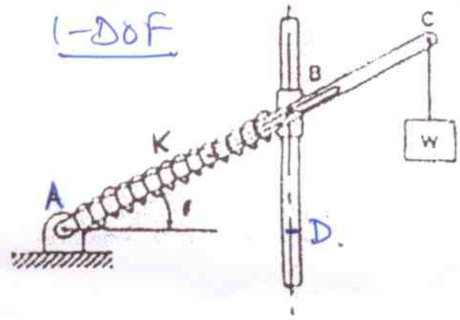
1

1. Rod ABC is connected through a pin and slot to a sleeve which slides on a vertical rod. Before the weight W of 100 N is applied at C, the rod is inclined at an angle of 45°. If K of the spring is 8000N/m. what is the angle θ for equilibrium? The length of AB is 300 mm and the length of BC is 200 mm when θ = 45°. Neglect friction and all weights other than W (Note: The force from the spring is K times its contraction.)



Spring unstretched at $AB = 300$, $\theta = 45°$.

$$AD = \frac{0.3}{\sqrt{2}}$$

$$V = \frac{1}{2}k\left(0.3 - \frac{0.3}{\sqrt{2}}\frac{1}{\cos\theta}\right)^2 + W(0.5)\sin\theta$$

$$\frac{\partial V}{\partial \theta} = 0.3^2 k\left(1 - \frac{1}{\sqrt{2}}\sec\theta\right)\left(-\frac{1}{\sqrt{2}}\tan\theta\sec\theta\right) + 0.5W\cos\theta$$

$$= -509.1168\tan\theta\sec\theta + 360\tan\theta\sec^2\theta + 50\cos\theta \rightarrow (1)$$

$= 0$ for equilibrium.

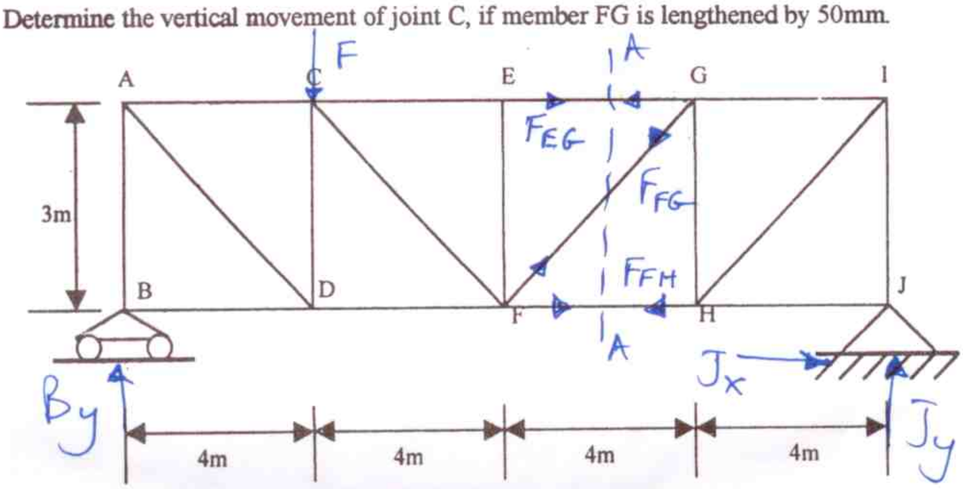
$$\frac{\partial^2 V}{\partial \theta^2} = -509.1168\left(\sec^3\theta + \tan^2\theta\sec\theta\right) + 360\left(\sec^4\theta + 2\tan^2\theta\sec^2\theta\right) - 50\sin\theta \rightarrow (2).$$

Solve (1) by Newton Rhapson (i.e, $\Delta\theta_i = -\left.\frac{\partial V/\partial\theta}{\partial^2 V/\partial\theta^2}\right|_{\theta_i}$, $\theta_{i+1} = \theta_i + \Delta\theta_i$)
use initial guess as $\theta = 1°$, get converged solution $\underline{\theta = 19.221°}$
or can do by trial & error method.

Extra: (Check) $\left.\frac{\partial^2 V}{\partial\theta^2}\right|_{\theta = 19.221} = -135.71$, ie, < 0 so Unstable equilibrium.

2

2. Determine the vertical movement of joint C, if member FG is lengthened by 50mm.



Apply vertical load at C, solve for F_{FG} by method of sections. Then with this vertical load and F_{FG} at joints G & F, apply principle of virtual work with given extension, to get vertical displacement of joint C. NOTE: This method will work only when "infinitesimally" small extensions of members are considered, since we assume that the member forces stay constant during those "infinitesimally" small extensions of the members. ②

method of sections:

$\Sigma M_B = 0 \Rightarrow J_y = \frac{4F}{16} = \frac{F}{4}$

$\Sigma F_y = 0$ for FBD to right of section AA gives,

$F_{FG} \frac{3}{5} = J_y \Rightarrow F_{FG} = \frac{5}{12} F$

Virtual work:

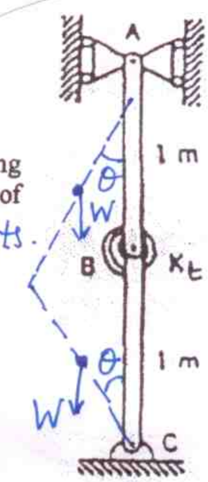
$\delta U = 0 = (-F)\,\delta_C + (-F_{FG})(50\,mm) = -\cancel{F}\delta_C - \frac{5}{12}\cancel{F}(50)$

$\Rightarrow \delta_C = -\frac{5}{12}(50) = \underline{\underline{-20.833\,mm}}$ (i.e 20.833 mm ↓).

③.

3. Two identical rods are pinned together at B and are pinned at A and C. At B, there is a torsional spring with rotational stiffness 500 Nm/rad. What is the max weight W that each rod can have for a case of stable equilibrium when the rods are collinear?

(Weights of AB, BC, act at midpts. as shown)



NOTE: When BC and AB rotate by θ, each, then torsional spring rotates (ie, deforms) by $\theta + \theta = 2\theta$.

$V = \frac{1}{2} k_T (2\theta)^2 + W(0.5\cos\theta + 1.5\cos\theta)$

$\frac{\partial V}{\partial \theta} = 4k_T\theta - 2W\sin\theta = 0 \Rightarrow \theta = 0^\circ$ is equilibrium position.

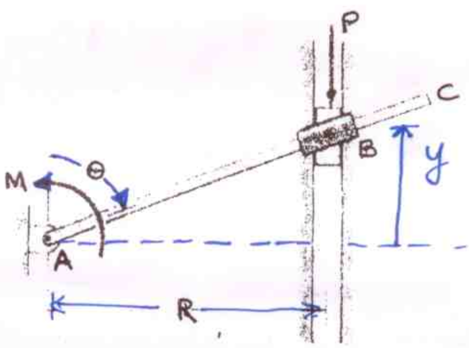
$\frac{\partial^2 V}{\partial \theta^2} = 4k_T - 2W\cos\theta$

For $\frac{\partial^2 V}{\partial \theta^2} > 0$, we need $4k_T > 2W$, ie, $W < 2k_T$

$\underline{\underline{W < 1000\,N}}$

P4

4. Collar B may slide along rod AC and is attached by a pin to a block which may slide in the vertical slot. Derive an expression for the magnitude of the couple M required to maintain equilibrium.



$$\delta U = 0 = (-M)\,\delta\theta + (-P)\,\delta y$$

$$y = \frac{R}{\tan\theta} \Rightarrow \delta y = -\frac{R}{\tan^2\theta}\sec^2\theta = -\frac{R}{\sin^2\theta}$$

$$\Rightarrow (-M)\cancel{\delta\theta} + (-P)\left(-\frac{R}{\sin^2\theta}\right)\cancel{\delta\theta} = 0$$

$$\Rightarrow \underline{\underline{M = \frac{PR}{\sin^2\theta}}}$$