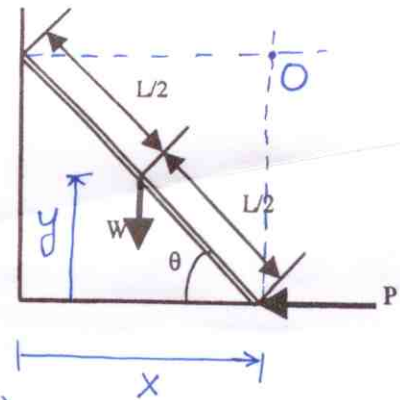
Tutorial # 7 CE 102

P.1 1. Assuming frictionless contacts, determine the magnitude of P for equilibrium.

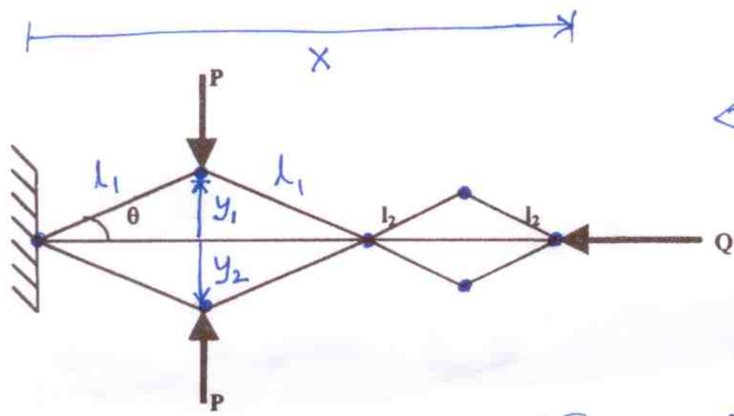
1-DOF (θ), $x = L\cos\theta$, $y = \frac{L}{2}\sin\theta$ AFD $\Rightarrow$



$$\delta U = (-P)(\delta x) + (-W)(\delta y) = 0$$
$$= (-P)(-\cancel{L}\sin\theta\,\delta\theta) + (-W)\left(\frac{\cancel{L}}{2}\cos\theta\,\delta\theta\right) = 0$$
$$\Rightarrow P = \frac{W}{2}\cot\theta$$

(get same result from $\Sigma M_o = 0$), so no substantial advantage of virtual work method in this problem.

P.2. 2. What is the relation among P, Q and θ for equilibrium.



⟸ AFD 1DOF (θ)

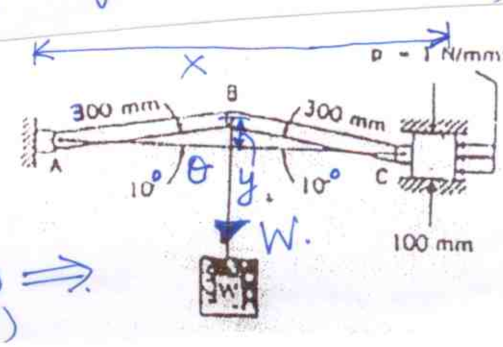
$y_1 = l_1\sin\theta = -y_2$

$x = 2(l_1 + l_2)\cos\theta$

$$\delta U = (-P)(\delta y_1) + (P)(\delta y_2) + (-Q)(\delta x) = 0$$
$$= -P l_1\cos\theta\,\delta\theta - P l_1\cos\theta\,\delta\theta + Q\,2(l_1+l_2)\sin\theta\,\delta\theta = 0$$
$$\Rightarrow \frac{P}{Q} = \frac{(l_1+l_2)}{l_1}\tan\theta$$

(Equilibrium by N.L. approach is tedious — too many equations from where you eliminate until you get relation in P, Q, R.)

P.3 3. The pressure p driving a piston of diameter 100 mm is 1 N/mm^2. At the configuration shown, what weight W will the system hold if friction is neglected?



1-DOF (θ), $\begin{pmatrix} x = 600\cos\theta, \\ y = 300\sin\theta \end{pmatrix}$ AFD $\Rightarrow$ (pA, W)

$$\delta U = (-W)(\delta y) + (-pA)(\delta x) = -W\cdot 300\cos\theta\,\delta\theta - 1 * \frac{\pi\,100^2}{4}(-600\sin\theta\,\delta\theta) = 0$$

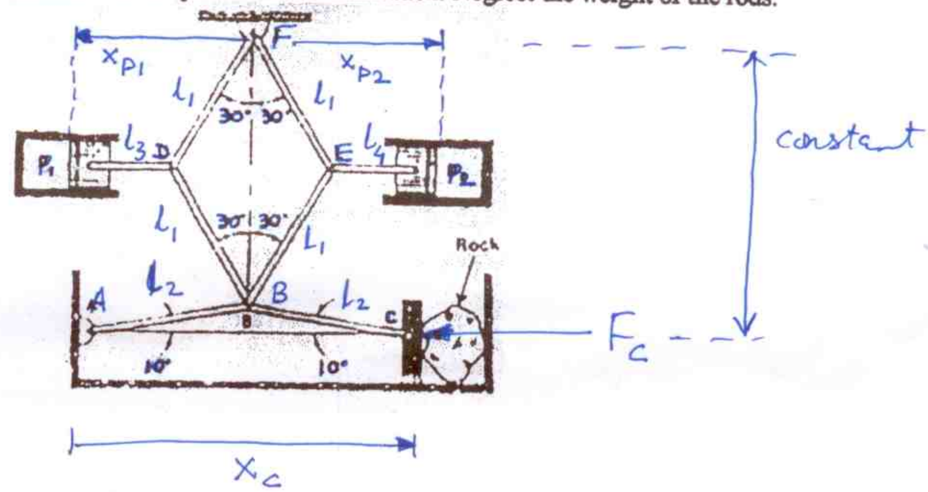
$$\Rightarrow W = \pi \times \frac{100^2}{4} \times \frac{600}{300} \tan 10^\circ = 2769.7\,N$$ ◄ (N.-L. more tedious)

P.4

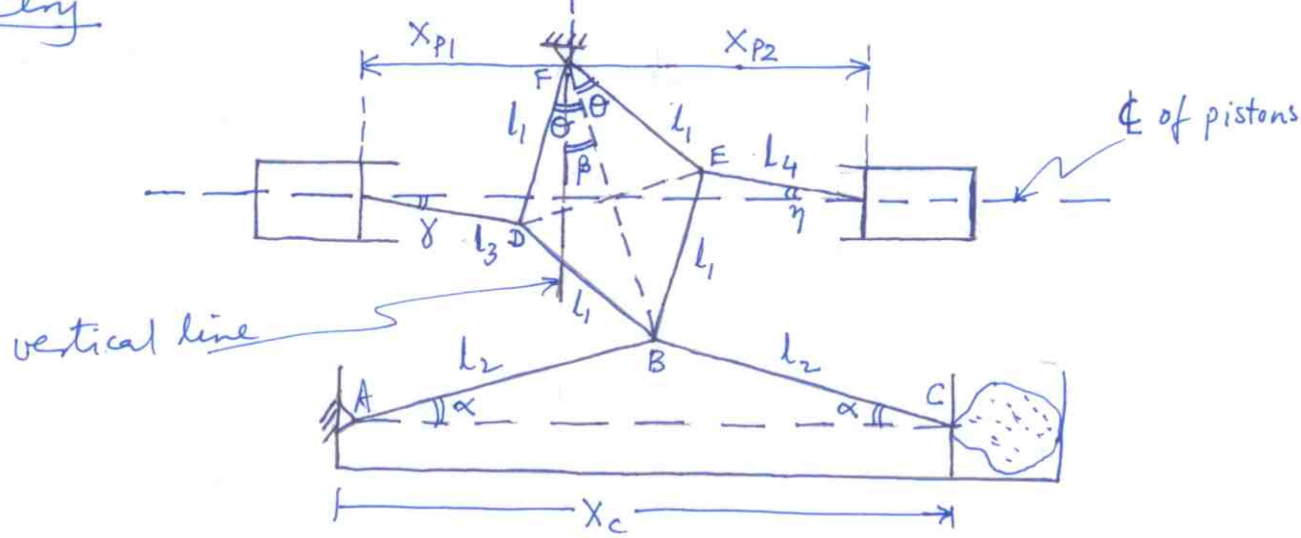
4. Find the force delivered at C in a horizontal direction to crush the rock pressure $p_1 = 100$ Mpa and $p_2 = 60$ Mpa (measured above atmospheric pressure). The diameters of the pistons are 100mm each. Neglect the weight of the rods.

AFD involves p_1A, p_2A, F_c

1-DOF problem.



Geometry



Kinematics (i.e., geometrical relations)

$$2 l_1 \cos\theta \cos\beta + l_2 \sin\alpha = 2 l_1 \cos 30 + l_2 \sin 10 = \text{const} \longrightarrow (a)$$

$$2 l_1 \cos\theta \sin\beta = l_2 (\cos\alpha - \cos 10) \longrightarrow (b)$$

$$\sin\gamma = \frac{l_1 \cos(\theta - \beta) - l_1 \cos 30^\circ}{l_3} \longrightarrow (c)$$

$$\sin\eta = \frac{l_1 \cos 30 - l_1 \cos(\theta + \beta)}{l_4} \longrightarrow (d)$$

$$x_{p1} = - l_1 \sin(\theta - \beta) - l_3 \cos\gamma \longrightarrow (e)$$

$$x_{p2} = l_1 \sin(\theta + \beta) + l_4 \cos\eta \longrightarrow (f).$$

(a), (b), (c), (d) can be viewed as 4 eqns with unknowns $\alpha, \beta, \gamma, \eta$ and θ given. Hence 1-DOF system verified.

Do virtual differential of (a) – (f) :-

(a) $\rightarrow$ $-2l_1 \sin\theta\cos\beta\,\delta\theta - 2l_1\cos\theta\sin\beta\,\delta\beta + l_2\cos\alpha\,\delta\alpha = 0 \rightarrow (1)$

(b) $\rightarrow$ $-2l_1\sin\theta\sin\beta\,\delta\theta + 2l_1\cos\theta\cos\beta\,\delta\beta + l_2\sin\alpha\,\delta\alpha = 0 \rightarrow (2)$

(c) $\rightarrow$ $\cos\gamma\,\delta\gamma = \frac{l_1}{l_3}\sin(\beta-\theta)(\delta\theta-\delta\beta) \rightarrow (3)$

(d) $\rightarrow$ $\cos\eta\,\delta\eta = \frac{l_1}{l_4}\sin(\theta+\beta)(\delta\theta+\delta\beta) \rightarrow (4)$

(e) $\rightarrow$ $\delta x_{p_1} = -l_1\cos(\theta-\beta)(\delta\theta-\delta\beta) + l_3\sin\gamma\,\delta\gamma \rightarrow (5)$

(f) $\rightarrow$ $\delta x_{p_2} = l_1\cos(\theta+\beta)(\delta\theta+\delta\beta) - l_4\sin\eta\,\delta\eta \rightarrow (6)$

For given θ, (a)–(d) can be solved for $\alpha, \beta, \gamma, \eta$. Then (1),(2),(3),(4) can be solved for $\delta\alpha, \delta\beta, \delta\gamma, \delta\eta$ in terms of $\delta\theta$, and the result put into (5),(6) gives δx_{p_1} & δx_{p_2} in terms of $\delta\theta$. Then principle of virtual work is,

$$(p_1 A)\,\delta x_{p_1} + (p_2 A)\,\delta x_{p_2} + (-F_c)\,\delta x_c = 0 \,.\rightarrow (7)$$

Now $\underbrace{x_c = 2l_2\cos\alpha}_{\text{must measure}} \Rightarrow \delta x_c = -2l_2\sin\alpha\,\delta\alpha \rightarrow (8)$

must measure x_c w.r.t. fixed pt (B is not fixed pt, so measure w.r.t. A).

$\delta\alpha$ known in terms of $\delta\theta$

Thus all the virtual displacements, ie, $\delta x_{p_1}, \delta x_{p_2}, \delta x_c$, in (7) are expressable as linear function of $\delta\theta$. We wont find virtual displacements for a general θ (since that is too cumbersome) but for the given data, ie $\theta = 30^\circ$, $\alpha = 10^\circ$, $\beta = \gamma = \eta = 0^\circ$. Put these values in (1) – (6),(8), and get,

(1) $\rightarrow$ $-2l_1\sin 30\,\delta\theta + l_2\cos 10\,\delta\alpha = 0 \rightarrow \delta\alpha = 2\frac{l_1}{l_2}\frac{\sin 30}{\cos 10}\delta\theta$

(2) $\rightarrow$ $2l_1\cos 30\,\delta\beta + l_2\sin 10\,\delta\alpha = 0 \rightarrow \delta\beta = -\frac{l_2}{2l_1}\frac{\sin 10}{\cos 30}\delta\alpha$

$\delta\beta = -\tan 10\tan 30\,\delta\theta$

(3) $\xrightarrow{\&(2)}$ $\delta\gamma = -\frac{l_1}{l_3}\sin 30(\tan 10\tan 30 + 1)\,\delta\theta \rightarrow$ not required

(4) $\xrightarrow{\&(2)}$ $\delta\eta = \frac{l_1}{l_4} \sin 30\ (1 - \tan 10 \tan 30)\ \delta\theta$ $\rightarrow$ not actually required (9)

(5) $\xrightarrow{\&(2)}$ $\delta x_{p1} = -l_1 \cos 30 \left(1 + \tan 10 \tan 30\right) \delta\theta \longrightarrow$ (I)

(6) $\xrightarrow{\&(2)}$ $\delta x_{p2} = l_1 \cos 30 \left(1 - \tan 10 \tan 30\right) \delta\theta \longrightarrow$ (II)

(8) $\xrightarrow{\&(1)}$ $\delta x_c = -2\cancel{l_2} \sin 10\ \frac{2 l_1}{\cancel{l_2}} \frac{\sin 30}{\cos 10}\ \delta\theta = -4 l_1 \tan 10 \sin 30\ \delta\theta \rightarrow$ (III)

Put (I), (II), (III) in (7) :

$$\left(100 * 10^6 * \frac{\pi (0.1)^2}{4}\right)\left(-\cancel{l_1} \cos 30 \left[1 + \tan 10 \tan 30\right] \cancel{\delta\theta}\right)$$

$$+ \left(-60 * 10^6 * \frac{\pi (0.1)^2}{4}\right)\left(\cancel{l_1} \cos 30 \left[1 - \tan 10 \tan 30\right] \cancel{\delta\theta}\right)$$

$$+ \left(-F_c\right)\left(-4 \cancel{l_1} \tan 10 \sin 30\right) \cancel{\delta\theta} = 0$$

$$\Rightarrow F_c = \frac{\frac{\pi (0.1)^2}{4} \cos 30 * 10^6 \left[160 + 40 \tan 10 \tan 30\right]}{4 \tan 10 \sin 30}$$

$$= \frac{\pi * 0.1^2}{16} \cot 10 \cot 30 \left(160 + 40 \tan 10 \tan 30\right) * 10^6 \text{ N}$$

$F_c = 3164.5$ kN.