Use of Singularity functions to find deflections

Definitions:

$$\langle x-a\rangle^n = 0, \quad x<a$$
$$= (x-a)^n, \quad x>a \quad \Bigg\} \; n\geq 0$$

$$\langle x-a\rangle^n = 0, \quad x\neq a$$
$$= \text{not defined at } x=a \quad \Bigg\} \; n<0$$

$$\int \langle x-a\rangle^n dx = \frac{\langle x-a\rangle^{n+1}}{n+1}, \quad n\geq 0$$
$$= \langle x-a\rangle^{n+1}, \quad n<0$$

Usage:

If point load $P(\downarrow)$ applied at $x=a$, and point moment $m(\circlearrowleft)$ applied at $x=b$, consider, $W(x) = P\langle x-a\rangle^{-1} + m\langle x-b\rangle^{-2}$

This is the distributed load representation of point loads.

Now $$EIy^{IV} = -W(x) = -P\langle x-a\rangle^{-1} - m\langle x-b\rangle^{-2}$$
$$EIy^{III} = -P\langle x-a\rangle^{0} - m\langle x-b\rangle^{-1}$$
$$EIy^{II} = -P\langle x-a\rangle^{1} - m\langle x-b\rangle^{0}$$

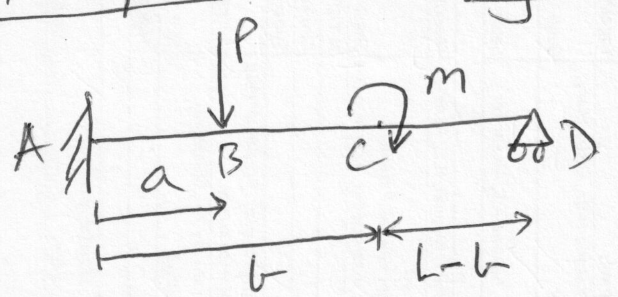
$\therefore$ $V = EIy^{III}$, this shows that shear force decreases by $-P$ when we cross $x=a$

$\therefore$ $M = EIy^{II}$, this shows that bending moment decreases by m as we cross $x=b$.

Both these conclusions are consistent with following FBD's

So for downward P and counterclockwise m, we have negative signs in w, ie −P, −m.

Example



By 4th order method.

$$w(x) = -P\langle x-a\rangle^{-1} + m\langle x-b\rangle^{-2}$$

$$EIy^{IV} = -w(x) = P\langle x-a\rangle^{-1} - m\langle x-b\rangle^{-2}$$

$$EIy^{III} = P\langle x-a\rangle^{0} - m\langle x-b\rangle^{-1} + C_1$$

$$EIy^{II} = P\langle x-a\rangle^{1} - m\langle x-b\rangle^{0} + C_1x + C_2$$

$$EIy^{I} = \frac{P}{2}\langle x-a\rangle^{2} - m\langle x-b\rangle^{1} + \frac{C_1}{2}x^2 + C_2x + C_3$$

$$EIy = \frac{P}{6}\langle x-a\rangle^{3} - \frac{m}{2}\langle x-b\rangle^{2} + \frac{C_1}{6}x^3 + \frac{C_2}{2}x^2 + C_3x + C_4$$

$$EIy\big|_{x=0} = 0 \Rightarrow C_4 = 0$$

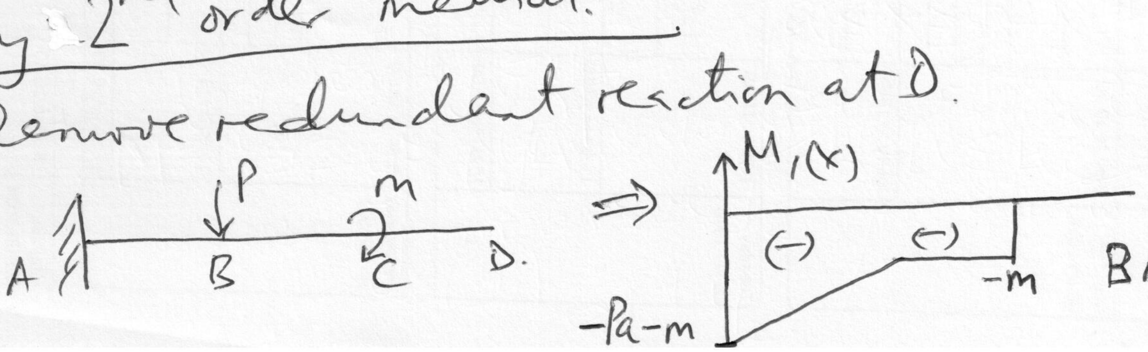
$$EIy^{I}\big|_{x=0} = 0 \Rightarrow C_3 = 0$$

$$\left\{\begin{array}{l} EIy\big|_{x=L} = 0 \Rightarrow \frac{P}{6}(L-a)^3 - \frac{m}{2}(L-b)^2 + \frac{C_1}{6}L^3 + \frac{C_2}{2}L^2 \\ EIy^{II}\big|_{x=L} = 0 \Rightarrow P(L-a) - m + C_1L + C_2 \end{array}\right.$$

solve for C_1, C_2 and hence get $EIy(x)$.

By 2nd order method.

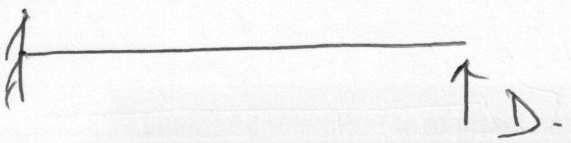
Remove redundant reaction at D.

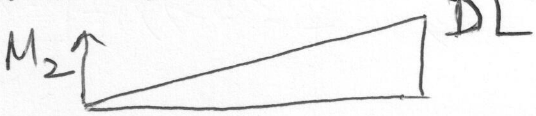


Let y_1 be deflection without redundant.

$EI y_1'' = M_1(x)$ → Need to double-integrate for 3 reg , ie AB, BC, CD, and get 6 constants of integration, and then match displacements & slopes at B^- & B^+ and C^- & C^+

Then introduce redundant at D





Let y_2 be deflection with redundant only applied.

$$EI y_2'' = Dx \Rightarrow EI y_2 = \frac{Dx^3}{6} + C_1 x + C_2$$

<u>Compatibility</u>.

$$(y_1)_{CD}\Big|_{x=L} - y_2\Big|_{x=L} = 0 \rightarrow \text{solve for } D.$$

$$\left.\begin{aligned} y &= (y_1)_{AB} + y_2 \\ &= (y_1)_{BC} + y_2 \\ &= (y_1)_{CD} + y_2 \end{aligned}\right\} \begin{aligned} &\longrightarrow \text{in } 0 \le x \le a \\ &\xrightarrow{\text{with D } \cancel{\text{included}}} \text{in } a \le x \le b \\ &\longrightarrow \text{in } b \le x \le L \end{aligned}$$

<u>TOO TEDIOUS</u>!! So 4th order method is better.