the figure has a uniformly distributed mass of 3 kg and has a 5 kg collar at its end that is supported on a dashpot with a damping constant c = 8 kg/sec. Develop the equation of motion and determine the natural undamped period of vibration for the system. What is the damping ratio and the damped period of vibration? Solve first by ignoring rod mass and then by considering it.

Assume static position shown. So weight of horizontal leg & 5kg mass at C does not enter into the moment term in EoM (∵ it balances static spring force in moment about B).



$$I_B\ddot{\theta} = \sum M_B \Rightarrow \left[5(0.2)^2 + 2\left(\frac{0.2^2}{3}\right) + 1\left(\frac{0.1^2}{3}\right)\right]\ddot{\theta} = -8(0.2^2)\dot{\theta} - 400(0.1^2)\theta - 1g(0.05\theta)$$

$$\Rightarrow 0.23\ddot{\theta} + 0.32\dot{\theta} + 4.4905\theta = 0.$$

$$T_n = \frac{2\pi}{\omega_n} = \frac{2\pi}{\sqrt{4.4905/0.23}} = 1.422 \text{ sec}$$

$$\zeta = 0.32/(2 * 0.23 * \sqrt{4.4905/0.23}) = 0.1574$$ ⇒ underdamped so periodic motion.

$$T_d = \frac{2\pi}{\omega_d} = \frac{2\pi}{\omega_n\sqrt{1-\zeta^2}} = 1.4399 \text{ sec.}$$

If you ignore rod mass, we have eom

$$5(0.2^2)\ddot{\theta} + 8(0.2^2)\dot{\theta} + 400(0.1^2)\theta = 0$$

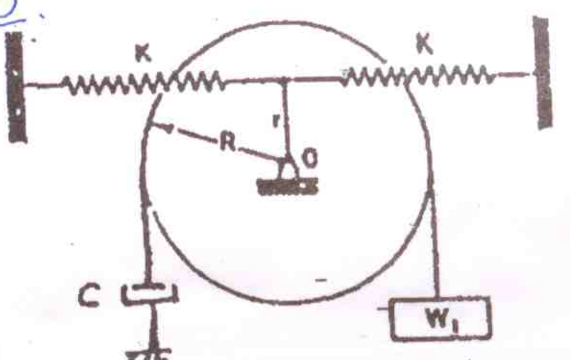
$$\Rightarrow 0.2\ddot{\theta} + 0.32\dot{\theta} + 4\theta = 0$$

$$T_n = \frac{2\pi}{\omega_n} = \frac{2\pi}{\sqrt{4/0.2}} = 1.405 \text{ sec}$$

$$\zeta = 0.32/(2 * 0.2 * \sqrt{4/0.2}) = 0.1789$$ ⇒ underdamped, so vibrations exist. (i.e, periodic motion)

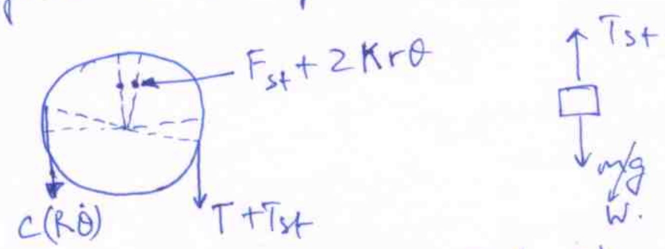
$$T_d = \frac{2\pi}{\omega_d} = 1.428 \text{ sec.}$$

P3.



3. A cylinder of mass M and radius R which rotates without friction about O, is connected to identical springs and a dashpot and supports a weight W_L as shown in figure. Consider $R = 250$ mm, $r = 150$ mm, $M = 10$ kg, $W_L = 50$ N, $K = 600$ N/m and $C = 8$ N-sec/m. Determine the natural frequency for small oscillations. If the cylinder is given an initial angular rotation of 0.01°, what is the maximum angular velocity of the cylinder? After 20 cycles of vibration, what is the rotational amplitude?

- Assume static position shown. Refer to P.4. of Tute-9. The only change in FBD of disk is that damping restoring force also appears



So eom becomes (refer Tute 9),

$$\left(\frac{MR^2}{2} + mR^2\right)\ddot{\theta} + cR^2\dot{\theta} + 2Kr^2\theta = 0.$$

$$\omega_n = \sqrt{\frac{2Kr^2}{\left(\frac{MR^2}{2} + mR^2\right)}} = \sqrt{\frac{2 * 600 * 0.15^2}{\frac{10 * 0.25^2}{2} + \frac{50 * 0.25^2}{9.81}}} = 6.5411 \text{ rad/sec.}$$

$$\zeta = \frac{c\cancel{R^2}}{2\left(\frac{M}{2} + m\right)\cancel{R^2}\,\omega_n} = 0.06057 \Rightarrow \text{underdamped.}$$

$$\omega_d = \omega_n\sqrt{1 - \zeta^2} = 6.5291.$$

Max velocity :

$$\theta = C\sin(\omega_d t + \psi)\, e^{-\zeta\omega_n t}$$

$$\dot{\theta} = \left[C\omega_d\cos(\omega_d t + \psi) - C\zeta\omega_n\sin(\omega_d t + \psi)\right] e^{-\zeta\omega_n t}$$

$$\ddot{\theta} = \left[-C\omega_d^2\sin(\omega_d t + \psi) - 2C\zeta\omega_n\omega_d\cos(\omega_d t + \psi) + C\zeta^2\omega_n^2\sin(\omega_d t + \psi)\right] e^{-\zeta\omega_n t}$$

$\ddot{\theta} = 0$ for $\tan(\omega_d t + \psi) = \dfrac{2\zeta\sqrt{1-\zeta^2}}{2\zeta^2 - 1}$

$$C = \left[x_0^2 + \left(\frac{\cancelto{0}{\dot{x}_0} + \zeta\omega_n x_0}{\omega_d}\right)^2\right]^{1/2} = 1.748539 * 10^{-4} \text{ rad}$$

$$\psi = \tan^{-1}\left(\frac{x_0 \omega_d}{\cancelto{0}{\dot{x}_0} + \zeta\omega_n x_0}\right) = 1.510193 \text{ rad.}$$

For $\ddot{\theta} = 0$, $\omega_d t + \psi = 0.121205 + n\pi$

Choose smallest n for which $t > 0$, ie $n = 1$

$\Rightarrow t = \dfrac{0.121205 + \pi - \psi}{\omega_d} = 0.2313$ secs. $\rightarrow$ time at which max velocity occurs

$\dot{\theta}(t = 0.2313) = -3.4861$ rad/sec. $= \dot{\theta}_{max}$

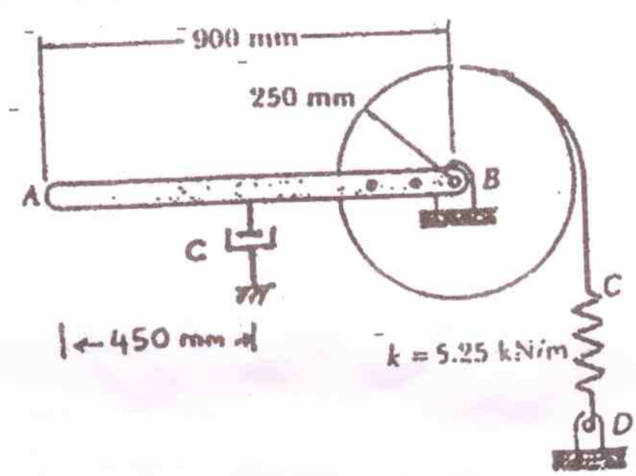
Rotational "amplitude" after 20 cycles:

$$\theta = C \sin(\omega_d t + \psi) e^{-\zeta\omega_n t}$$

Rot. Amplitude $= Ce^{-\zeta\omega_n t} = Ce^{-\left(\zeta\omega_n 20 * \frac{2\pi}{\omega_d}\right)} = 8.5355 * 10^{-8}$ rad.

P. 4.

4. A 7 kg slender rod *AB* is riveted to a 5 kg uniform disk. A belt attaches the rim of the disk to a spring that holds the rod at rest in the position shown. Determine the undamped period of vibration. If end *A* of the rod is moved 18mm down and released, determine the maximum velocity of end *A*. After 10 cycles of vibration, the maximum amplitude of end *A* of the rod is found to be 12 mm. Determine (a) the damping ratio of the system, and (b) the value of the damping constant *c*.



Add damping term to P. 2, Tute 9, and get eom as,

$$\underbrace{\left[7\left(\frac{0.9^2}{3}\right) + 5\left(\frac{0.25^2}{2}\right)\right]}_{m}\ddot{\theta} + c\,0.45^2\dot{\theta} + \underbrace{5.25 * 10^3 * 0.25^2}_{k}\,\theta = 0.$$

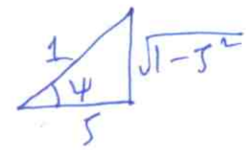
$\omega_n = 12.6631$ rad/s (from P. 2 Tute 9)

Amplitude after 10 cycles $= 12\,mm = 900\,C e^{-\zeta\omega_n 10 T_d}$

$x_0 = 900\,C \sin\psi$, $\psi = \tan^{-1}\left(\dfrac{\theta_0 \omega_d}{\cancel{\dot{\theta}_0} + \zeta\omega_n\theta_0}\right) = \tan^{-1}\left(\dfrac{\sqrt{1-\zeta^2}}{\zeta}\right)$

$\dot{\theta}_0 \to 0$

$\Rightarrow x_0 = 900\,C \dfrac{\sqrt{1-\zeta^2}}{1} = 900\,C\sqrt{1-\zeta^2} = 18\,mm$ (given)



$$\Rightarrow \frac{12}{18} = \frac{\cancel{900}\,\cancel{C}\, e^{-\left(\zeta\omega_n 10 \times \frac{2\pi}{\omega_d}\right)}}{\cancel{900}\,\cancel{C}\sqrt{1-\zeta^2}}$$

$$\frac{12}{18} = \frac{e^{-40\pi\zeta/\sqrt{1-\zeta^2}}}{\sqrt{1-\zeta^2}} \quad (1)$$

$\to$ solution of ζ is to be done numerically.

However, we will approximate $\zeta \ll 1$, hence

$$\frac{12}{18} = e^{-40\pi\zeta} \qquad \Rightarrow -\frac{1}{40\pi}\ln\left(\frac{12}{18}\right) = \zeta$$

$\Rightarrow \zeta = 0.0032265$ $\to$ substitute in (1) and verify that this is an excellent approximation.

$C = 2\zeta m\omega_n = 0.16721$

Alternatively, we can assume $x_0 \simeq 900\,C$ since $\psi \simeq \frac{\pi}{2}$ for $\zeta \ll 1$. Then,

$$\frac{12}{18} = \frac{\cancel{900\,C}\, e^{-(40\pi\zeta/\sqrt{1-\zeta^2})}}{\cancel{900\,C}}$$

$$\underbrace{-\frac{1}{40\pi}\ln\left(\frac{12}{18}\right)}_{a} = \frac{\zeta}{\sqrt{1-\zeta^2}} \Rightarrow a\zeta^2 + \zeta - a = 0$$

$$\zeta = \frac{-1 \pm \sqrt{1+4a^2}}{2a} = 0.0032266 \text{ or } -309.92$$

same as before

discard, $\zeta > 0$.

Max velocity. $C = \left[\theta_0^2 + \left(\dfrac{\zeta\omega_n\theta_0}{\omega_d}\right)^2\right]^{1/2}$, $\omega_d = \sqrt{1-\zeta^2}\,\omega_n = 12.663$,

$\theta_0 = \dfrac{18}{900}$, $C = 0.02$ (i.e., "nearly" θ_0 since $\zeta \ll 1$).

$\psi = \tan^{-1}\left(\sqrt{1-\zeta^2}/\zeta\right) = 1.5675697 \simeq \pi/2$.

For max velocity (see P.3),

$$\omega_d t + \psi = \tan^{-1}\left(\frac{2\zeta\sqrt{1-\zeta^2}}{2\zeta^2-1}\right) = -0.00645318 + n\pi$$

Smallest n giving $t > 0$ is $n=1$ for which

$$t = \frac{-0.00645318 + \pi - 1.5675697}{12.663}$$

$\dot{\theta}_{max} = -0.251984$ rad/sec

$v_{max} = 900\,\dot{\theta}_{max} = -226.7859$ mm/sec.