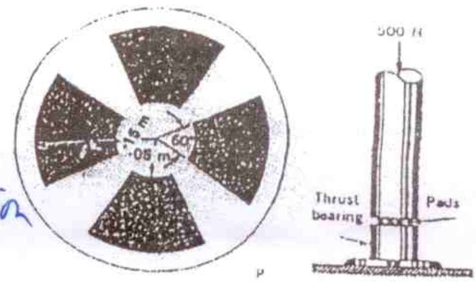
P.1 1. The support end of a dry thrust bearing is shown in the figure. Four pads form the contact surface. If a shaft creates a 500 N thrust uniformly distributed over the pads, what is the resisting torque for a coefficient of friction of 0.1?



Assume uniform pressure distribution on pads.

$\Rightarrow dN$ = normal force on area element

$$= \frac{P}{A} r\,d\theta\,dr = \frac{P}{\frac{240}{360}\pi(R_o^2 - R_i^2)} r\,d\theta\,dr$$

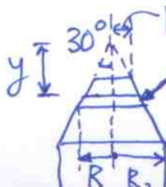
$$dF = \mu_s dN$$

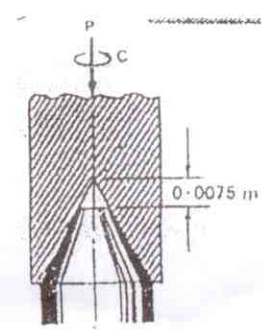
$$dM = dF * r = r\mu_s \frac{P}{\frac{2}{3}\pi(R_o^2 - R_i^2)} r\,d\theta\,dr$$

$$M = \int_0^{\frac{2}{3}(2\pi)}\int_{R_i}^{R_o} dM = \frac{P\mu_s}{\frac{2}{3}\pi}\left(\frac{R_o^3 - R_i^3}{3}\right)\frac{2}{3}(2\pi)\frac{1}{R_o^2 - R_i^2}$$

$$= \frac{2}{3} P\mu_s \frac{R_o^3 - R_i^3}{R_o^2 - R_i^2} = 5.4167 \text{ N.m}$$

P.2. 2. Compute the frictional torque needed to rotate the truncated cone relative to the cone has a 0.05 m diameter base and a 60° cone angle, and is cut off 0.0075 m from the cone tip. The coefficient of dynamic friction is 0.2.





Assume uniform pressure

$$p = \frac{dN}{dA}$$

$$\Sigma F_y: \quad P = \left(\int_A p\,dA\right)\sin 30^\circ = p\sin 30^\circ * A \quad (\because p \text{ is uniform})$$

$$\Rightarrow p = \frac{P}{A\sin 30^\circ}$$

$$A = \int dA = \int 2\pi R \frac{dy}{\cos 30^\circ}$$

$$y = \cot 30^\circ R \Rightarrow dy = \cot 30^\circ dR$$

$$\Rightarrow A = \frac{2\pi}{\sin 30^\circ}\int_{R_1}^{R_2} R\,dR = \frac{\cancel{2}\pi}{\sin 30^\circ}\cdot\left(\frac{R_2^2 - R_1^2}{\cancel{2}}\right)$$

②

$$dM = dF.R = \mu dN.R = \mu p\,dA.R = \mu p \underbrace{\left(R\,d\theta \frac{dy}{\cos 30^\circ}\right)}_{dA} R$$

$$= \mu \frac{P}{A \sin 30^\circ} R^2 d\theta \frac{\cot 30^\circ\, dR}{\cos 30^\circ}$$

$$= \mu \frac{P}{\pi(R_2^2 - R_1^2)} \frac{1}{\sin 30^\circ} d\theta\, R^2 dR$$

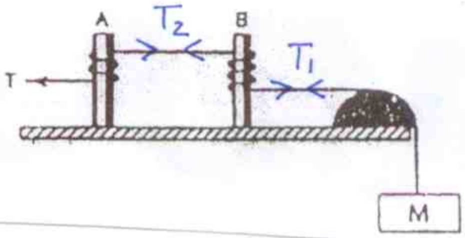
$$\Rightarrow M = \int_0^{2\pi}\int_{R_1}^{R_2} dM = \mu \frac{P}{\cancel{\pi}(R_2^2 - R_1^2)} \frac{1}{\sin 30^\circ} 2\cancel{\pi}\left(\frac{R_2^3 - R_1^3}{3}\right)$$

$$= \frac{2}{3}\frac{\mu P}{\sin 30^\circ}\left(\frac{R_2^3 - R_1^3}{R_2^2 - R_1^2}\right) = \frac{2}{3} * \frac{0.2}{\sin 30^\circ}\left[\frac{0.025^3 - (0.0075 \tan 30^\circ)^3}{0.025^2 - (0.0075 \tan 30)^2}\right] * P$$

$$= 0.006837 P \text{ N.m}$$

P.3.

3. A cord is wrapped twice around one capstan *A* and three times around a second capstan *B*. Finally the cord goes over a half barrel section and supports a mass of 500 kg. What is the force **T** required to maintain this load? Take the coefficient of static friction = 0.1 for all surfaces of contact.



$Mg > T_1 > T_2 > T$ for impending downward motion of **M**.

$$\Rightarrow \frac{Mg}{T_1} = e^{0.1\frac{\pi}{2}}, \quad \frac{T_1}{T_2} = e^{0.1(6\pi)}, \quad \frac{T_2}{T} = e^{0.1(4\pi)}$$

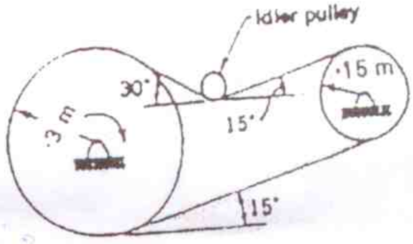
$$\Rightarrow T = e^{-0.4\pi} e^{-0.6\pi} Mg\, e^{-0.05\pi} = Mg\, e^{-1.05\pi} = 181.15\text{ N}$$

Extra: By reciprocating the above relations you get, for impending upward motion of M, $T = Mg\,e^{1.05\pi} = 132811\text{N}$.

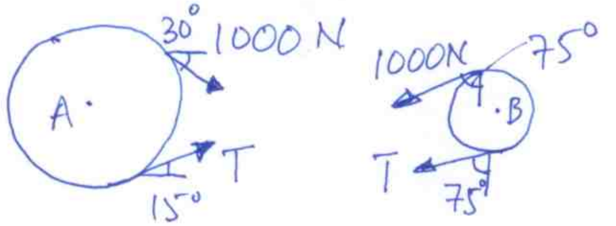
For $181.15 < T < 132811$ the load is held in place but slip is not impending.

1

4. An idler pulley is used to increase the angle of wrap for the pulleys shown in the figure. If the tension in the slack side is 1000 N, find the maximum torque that can be transmitted by the pulleys for a coefficient of friction of 0.3.



Assume idler pulley is small.
$\Rightarrow$ slack slide tension is same for both driving & driven pulleys



$\beta_A = 180 + 30 + 15 = 225^\circ$
$\beta_B = 180^\circ$

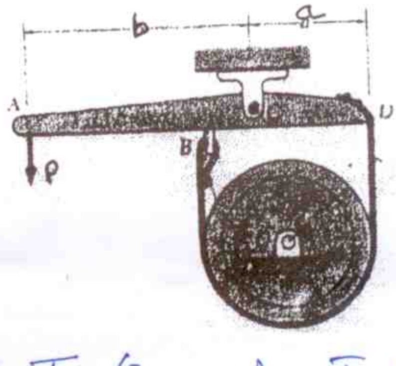
$\because \beta_B < \beta_A$, impending slip is on driven pulley at the max torque transmission state.

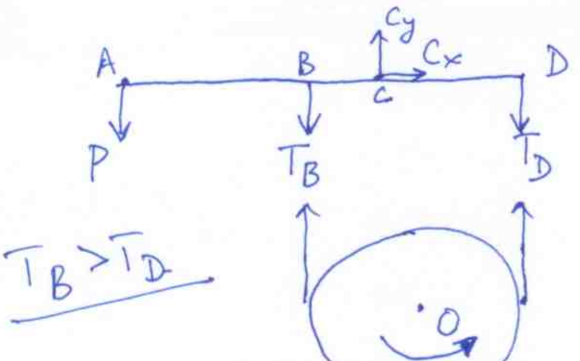
$\Rightarrow \frac{T}{1000} = e^{0.3\beta_B} \Rightarrow T = 2566.33$ N.

So when imp slip at B pulley, A pulley is below imp slip state.

Torque transmitted (from A to B) $= M = (T - 1000) * 0.15 = 1566.33 * 0.15$
$= 234.95$ N.m

P.5. 5. A brake drum of radius r = 150 mm is rotating counterclockwise when a force **P** of magnitude 60 N is applied at A. Knowing that the coefficient of kinetic friction is 0.40, determine
   (a) the moment about O of the friction forces applied to the drum when a = 250 mm, and b = 300 mm, and
   (b) the maximum value of the coefficient of kinetic friction for which the brake is not self-locking.





$T_B > T_D$

$\Sigma M_C: Pb + T_B(2r - a) - T_D(a) = 0 \rightarrow ①$

Imp slip : $\frac{T_B}{T_D} = e^{\mu\pi} \rightarrow ②$

①, ② $\Rightarrow 60(300) + T_D e^{0.4\pi}(300 - 250) - T_D(250) = 0$

$\Rightarrow T_D = 242.19, \; T_B = 850.97$

$M_O = (T_B - T_D) * 150 = 91.316$ N.m

If self locking, then when P is removed ($P=0$) (4)
the braking action should still be present, ie.,
leverarm ABCD should remain horizontal in static
equilibrium under action of T_B and T_D alone,
thus keeping the rope taut so that (2) is still
valid.

$$①\big|_{P=0} \Rightarrow \frac{T_B}{T_D} = \frac{a}{2r-a} \rightarrow ①'$$

$$①', ② \Rightarrow \frac{a}{2r-a} = e^{\mu\pi} \Rightarrow \mu = \frac{1}{\pi}\ln\left(\frac{a}{2r-a}\right) = 0.5123.$$

What happens when $\mu > 0.5123$? In that case,
$T_B(2r-a) - T_D(a) = 0$ (i.e, T_B, T_D still maintain
moment equilibrium of lever ABCD). However,
$\frac{T_B}{T_D} < e^{\mu\pi}$, ie, the second eqn (②) is now
an inequality and the system is SID but
braking action still present w/o input P.