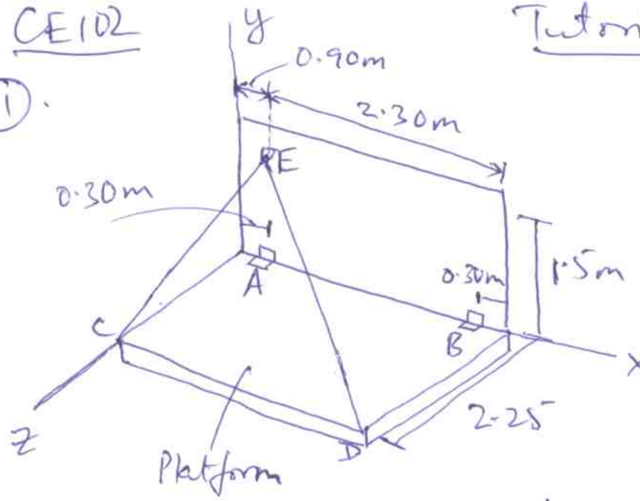
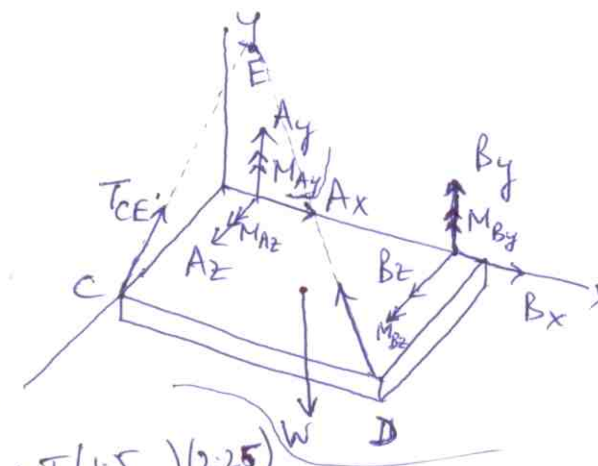
CE102 Tutorial 3 (1)

(1).



Frictionless hook E $\Rightarrow T_{CE} = T_{DE} = T$
+ massless rope



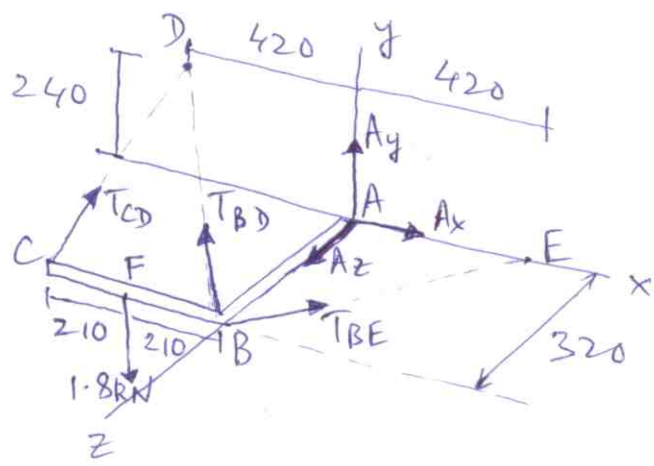
$$T_{CE} = T(0.9\underline{i} + 1.5\underline{j} - 2.25\underline{k})/2.85$$

$$T_{DE} = T(-2.3\underline{i} + 1.5\underline{j} - 2.25\underline{k})/3.55$$

$$\Sigma M_{xx} = 0 \Rightarrow T\left(\frac{1.5}{2.85}\right)(2.25) + T\left(\frac{1.5}{3.55}\right)(2.25) - W\left(\frac{2.25}{2}\right) = 0 \Rightarrow T = 1292.35\text{ N}$$

Longer way: 11 unknowns $(A_x, A_y, A_z, B_x, B_y, B_z, T)$ $(M_{Ay}, M_{Az}, M_{By}, M_{Bz})$ & 6 eqns
So SID. But you can write $\Sigma \underline{M} = 0$, $\Sigma \underline{F} = 0$
and solve for T (and some of the reactions).

(2)



6 unknowns $(A_x, A_y, A_z, T_{BE}, T_{BD}, T_{CD})$
$\Rightarrow$ S.D.

$$\underline{T}_{CD} = (240\underline{j} - 320\underline{k})/400$$

$$\underline{T}_{BD} = (-420\underline{i} + 240\underline{j} - 320\underline{k})/580$$

$$\underline{T}_{BE} = (420\underline{i} - 320\underline{k})/\sqrt{278800}$$

$\Sigma M_{AB} = 0$: only T_{CD} participates (besides load).

$$\Rightarrow T_{CD}\left(\frac{240}{400}\right)(420) - 1.8(210) = 0 \Rightarrow T_{CD} = 1.5\text{ kN}$$

$\Sigma M_{xx} = 0$: only T_{BD} participates (besides T_{CD} & load).

$$\Rightarrow T_{CD}\left(\frac{240}{400}\right)(320) + T_{BD}\left(\frac{240}{580}\right)(320) - 1.8(320) = 0 \Rightarrow T_{BD} = 2.175$$

$\Sigma M_{AD} = 0$: only T_{BE} participates (besides load).

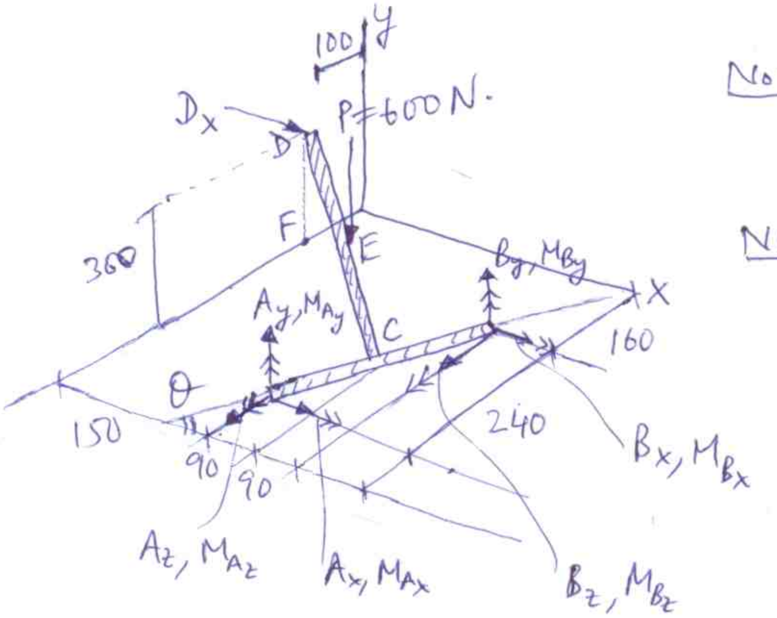
$$\Rightarrow \underset{\sim}{\ell}_{AD} \cdot \left[\underline{r}_{AF} \times (-1.8\underline{j}) + \underline{r}_{AB} \times \underline{T}_{BE}\right] = 0$$

$$\Rightarrow \frac{(-420\underline{i}+240\underline{j})}{\sqrt{234000}} \cdot \left[(-210\underline{i}+320\underline{k})\times(-1.8\underline{j}) + T_{BE}(320\underline{k})\times\frac{(420\underline{i}-320\underline{k})}{\sqrt{278800}} \right] = 0$$

$$\left(\frac{-420}{\cancel{\sqrt{234000}}}\right)(\cancel{320}\times 1.8) + T_{BE}\left(\frac{240}{\cancel{\sqrt{234000}}}\right)\left(\frac{\cancel{320}\times 420}{\sqrt{278800}}\right) = 0$$

$$\Rightarrow T_{BE} = 3.9601\ kN$$

(3)



unit vector along AB

NOTE: $(M_{Ax}\underline{i}\ M_{Az}\underline{k})\cdot \underline{e}_{AB} = 0$
$(M_{Bx}\underline{i} + M_{Bz}\underline{k})\cdot \underline{e}_{AB} = 0$

NOTE: If either of the bearings at A or B do not provide axial thrust, then for that bearing the x and z comps of reaction are related such that their sum has no comp along $\underline{e}_{AB}$.

NOTE: only one of the bearings at A or B is required to provide axial thrust to balance the comp of D_x along $\underline{e}_{AB}$ / NOTE: SID $\because$ reactions > 6.

$\Sigma M_{AB} = 0$ gives D_x $\because$ only D_x participates (besides P).

$$\Rightarrow D_x \sin\theta\,(300) - P\left(\frac{d}{2}\right) = 0$$

$\theta = \tan^{-1}\left(\frac{240}{180}\right) = 53.13°$

where $d = \perp^{ar}$ dist from F to AB.

$AB = 300\ m$
$\sin\theta = \frac{240}{300} = 0.8$

$$\underline{AF} = -150\underline{i} - (240+160-100)\underline{k} = -150\underline{i} - 300\underline{k}$$

$$\underline{e}_{AB} = (180\underline{i} - 240\underline{k})/300 = 0.6\underline{i} - 0.8\underline{k}$$

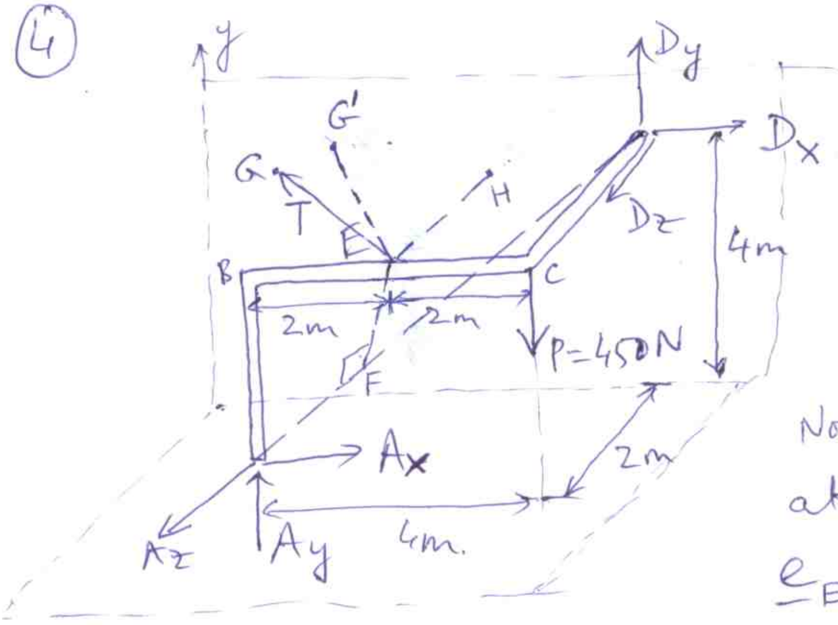
$$d = \sqrt{(AF)^2 - (\underline{AF}\cdot\underline{e}_{AB})^2} = \sqrt{112500 - (-150\times 0.6 + 300\times 0.8)^2}$$
$$= 300$$

$$\Rightarrow D_x = 600\left(\frac{300}{2}\right)\Big/(300(0.8)) = 375\ N.$$

Can also do by vector approach, i.e.,

$\Sigma M_{AB} = \underline{e}_{AB}\cdot\left[\underline{r}_{AD}\times D_x\underline{i} + \underline{r}_{AE}\times(-P\underline{j})\right] \longrightarrow$ will give the result

(4)



(METHOD 1 — more physical)

7 reactions (3 each at A & D + tension T)

So SID.

To find T do $\sum M_{AD} = 0$

Now consider three mutually $\perp$ar directions at pt E defined by $\underline{e}_{EH}$, $\underline{e}_{EF}$, $\underline{e}_{EG'}$, where $\underline{e}_{EH}$ is parallel to $\underline{e}_{AD}$, as shown.

Then only the $\underline{e}_{EG'}$ component of T contributes to moment about AD axis, so T will be minimum if G coincides with G' (since then T will be wholly in $\underline{e}_{EG'}$ direction so the $\underline{e}_{EF}$ and $\underline{e}_{EH}$ comp's of T are zero, and hence not wasted during process of providing equilibrium).

$$\underline{e}_{AD} = (4\underline{i} + 4\underline{j} - 2\underline{k})/6 = \tfrac{2}{3}\underline{i} + \tfrac{2}{3}\underline{j} - \tfrac{1}{3}\underline{k} = \underline{e}_{EH}$$

$$AF = \underline{AE}\cdot\underline{e}_{AD} = (2\underline{i}+4\underline{j})\cdot\left(\tfrac{2}{3}\underline{i} + \tfrac{2}{3}\underline{j} - \tfrac{1}{3}\underline{k}\right) = \tfrac{4}{3} + \tfrac{8}{3} = 4$$

$$AD = \sqrt{4^2+4^2+2^2} = 6, \Rightarrow DF = AD - AF = 2.$$

$$x_F = (\underbrace{x_A}_{-4} - \underbrace{x_D}_{0})\,2/6 = -4/3, \quad y_F = (\underbrace{y_A}_{-4} - \underbrace{y_D}_{0})\,2/6 = -4/3, \quad z_F = (\overset{2}{z_A} - \overset{0}{z_D})\tfrac{2}{6} = 2/3$$

$$F \Rightarrow (-4/3, -4/3, 2/3)$$

$$\underline{e}_{EF} = \left[\left(-\tfrac{4}{3}+2\right)\underline{i} - \tfrac{4}{3}\underline{j} + \left(\tfrac{2}{3}-2\right)\underline{k}\right]/2 = \tfrac{1}{3}\underline{i} - \tfrac{2}{3}\underline{j} - \tfrac{2}{3}\underline{k}$$

$$\underline{e}_{EG'} = \underline{e}_{EH}\times\underline{e}_{EF} = \left[\left(\tfrac{2}{3}\right)\left(-\tfrac{2}{3}\right) - \left(-\tfrac{1}{3}\right)\left(-\tfrac{2}{3}\right)\right]\underline{i} + \left[\left(-\tfrac{1}{3}\right)\left(\tfrac{1}{3}\right) - \left(\tfrac{2}{3}\right)\left(-\tfrac{2}{3}\right)\right]\underline{j}$$
$$+ \left[\left(\tfrac{2}{3}\right)\left(-\tfrac{2}{3}\right) - \left(\tfrac{2}{3}\right)\left(\tfrac{1}{3}\right)\right]\underline{k} = -\tfrac{6}{9}\underline{i} + \tfrac{3}{9}\underline{j} - \tfrac{6}{9}\underline{k}$$

$$\underline{e}_{EG'} = -\tfrac{2}{3}\underline{i} + \tfrac{1}{3}\underline{j} - \tfrac{2}{3}\underline{k}$$

So, parametric form of equation of line thro E and in direction of $\underline{e}_{EG'}$ is,

$$\underline{r} = \left(a - \tfrac{2}{3}t\right)\underline{i} + \left(b + \tfrac{1}{3}t\right)\underline{j} + \left(c - \tfrac{2}{3}t\right)\underline{k} \quad \left(\rightarrow \text{gives } \frac{d\underline{r}}{dt} = \underline{e}_{EG'}\right).$$

Say $t=0$ at point E (where t is the parameter),

$\Rightarrow a=-2,\ b=0,\ c=2$

$\Rightarrow \underline{r} = (-2-\frac{2}{3}t)\,\underline{i} + \frac{1}{3}t\,\underline{j} + (2-\frac{2}{3}t)\,\underline{k}$

For $z=0$ (ie point of intersection, G', of line $\underline{r}$ with the wall)

$2-\frac{2}{3}t = 0 \Rightarrow t=3.$

$\Rightarrow x = -2-\frac{2}{3}(3) = -4, \quad y = \frac{1}{3}(3) = 1$

$\Rightarrow G' \rightarrow (-4, 1, 0)$ for T_{min}.

$T = T_{min}\left(-\frac{2}{3}\underline{i} + \frac{1}{3}\underline{j} - \frac{2}{3}\underline{k}\right) \quad (\text{ie } T_{min}\,\underline{e}_{EG'})$

$\Sigma M_{AD} = 0 \Rightarrow \underline{e}_{AD}\cdot\left[\underline{r}_{AE}\times\underline{T} + \underline{r}_{AC}\times(-P\underline{j})\right] = 0 \longrightarrow (*)$

$\Rightarrow \left(\frac{2}{3}\underline{i} + \frac{2}{3}\underline{j} - \frac{1}{3}\underline{k}\right)\cdot\left[(2\underline{i}+4\underline{j})\times\left(-\frac{2}{3}\underline{i}+\frac{1}{3}\underline{j}-\frac{2}{3}\underline{k}\right)T_{min} - P(4\underline{i}+4\underline{j})\times(\underline{j})\right] = 0.$

$(= T_{min} * EF)$

$\Rightarrow T_{min}\sqrt{20-16} = \frac{4}{3}P \Rightarrow T_{min} = \frac{2\times 450}{3} = 300\text{ N}.$

(less physical)

Another way: Using $(*)$ above,

$\underline{e}_{AD}\cdot(\underline{r}_{AE}\times\underline{T}) = P\,\underline{e}_{AD}\cdot(\underline{r}_{AC}\times\underline{j})$

$\Rightarrow \underline{T}\cdot(\underline{e}_{AD}\times\underline{r}_{AE}) = P\left(\frac{2}{3}\underline{i}+\frac{2}{3}\underline{j}-\frac{1}{3}\underline{k}\right)\cdot\left[(4\underline{i}+4\underline{j})\times\underline{j}\right]$

$\underline{T}\cdot\left[\frac{4}{3}\underline{i} - \frac{2}{3}\underline{j} + \left(\frac{8}{3}-\frac{4}{3}\right)\underline{k}\right] = -\frac{4}{3}P \longrightarrow (**)$

If T is minimum then $\underline{T}$ should lie along $(\underline{e}_{AD}\times\underline{r}_{AE})$

ie $\underline{T} = T_{min}\left(4/3\,\underline{i} - 2/3\,\underline{j} + 4/3\,\underline{k}\right)\Big/\sqrt{\frac{36}{9}} = T_{min}\left(\frac{2}{3}\underline{i} - \frac{1}{3}\underline{j} + \frac{2}{3}\underline{k}\right)$

$(**) \Rightarrow T_{min}\left(\frac{8}{9}+\frac{2}{9}+\frac{8}{9}\right) = -\frac{4}{3}(450) \Rightarrow T_{min} = \frac{-4(450)(3)}{18} = -300$

So $\underline{T}_{min}$ is magnitude 300 N in direction $\left(-\frac{2}{3}\underline{i}+\frac{1}{3}\underline{j}-\frac{2}{3}\underline{k}\right)$

The rest follows from first method. Same as $\underline{e}_{EG'}$ of first method.

Method 3 — (too cumbersome but straightforward)

$\underset{\sim}{T} = T\left((x+2)\,\underline{i} + y\,\underline{j} - 2\underline{k}\right)$ where $(x,y) \Leftarrow G'$

$$\underline{e}_{AD} \cdot (\underline{r}_{AE} \times \underline{T}) = T\,\frac{\left(-\frac{2}{3}y - \frac{8}{3} + \frac{4}{3}(x+2)\right)}{\sqrt{(x+2)^2 + y^2 + 4}} = T f(x,y)$$

So we wish to find max value of $f(x,y)$, then T will be minimum. Do $\frac{\partial f(x,y)}{\partial x} = 0$, $\frac{\partial f(x,y)}{\partial y} = 0$, get complicated equations (two of them) in terms of x, y & solve for x, y. (Won't work out in practical situation like test, exam)

may not