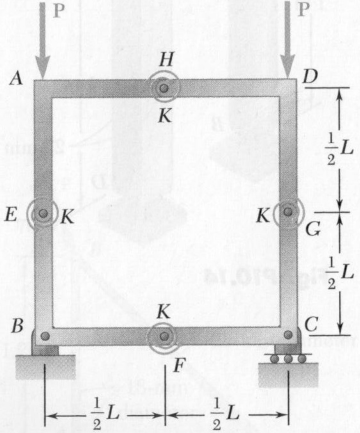
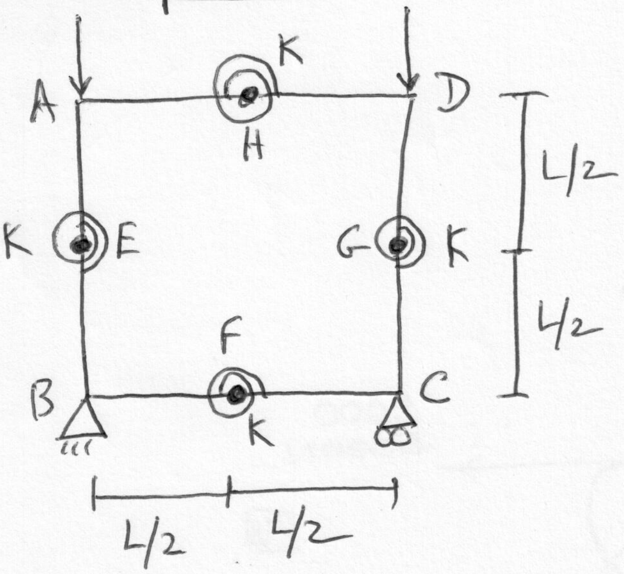
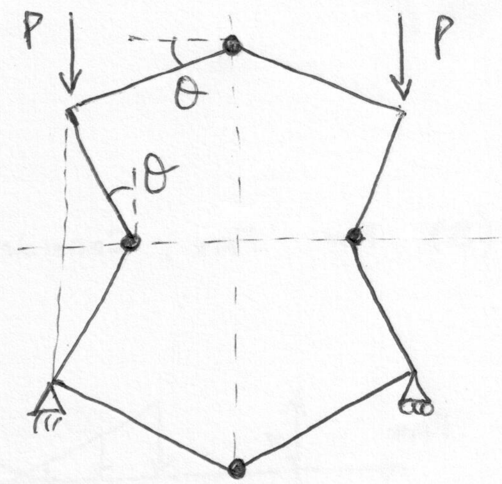
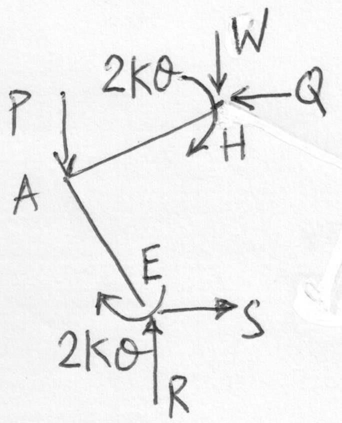


A frame consists of four L-shaped members connected by four torsional springs, each of constant K. Knowing that equal loads $\mathbf{P}$ are applied at points A and D as shown, determine the critical value P_{cr} of the loads applied to the frame.

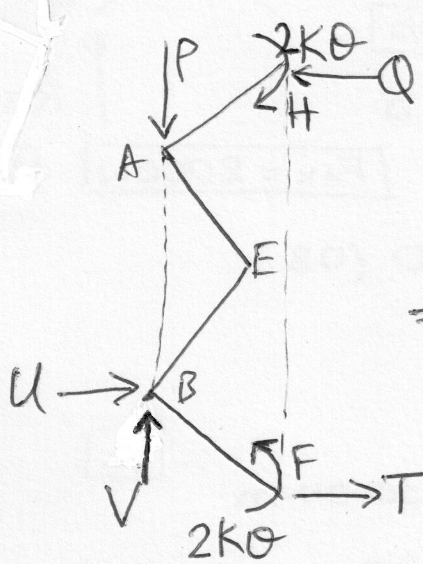
Example 1







$\Rightarrow$ Symmetry $\Rightarrow W=0$, $\Rightarrow R=P$.

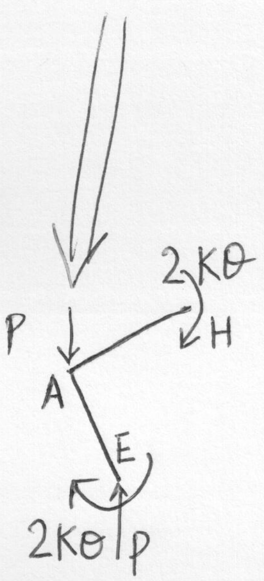


$\Rightarrow$ External equilibrium (for full structure)

$U=0,\ V=P.$

$\Rightarrow \underset{\substack{\text{Half} \\ \text{FBD (HAEBF)}}}{\sum M_B} = 0 \Rightarrow Q=T=0$

Thus from quarter FBD (HAE), $S=Q=0$.



$$\sum M_A = 0 = P\left(\frac{L}{2}\theta\right) = 2(2K\theta) \Rightarrow \boxed{P_{cr} = \frac{8K}{L}}$$

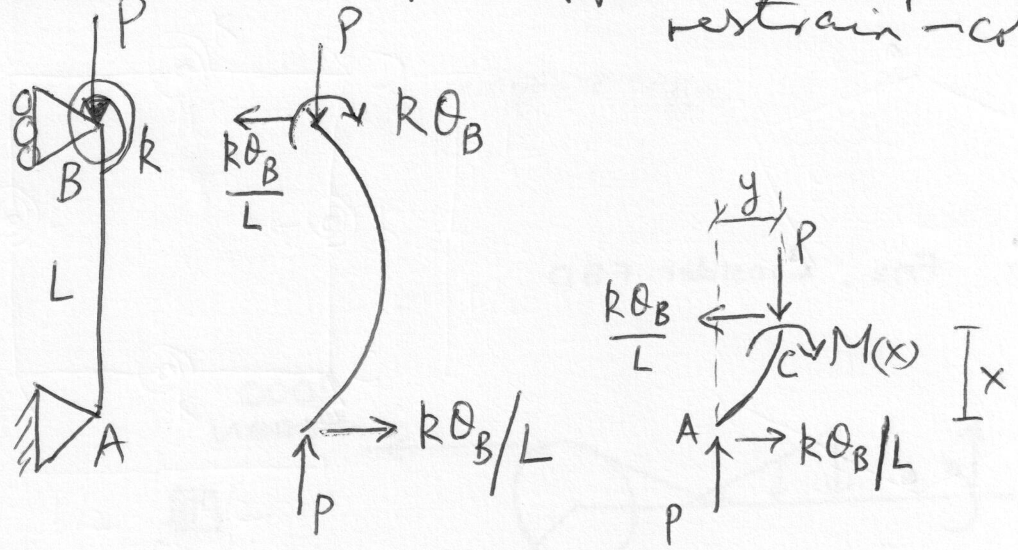
for $\theta \neq 0$.

By Potential energy, $4\left(\frac{1}{2}K(2\theta)^2\right) - 2\left(P\left[L - 2\cdot\frac{L}{2}\cos\theta\right]\right) = V$

$$\frac{\partial V}{\partial \theta} = 0 \Rightarrow 16K\theta - P2L\underbrace{\sin\theta}_{\simeq\theta}$$

$$\Rightarrow P_{cr} = \frac{8K}{L}$$

Example 2 Simply supported with partial restrain-column.



$$\Sigma M_c = 0 \Rightarrow Py + M(x) - \frac{k\theta_B}{L}x = 0$$

$$Py + EIy'' - \frac{k\theta_B}{L}x = 0, \quad \text{let } K^2 = \frac{P}{EI}$$

$$y = C_1 \cos Kx + C_2 \sin Kx + \frac{k\theta_B}{PL}x = 0$$

BC's

$$x=0,\ y=0 \rightarrow C_1 = 0$$

$$x=L,\ y=0 \rightarrow C_2 \sin KL + \frac{k\theta_B}{PL}L = 0.$$

$$x=L,\ y' = -\theta_B \rightarrow C_2 K \cos KL + \frac{k\theta_B}{PL} = -\theta_B$$

$$\begin{bmatrix} \sin KL & k/P \\ K\cos KL & 1 + \frac{k}{PL} \end{bmatrix} \begin{Bmatrix} C_2 \\ \theta_B \end{Bmatrix} = 0 \longrightarrow \underline{\text{EVP}}$$

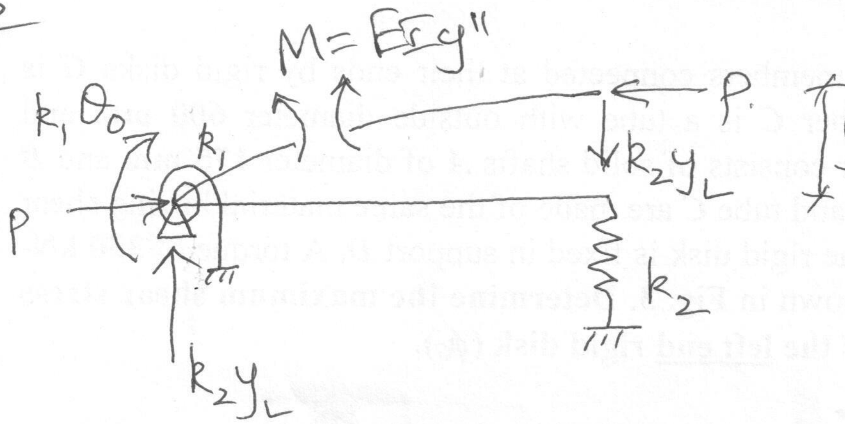
$\det[\ ] = 0$ for non-zero $\{C_2 \ \ \theta_B\}^T$.

$$\tan KL = \frac{Kk}{P}\left(1 + \frac{k}{PL}\right)^{-1} = \frac{KkL}{PL + k} = KL\left(\frac{k}{K^2EIL + k}\right)$$

$$\tan KL = KL\left(\frac{1}{\frac{EI}{k}K^2L + 1}\right) \rightarrow \text{solve numerically}$$

As $k \rightarrow \infty$ it approaches fixed-pinned/roller column
(see Case III of before) $\Leftarrow$

P3



$$Py + EIy'' - k_2 y_L x - k_1\theta_0 = 0 \quad , \quad k^2 = \frac{P}{EI}$$

$$y(x) = A\cos kx + B\sin kx + \frac{k_1\theta_0}{P} + \frac{k_2 y_L}{P}x$$

$$\begin{array}{l} y(0) = 0 \longrightarrow \\ y(L) = y_L \longrightarrow \\ y'(0) = \theta_0 \longrightarrow \\ k_1\theta_0 + k_2 y_L L - P y_L = 0 \longrightarrow \end{array} \begin{bmatrix} 1 & 0 & 1 & 0 \\ \cos kL & \sin kL & 1 & \left(-\frac{P}{k_2} + L\right) \\ 0 & k & -P/k_1 & 1 \\ 0 & 0 & P & \left(PL - \frac{P^2}{k_2}\right) \end{bmatrix} \begin{Bmatrix} A \\ B \\ k_1\theta_0/P \\ k_2 y_L/P \end{Bmatrix} = 0$$

det = 0 or solve simultaneously

$$A = -k_1\theta_0/P \quad ; \quad \frac{k_2 y_L}{P} = -\frac{P}{(PL - P^2/k_2)} k_1\theta_0/P$$

$$B = \frac{1}{k}\left[\frac{P}{k_1} + \frac{1}{L - P/k_2}\right]\frac{k_1\theta_0}{P}$$

So characteristic equation is,

$$\underbrace{\frac{k_1\theta_0}{P}}_{\neq 0}\left\{-\cos kL + \sin kL \frac{1}{k}\left[\frac{k^2 EI}{k_1} + \frac{1}{L - k^2 EI/k_2}\right] + 1 - \overbrace{\cancel{\left[-\frac{k^2 EI}{k_2} + L\right]\left[\frac{1}{L - \frac{k^2 EI}{k_2}}\right]}}^{=1}\right\} = 0$$

$$\Rightarrow \tan kL = k\left[\frac{k^2 EI}{k_1} + \frac{1}{L - k^2 EI/k_2}\right]^{-1}$$

For $k_1 \to \infty$, $k_2 \to \infty$, ie  $\leftarrow P$, get $\tan kL = kL$

For $k_1 \to 0$, $k_2 \to \infty$, ie  $\leftarrow P$, get $\tan kL = 0$, $kL = n\pi$

For $k_1 \to \infty$, $k_2 \to 0$, ie  $\leftarrow P$, get $\tan kL \to \infty$, $kL = (2n+1)\frac{\pi}{2}$

$n = 0, 1, 2, \ldots$