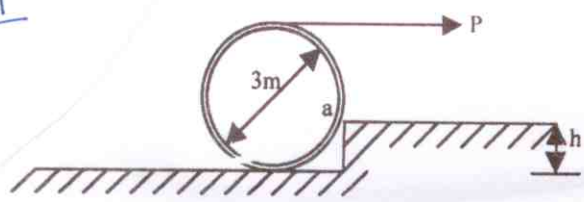
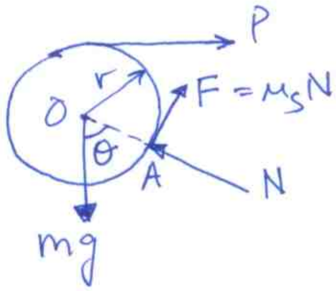
P.1



1. What is the height h of the step shown in the figure so that the force **P** will roll the 25 kg cylinder over the step at the same time that there is impending slippage at *a*? Take coefficient of static friction = 0.3.



$$\Sigma M_o : \quad P\cancel{r} = F\cancel{r} \longrightarrow (1)$$

$$\Sigma F_x : \quad P - N\sin\theta + F\cos\theta = 0 \rightarrow (2)$$

$$\Sigma F_y : \quad mg - N\cos\theta - F\sin\theta = 0 \rightarrow (3)$$

$$(2)^2 + (3)^2 \rightarrow P^2 + (mg)^2 = N^2 + F^2$$

$$\Rightarrow N = mg \quad (\because F = P \text{ from } (1)).$$

Put $F = \mu_s N$ (impending slip). Unknowns in (1), (2), (3) are θ, P, N. $N = mg$ already obtained.

$$(1), (2) \rightarrow \mu_s \cancel{N} - \cancel{N}\sin\theta + \mu_s \cancel{N}\cos\theta = 0 \longrightarrow (a)$$

$$N = mg \;\&\; (3) \rightarrow \cancel{N} - \cancel{N}\cos\theta - \mu_s \cancel{N}\sin\theta = 0 \longrightarrow (b)$$

$$(a), (b) \longrightarrow \sin\theta\,(1 + \mu_s^2) = 2\mu_s$$

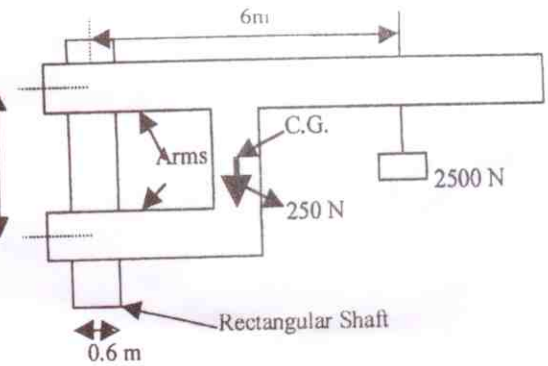
$$\Rightarrow \theta = 33.39^\circ$$

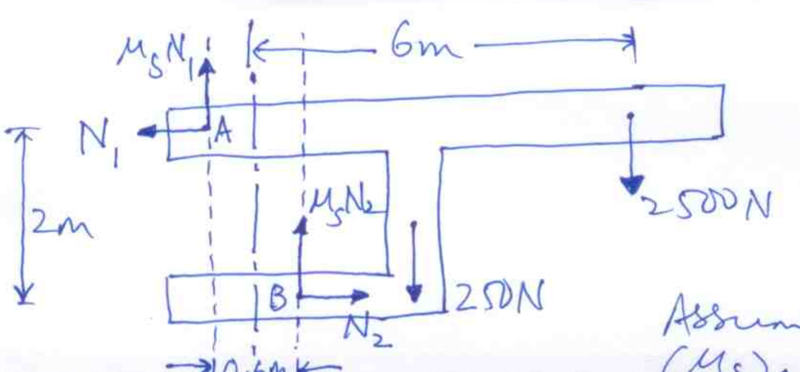
$$\Rightarrow h = 1.5(1 - \cos\theta) = 0.2477\,m.$$

NOTE: The static equilibrium condition is valid only at instant shown. During the rotation about A, it is not in static equilibrium.

P.2.

2. What is the minimum coefficient of friction required just to maintain the bracket and its 250 kg load? The centre of gravity is 1.8 m from the shaft cenreline.





Assume impending slip for finding

Impending slip downward. Bracket contacts shaft on left side upper (i.e., A) and right side lower (i.e., B). Point contact at ℄ of bracket assumed.

$$\Sigma F_x : \quad N_1 = N_2 \longrightarrow \text{(i)}$$

$$\Sigma F_y : \quad 2500 + 250 - \mu_s (N_1 + N_2) = 0 \longrightarrow \text{(ii)}$$

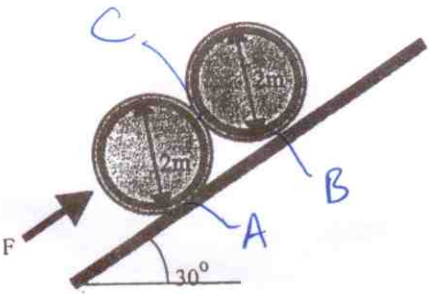
$$\Sigma M_A : \quad 2500(6.3) + 250(2.1) - N_2(2) - \mu_s N_2 (0.6) = 0 \longrightarrow \text{(iii)}$$

$$\text{(i)} - \text{(iii)} \longrightarrow \mu_s (2500 * 6.3 + 250 * 2.1) - 2750 - 0.6 \mu_s \left(\frac{2750}{2}\right) = 0$$

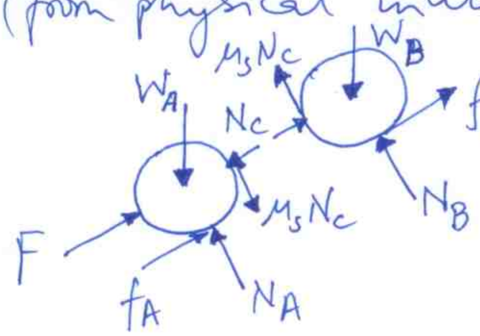
$$\Rightarrow \mu_s = 0.178 \text{ (min } \mu_s \text{ required).}$$

P. 3.

3. What is the force **F** to hold two cylinders, each having a mass of 50 kg. Take coefficient of static friction = 0.2 for all surfaces of contact.



Assume slip impends at C only (from physical intuition).



$\longrightarrow$ 6 unknowns ($F, N_A, f_A, N_B, f_B, N_C$) $\Rightarrow$ S.D.

$\mu_s = 0.2$ (given)

$$\Sigma M_A : \quad F(1) - N_C(1) + \mu_s N_C (1) - W_A (1 \sin 30) = 0 \longrightarrow ①$$

$$\Sigma M_B : \quad N_C(1) + \mu_s N_C(1) - W_B (1 \sin 30) = 0 \longrightarrow ②$$

$$\Rightarrow N_C = W_B \sin 30 / (1 + \mu_s) = 204.375 \text{ N.}$$

$$①, ② \longrightarrow F = N_C - \mu_s N_C + W_A \sin 30 = (1 - \mu_s) \frac{W_B \sin 30}{(1 + \mu_s)} + W_A \sin 30$$

$$(\because W_A = W_B) \quad = W_A \sin 30 \frac{2}{(1 + \mu_s)} = 408.75 \text{ N.}$$

Check: That $f_A \leq \mu_s N_A$, $f_B \leq \mu_s N_B$ as assumed.

(3)

$f_A = -F + W_A \sin 30 + N_C = 40.875$ N.

$N_A = W_A \cos 30 + \mu_s N_C = 465.66$ N.

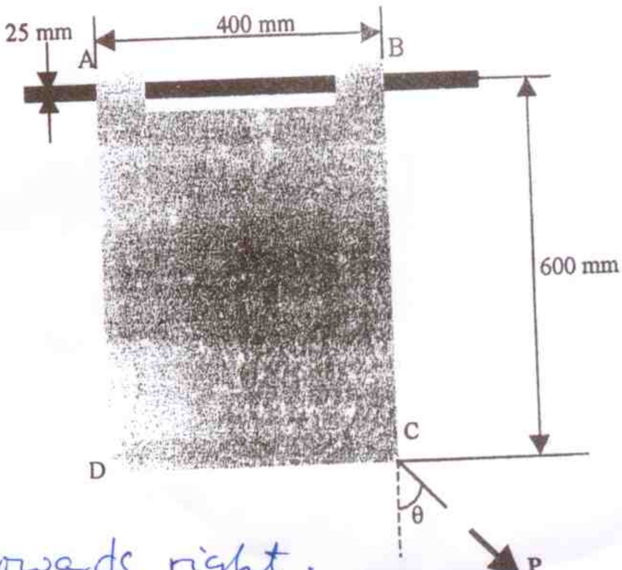
$N_B = (W_A + W_B)\cos 30 - N_A = 383.91$ N.

$f_B = -F - f_A + (W_A + W_B)\sin 30 = 40.875$ N.

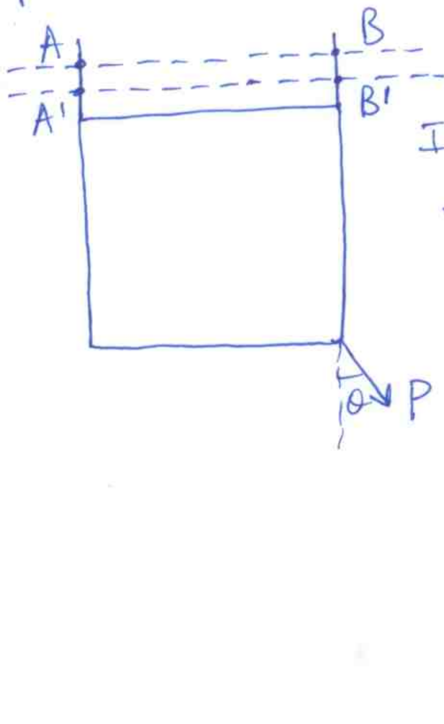
Thus $f_A < \mu_s N_A$, $f_B < \mu_s N_B$ so OK, i.e., assumption is true.

P.4

4. A light metal panel is welded to two short sleeves of 25-mm inside diameter which may slide on a horizontal rod. The coefficient of friction between the sleeves and the rod is 0.40. A cord attached to the corner C is used to move the panel along the rod. Determine the range of values of θ for which the panel will start moving to the right.



∵ sleeves are short, assume point contact at A & B. or A' & B'

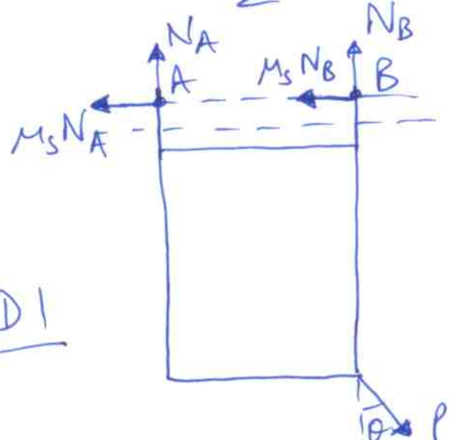


Impending slip towards right.

Take moments about B or B', as the case may be. Then we see that $\forall \theta$, contact is at A since moment due to N_A should counter that due to $P\sin\theta$ otherwise moments wont get balanced. (Note: moment due to friction at A, ie due to f_A which has max value of $\mu_s N_A$ will be less than that due to N_A since $\mu_s < 1$ and lever arm of 25 mm < lever arm of 400 mm, so moment due to friction at A cant balance that due to $P\sin\theta$). Now take moments about A or A'. From moment balance you see that when θ is such that moments of $P\cos\theta$, $P\sin\theta$, and $F_B\underbrace{(\mu_s N_B)}$ (max value), taken about A or A' (as the case may be), get balanced (i.e., pass thru a zero) then N_B will be about to reverse direction.

<u>Conclusion</u>: N_A always upward. N_B is upward for $0 < \theta < \theta_{min}$ and downward for $\theta_{max} < \theta < \pi/2$. Slip impends at θ_{min} & θ_{max}. No slip between $0 < \theta < \theta_{min}$ and $\theta_{max} < \theta < \frac{\pi}{2}$. Slip between $\theta_{min} < \theta < \theta_{max}$

<u>θ_{min}</u>:



<u>FBD1</u>

$$P\cos\theta = N_A + N_B \longrightarrow \text{(i)}$$

$$P\sin\theta = \mu_s (N_A + N_B) \longrightarrow \text{(ii)}$$

$$\Rightarrow \tan\theta = \mu_s \Rightarrow \theta_{min} = \tan^{-1}(\mu_s) = 21.801^\circ$$

Moment eqn (ΣM_B):

$$P\sin\theta\,(612.5) = N_A\,(400) \longrightarrow \text{(iii)}$$

Unknowns in (i), (ii), (iii) are N_A, N_B, θ (P is assumed known, can be anything).

EXTRA (for understanding)

<u>θ_{max}</u>:

<u>FBD2</u>

$$P\cos\theta + N_B - N_A = 0 \longrightarrow \text{(i)}$$

$$P\sin\theta - \mu_s (N_A + N_B) = 0 \longrightarrow \text{(ii)}$$

$$\Rightarrow \tan\theta = \frac{\mu_s (N_A + N_B)}{(N_A - N_B)} \longrightarrow \text{(a)}$$

So we need N_A, N_B (in terms of P). Use moment eqn.

$$P\sin\theta\,(600 - 12.5) + \mu_s N_A\,(25) - N_A\,(400) = 0 \longrightarrow \text{(iii)}$$

$$\Rightarrow N_A = \frac{P\sin\theta\,(587.5)}{400 - 25\mu_s} \longrightarrow \text{(b)}$$

(a), (b), (1):

$$\Rightarrow \tan\theta = \frac{\mu_s\left[2\cancel{P}\sin\theta\frac{(587.5)}{(390)} - \cancel{P}\cos\theta\right]}{\cancel{P}\cos\theta}$$

$$\Rightarrow \tan\theta = 2\mu_s\left(\frac{587.5}{390}\right)\tan\theta - \mu_s \Rightarrow \tan\theta = 1.95$$

$$\theta_{max} = 62.85$$

So slip occurs towards right for $21.801^\circ < \theta < 62.85^\circ$

Note: For θ outside this range, for slip to be impending you will get $\mu_s < 0.4$ (unknowns will be N_A, N_B, μ_s, input is θ) using FBD1 for for $\theta < 21.801^\circ$ and FBD 2 for $\theta > 62.85^\circ$.