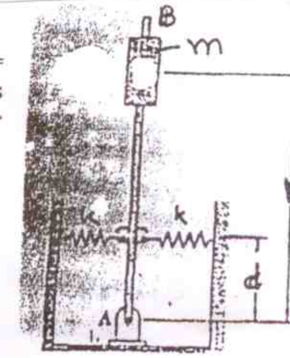
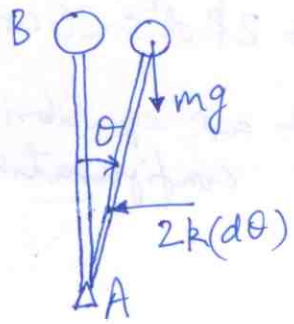
P. 1.

1. Rod AB is attached to a hinge at A and to two springs, each of constant k. If h = 700 mm, d = 300 mm, and m = 20 kg, determine the value of k for which the period of small oscillations is (a) 1 sec, (b) infinite. Neglect the mass of the rod and assume that each spring can act in either **tension or compression.**





$$\Sigma I_A \ddot{\theta} = \Sigma M_A$$

$$(mh^2)\ddot{\theta} = -2k(d\theta)d + mg(h\theta)$$

$$(mh^2)\ddot{\theta} + (2kd^2 - mgh)\theta = 0.$$

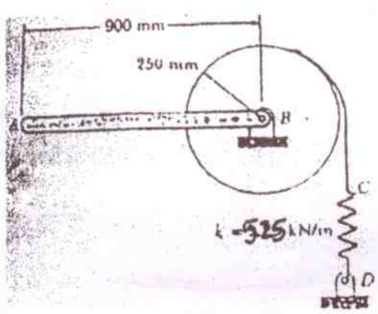
$$\omega_n = \sqrt{\frac{2kd^2 - mgh}{mh^2}}, \quad T_n = \frac{2\pi}{\omega_n}$$

(see over)

For $T_n = \infty$, $\omega_n = 0$, i-e, $k = mgh/2d^2 = 763$ N/m.

For $T_n = 1$, $\omega_n = 2\pi$, ie, $k = \dfrac{4\pi^2 mh^2 + mgh}{2d^2} = 2912.4$ N/m

P. 2



2. A 7-kg slender rod AB is riveted to a 5-kg uniform disk. A belt attaches the rim of the disk to a spring that holds the rod at rest in the position shown. If end A of the rod is moved 18 mm down and released, determine (a) the period of vibration, and (b) the maximum velocity of end A.

Assume static position shown. So weight of rod + disk balance static moment due to static spring force — hence not included in e.o.m.

$$I_B\ddot{\theta} = \Sigma M_B \Rightarrow \left[\frac{7(0.9)^2}{3} + \frac{5(0.25)^2}{2}\right]\ddot{\theta} = -5.25\times10^3(0.25\theta)(0.25)$$

EoM.

$$\ddot{\theta} + \omega_n^2\theta = 0, \quad \omega_n^2 = \frac{5.25\times10^3 \times 0.25^2}{\frac{7}{3}(0.9)^2 + \frac{5}{2}(0.25)^2}$$

$\Rightarrow \omega_n = 12.6631$ rad/s, $T_n = \dfrac{2\pi}{\omega_n} = 0.49618$ sec

$\theta = C\sin(\omega_n t + \psi)$, $\dot{\theta} = C\omega_n\cos(\omega_n t + \psi)$

$\dot{\theta}$ max occurs for $\omega_n t + \psi = 2n\pi$, $n = 0, 1, 2, \cdots$, and equals $C\omega_n$

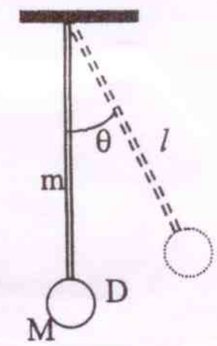
$$C = \left(\theta_0^2 + \frac{\dot{\theta}_0^2}{\omega_n^2}\right)^{1/2} = \theta_0 = \frac{18}{900} \text{ rad}$$

$$\dot{\theta}_{max} = \frac{18}{900} * 12.6631 = 0.25326 \text{ rad/sec}$$

$$(v_A)_{max} = 0.9\,\dot{\theta}_{max} = 0.227935 \text{ m/sec}$$

P.3

3. Determine the natural frequency of the pendulum shown for small oscillations. Consider the rod mass m and the bob to be a sphere of diameter D and mass M.

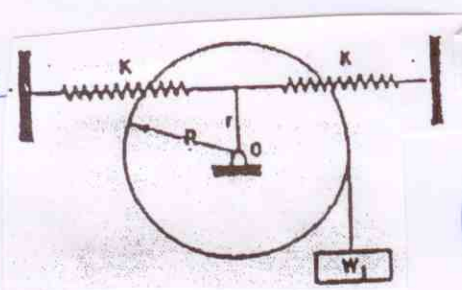


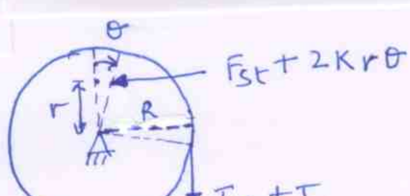
$$I_0\ddot{\theta} = \sum M_0 \Rightarrow \left[\frac{ml^2}{3} + \frac{2}{5}M\frac{D^2}{4} + M\left(l+\frac{D}{2}\right)^2\right]\ddot{\theta} = -mg\frac{l}{2}\theta - Mg\left(L+\frac{D}{2}\right)\theta \rightarrow \text{EOM}$$

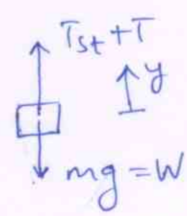
$$\omega_n = \sqrt{\frac{mg\frac{l}{2} + Mg\left(l+\frac{D}{2}\right)}{\frac{ml^2}{3} + Ml^2 + MlD + \frac{7}{5}M\frac{D^2}{4}}}$$

P.4.

A cylinder of mass M and radius R is connected to identical springs and rotates without friction about O. Determine the natural frequency of small oscillations.







Assume static equilibrium shown.

$\sum M_0 = 0$ for static case, from disk FBD,

$\Rightarrow F_{st}\,r = T_{st}\,R \rightarrow (1)$, F_{st}, T_{st} are spring force & tension for static case.

$\sum M_0 = I_0\ddot{\theta}$ for dynamic equilibrium. For small motion about static equilibrium,

$$\frac{MR^2}{2}\ddot{\theta} = -(\cancel{F_{st}} + 2Kr\theta)r + (T + \cancel{T_{st}})R \rightarrow (2)$$

cancel out due to (1).

Now look at FBD of suspended weight.

$\Sigma F_y = 0$ for static equilibrium.

$\Rightarrow W = T_{st} \longrightarrow (3)$

$\Sigma F_y = m\ddot{y}$ for dynamic equilibrium.

$\Rightarrow -T - \cancel{T_{st}} + \cancel{W} = m\ddot{y} \rightarrow (4)$ (ie $y \downarrow$ +ve)

from (3)

Kinematics for small θ gives,

$$y = R\theta \rightarrow (5)$$

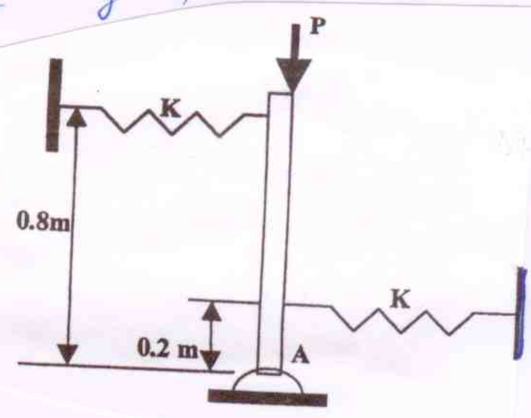
$$(4),(5) \longrightarrow T = -mR\ddot{\theta} \longrightarrow (6)$$

$$(6),(2) \longrightarrow \left(\frac{MR^2}{2} + mR^2\right)\ddot{\theta} + 2Kr^2\theta = 0$$

$$\omega_n = \sqrt{\frac{2Kr^2}{\left(\frac{MR^2}{2} + mR^2\right)}} = \sqrt{\frac{2Kr^2}{\left(\frac{MR^2}{2} + \frac{W}{g}R^2\right)}}$$

P.5

5. **A 1-m rod weighing 60 N is maintained in a vertical position by two identical springs having each a spring constant of 50 N/mm. Determine the vertical force *P* that will cause the natural frequency of the rod to approach zero for small oscillations.**



$$\Sigma M_A = I_A\ddot{\theta} \Rightarrow mg\left(\frac{L}{2}\theta\right) + P(L\theta) - K(0.8\theta)0.8 - K(0.2\theta)0.2 = \frac{ml^2}{3}\ddot{\theta}$$

$$\Rightarrow \frac{60}{9.81}\frac{(1)^2}{3}\ddot{\theta} + \left[K(0.8^2 + 0.2^2) - \frac{mg}{2} - P\right]\theta = 0$$

$$\omega_n = \sqrt{\frac{\left[0.68K - \frac{mg}{2} - P\right]9.81}{20}}$$

$$\omega_n = 0 \Rightarrow P = 0.68K - \frac{mg}{2} = 33970\ \text{N}.$$

Extra: What does $\omega_n = 0$ mean physically? The rod will have ∞ T_n, ie give it a small disturbance θ and it will never return to its original $\theta = 0°$ configuration. By virtual work & Potential energy we have,

$$V = \tfrac{1}{2}K\left[(0.8\sin\theta)^2 + (0.2\sin\theta)^2\right] + Pl\cos\theta + mg\frac{l}{2}\cos\theta$$

(4)

EQUILIBRIUM

$$\frac{\partial V}{\partial \theta} = 0 = K\left(0.8^2 \sin\theta\cos\theta + 0.2^2 \sin\theta\cos\theta\right) - Pl\sin\theta - mg\,l/2\,\sin\theta$$

$\Rightarrow \sin\theta = 0$ and $\cos\theta = \left(P + \frac{mg}{2}\right)l / (0.68K)$ are the two equilibrium configurations.

STABILITY OF EQUILIBRIUM

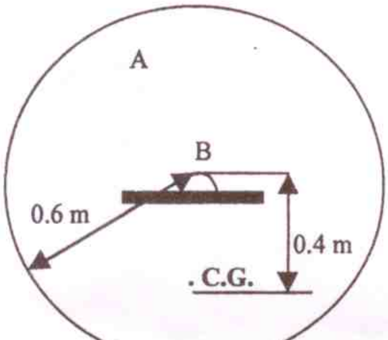
$$\frac{\partial^2 V}{\partial \theta^2} = 0.68K\left(\cos^2\theta - \sin^2\theta\right) - Pl\cos\theta - mg\frac{l}{2}\cos\theta$$

$$\left.\frac{\partial^2 V}{\partial \theta^2}\right|_{\theta=0^\circ} = 0.68K - Pl - \frac{mgl}{2} > 0 \text{ for } P < \frac{0.68K}{l} - \frac{mg}{2}$$

ie $\theta = 0^\circ$ configuration is stable for $P < 33970\,N$ $\longrightarrow$ same result as from vibration analysis.

Moral: $\omega_n = 0$ always represents a condition of loss of stability $\because T_n$ becomes ∞.

P.6



6. A disc weights 445 N and has a radius of gyration of 0.45 m about its axis of symmetry. Note that the centre of gravity does not coincide with the geometric centre. Determine the natural frequency of oscillation. If at the instant that the centre of gravity is directly below ***B***, the disc is rotating at a speed of 0.01 rad/sec counter- clockwise, determine the oscillation amplitude.

- This is an eccentric disk (ie mass distribution is non-uniform so CG does not coincide with geometric center). This typically occurs in rotating machinery – e.g. washing machines, and produces vibrations (forced vibrations). In the present problem, it is one of free vibration, where mg produces restoring moment due to its eccentricity from the geometric center.

$$I_B\ddot{\theta} = \Sigma M_B \Rightarrow \cancel{m}k_B^2\ddot{\theta} = -\cancel{m}g(0.4\theta)$$

$$\ddot{\theta} + \left(0.4g/k_B^2\right)\theta = 0.$$

$$\omega_n = \left[0.4 \times 9.81 / 0.45^2\right]^{1/2} = 4.402 \text{ rad/sec}$$

Initial condts: $\theta_0 = 0$, $\dot{\theta}_0 = 0.01$ $\Rightarrow$ $C = \left[\theta_0^2 + (\dot{\theta}_0/\omega_n)^2\right]^{1/2} = \frac{0.01}{4.402} = 0.002271$ rad

Tute 9 — P-4 (modified) → Assume position shown is for unstressed springs.

$$\frac{MR^2}{2}\ddot{\theta} = -2K(r\sin\theta)(r\cos\theta) + TR\cos\theta \rightarrow ①$$

$$\frac{W_1}{g}\ddot{x} = W_1 - T \longrightarrow ②$$

$$R\sin\theta = x \quad \text{(kinematics)} \longrightarrow ③$$

From ②, ③,

$$T = W_1\left(-R\ddot{\theta}\cos\theta + R\dot{\theta}^2\sin\theta + 1\right) \rightarrow ④$$

Put ④ in ①,

$$\frac{MR^2}{2}\ddot{\theta} + 2Kr^2\sin\theta\cos\theta + W_1\left(R\ddot{\theta}\cos\theta - R\dot{\theta}^2\sin\theta - 1\right)R\cos\theta = 0 \rightarrow ⑤$$

STATIC EQUILIBRIUM.

Put $\ddot{\theta} = \dot{\theta} = 0$,

$$\cos\theta\left[2Kr^2\sin\theta - W_1R\right] = 0.$$

$$\theta_{st_1} = \frac{\pi}{2}, \quad \theta_{st_2} = \sin^{-1}\frac{W_1R}{2Kr^2}$$

same

STABILITY.

$$V = 2 \times \frac{1}{2}K(r\sin\theta)^2 - W_1R\sin\theta$$

$$\frac{dV}{d\theta} = 2K(r\sin\theta)(r\cos\theta) - W_1R\cos\theta$$

$$\frac{d^2V}{d\theta^2} = 2Kr^2\cos2\theta + W_1R\sin\theta$$

$$\therefore \left.\frac{d^2V}{d\theta^2}\right|_{\theta_{st1}} > 0 \text{ for } W_1R > 2Kr^2$$

$$\left.\frac{d^2V}{d\theta^2}\right|_{\theta_{st2}} = 2Kr^2\left(1 - 2\left[\frac{W_1R}{2Kr^2}\right]^2\right) + W_1R\left(\frac{W_1R}{2Kr^2}\right)$$

$$\left.\frac{d^2V}{d\theta^2}\right|_{\theta_{st2}} = 2Kr^2 - \frac{(W_1R)^2}{2Kr^2} > 0 \text{ for } 2Kr^2 > W_1R$$

LINEARIZED MOTION ABOUT θ_{st}.

put $\theta = \theta_{st} + \alpha$, α small, in 5.
Neglect Higher order terms (quadratic & beyond in α, $\dot{\alpha}^2$ etc).

$$\frac{MR^2}{2}\ddot{\alpha} + Kr^2\left(\underline{\underline{\sin 2\theta_{st}}} + 2\cos 2\theta_{st}\,\alpha\right)$$
$$+ W_1\left(R\ddot{\alpha}\left[\cos\theta_{st} - \sin\theta_{st}\,\cancel{\alpha}\right] - R\cancel{\dot{\alpha}^2}\left[\sin\theta_{st} + \cos\theta_{st}\,\alpha\right] - 1\right)R\left(\underline{\underline{\cos\theta_{st}}} - \sin\theta_{st}\,\alpha\right) = 0.$$

neglect $\because \ddot{\alpha}\alpha$. neglect.

underlined terms cancel due to static equilibrium.

$$\Rightarrow \left[\frac{MR^2}{2} + W_1(R\cos\theta_{st})^2\right]\ddot{\alpha} + \left[2Kr^2\cos 2\theta_{st} + W_1R\sin\theta_{st}\right]\alpha = 0$$

6.

now evaluate these for θ_{st1}, θ_{st2} & get two different natural frequencies.