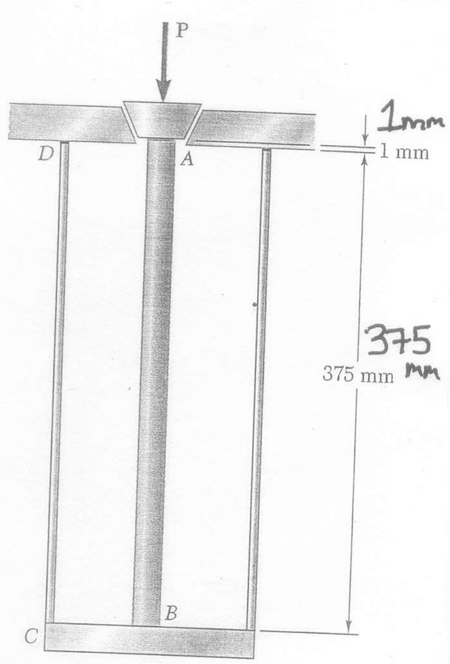
Ex 1 SID



AB → E = 105 GPa, cross section 140 mm^2 (1)

CD → E = 72 GPa, cross section 250 mm^2

CB → rigid end plate of cylinder CD.

Find P to close the 1 mm gap.

Equilibrium of whole (ie ABCD)

$\Rightarrow P_{CD} = P$ (tensile)

Equil of CB $\Rightarrow P_{AB} = P$ (Compressive)

$$\delta_{C/D} = \delta_C = \frac{P_{CD} L_{CD}}{(AE)_{CD}} = \frac{P(375)}{(250)(72 \times 10^3)} \ (\downarrow) \quad \text{(1)}$$

$\delta_B = \delta_C$ compatibility

$$\delta_{B/A} = \delta_B - \delta_A = \delta_C - 1 \quad \text{(2)}$$

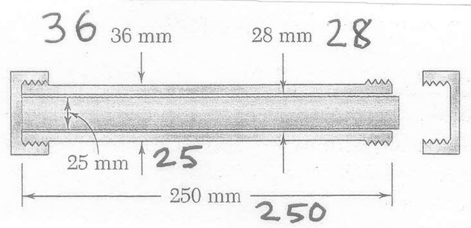
$$\delta_{B/A} = \frac{P_{AB} L_{AB}}{(AE)_{AB}} = \frac{(-P)(375)}{(140)(105 \times 10^3)} \quad \text{(3)}$$

(1), (2), (3),

$$375P\left(\frac{1}{250 \times 72 \times 10^3} + \frac{1}{140 \times 105 \times 10^3}\right) = 1$$

$P = 21578$ N

Ex 2 [SID]



Threaded tube → E = 70 GPa,
36 mm outer dia,
28 mm inner dia.
Screw thread → 1.5 mm pitch.
Inner rod → E = 105 GPa
25 mm dia.
Inner rod slightly longer than tube.
Threaded cover rotated 1/4 turn.
Find σ_{tube}, σ_{rod}, δ_{tube}, δ_{rod}.

Equilibrium of threaded cover $\Rightarrow -P_r = P_t = P$

Compatibility

$$\delta_t - \delta_r = \frac{1}{4} * 1.5 \rightarrow (1)$$

$$\delta_t = \frac{P_t L_t}{(AE)_t} = \frac{P(250)}{\pi\left(\frac{36^2-28^2}{4}\right) * 70*10^3} \rightarrow (2)$$

$$\delta_r = \frac{P_r L_r}{(AE)_r} = \frac{(-P)(250)}{\pi\left(\frac{25^2}{4}\right) * 105*10^3} \rightarrow (3)$$

(1), (2), (3),

$$\frac{250P}{\frac{\pi}{4} * 10^3}\left[\frac{1}{(36^2-28^2)(70)} + \frac{1}{(25^2)(105)}\right] = \frac{1.5}{4}$$

$P = 27309\ N = P_t = -P_r$

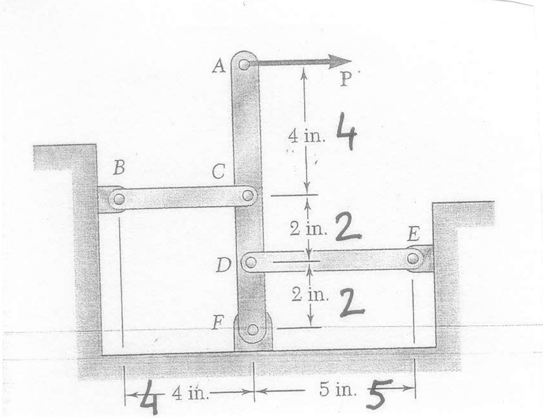
$\sigma_t = \frac{P_t}{A_t} = 67.91\ N/mm^2$, $\delta_t = \frac{P_t L_t}{(AE)_t} = 0.2425\ mm$

$\sigma_r = \frac{P_r}{A_r} = -55.63\ N/mm^2$, $\delta_r = \frac{P_r L_r}{(AE)_r} = -0.1325$

Ex 3 **SID**

BC, DE → E = 29*10^6 psi. (3)

$\frac{1}{2}'' \times \frac{1}{4}''$ cross section



P = 600 lb applied to rigid AF

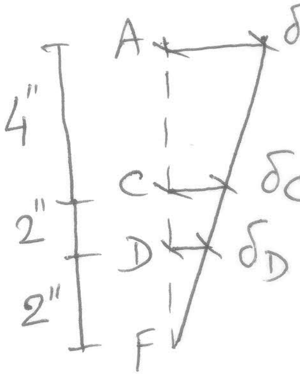
Find a) P_{BC}, P_{DE} b) δ_A

→ Consider P_{BC} (T), P_{DE} (C)

Equilibrium → $\sum M_F = 0 \Rightarrow (600)(8) = P_{BC}(4) + P_{DE}(2)$ → (1)

Compatibility $\delta_C = 2\delta_D$

$$\frac{P_{BC}(4)}{\cancel{(AE)}_{BC}} = \frac{2P_{DE}(5)}{\cancel{(AE)}_{DE}} \rightarrow (2)$$



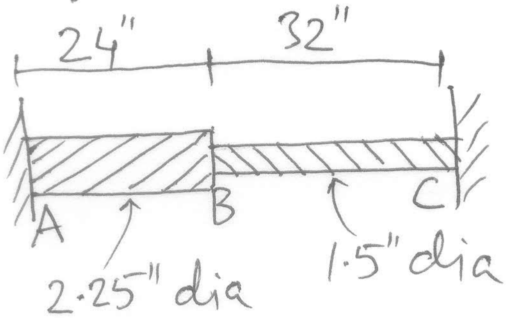
(1), (2) ⇒ $P_{DE} = \frac{(600)(8)}{12}$

$P_{DE} = 400$ lb (C), $P_{BC} = 1000$ lb (T)

$$\delta_A = \frac{8}{2}\delta_D = 4\frac{P_{DE}L_{DE}}{(AE)_{DE}} = \frac{4(400)(5)}{(\frac{1}{2})(\frac{1}{4})(29\times10^6)}$$

$\delta_A = 2.207\times10^{-3}$ in. (→).

Ex 4 SID – Thermal



AB → $E = 29 \times 10^6$ psi

$\alpha = 6.5 \times 10^{-6} / {}^\circ F$

BC → $E = 10.4 \times 10^6$ psi

$\alpha = 13.3 \times 10^{-6} / {}^\circ F$

Find: a) σ_{AB}, σ_{BC} due to $\Delta T = 70^\circ F$

b) δ_B.

(1) Release redundant P (←) at C:
(i.e. free expansion)

$$\underset{(\longrightarrow)}{\delta_{CT}} = (6.5 \times 10^{-6})(70)(24) + (13.3 \times 10^{-6})(70)(32) \rightarrow (1)$$

(2) Apply redundant P (←) at C:

$$\underset{(\longleftarrow)}{\delta_{CP}} = \frac{P_{BC} L_{BC}}{(AE)_{BC}} + \frac{P_{AB} L_{AB}}{(AE)_{AB}}$$

Equilibrium ⇒ $P_{BC} = P_{AB} = P \rightarrow (2)$

$$\Rightarrow \delta_{CP} = P\left(\frac{32}{\frac{\pi}{4}(1.5)^2(10.4 \times 10^6)} + \frac{24}{\frac{\pi}{4}(2.25)^2(29 \times 10^6)}\right) \rightarrow (3)$$

(3) Compatibility:

$$\delta_{CT} = \delta_{CP} \rightarrow (4)$$

(1),(3),(4) ⇒ $P = 20885.19$ lb $= P_{BC} = P_{AB}$

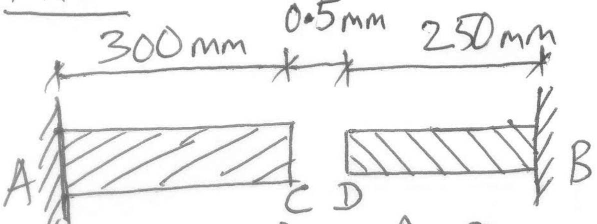
$$\sigma_{BC} = \frac{P_{BC}}{A_{BC}} = 11818.6 \text{ psi (C)}, \quad \sigma_{AB} = \frac{P_{AB}}{A_{AB}} = 5252.7 \text{ psi (C)}$$

$$\delta_B = \underset{(\longrightarrow)}{\delta_{BT}} + \underset{(\longleftarrow)}{\delta_{BP}} = \frac{-P_{AB} L_{AB}}{(AE)_{AB}} + \alpha_{AB} L_{AB} \Delta T$$

$$= \underset{(\longrightarrow)}{6.57 \times 10^{-3} \text{ in.}}$$

SID Thermal

Ex 5



$A = 2000\,mm^2$	$A = 800\,mm^2$
$E = 75\,GPa$	$E = 190\,GPa$
$\alpha = 23 * 10^{-6}/^\circ C$	$\alpha = 17.3 * 10^{-6}/^\circ C$

0.5 mm gap at 20°C (room temp)

When $T = 140^\circ C$,

Find a) normal stresses in each rod

b) change in their lengths.

(1) Free expansion (ie, remove constraint at CD).

$$\delta_{CT} = (23 * 10^{-6})(120^\circ)(300)\ (\rightarrow)$$

$$\delta_{DT} = (17.3 * 10^{-6})(120)(250)\ (\leftarrow)$$

$$(\delta_{C/D})_T = \delta_{CT} + \delta_{DT} = 1.347\ (\rightarrow) > 0.5 \text{ so gap closes}$$

(2) Apply constraint P at interface CD.

Equilibrium $\Rightarrow P_{CA} = P_{DB} = P$.

$$\delta_{CP} = \frac{P_{CA} L_{CA}}{(AE)_{CA}} = \frac{P(300)}{(2000)(75 * 10^3)}\ (\leftarrow)$$

$$\delta_{DP} = \frac{P_{DB} L_{DB}}{(AE)_{DB}} = \frac{P(250)}{(800)(190 * 10^3)}\ (\rightarrow)$$

$$(\delta_{C/D})_P = \delta_{CP} + \delta_{DP} = 3.6447 * 10^{-6} P\ (\leftarrow).$$

(3) Compatibility

$$(\delta_{C/D})_T + (\delta_{C/D})_P = 0.5\ (\rightarrow)$$

gap ← $(\delta_{C/D})_T$ $(\rightarrow)$; $(\delta_{C/D})_P$ $(\leftarrow)$ → overlap

$$1.347 - 3.6447 * 10^{-6} P = 0.5$$

$$\Rightarrow P = 232390\ N.$$

(Ex5 contd)

$$\sigma_{AC} = \frac{P_{AC}}{2000} = \frac{232390}{2000 * 10^{-6}} = 0.116195 \text{ GPa}$$

$$\sigma_{BD} = \frac{P_{BD}}{800} = \frac{232390}{800 * 10^{-6}} = 0.290487 \text{ GPa}$$

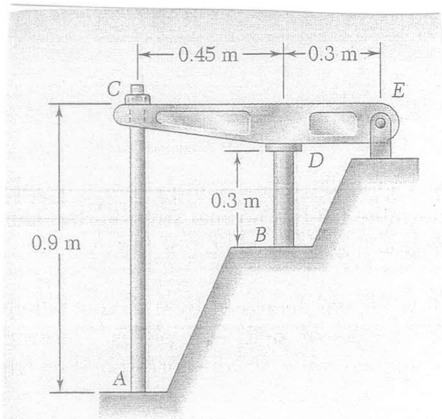
$$\Delta L_{AC} = (\Delta L_{AC})_T + (\Delta L_{AC})_P$$

$$= \delta_{CT} - \delta_{CP} = 0.828 - 0.46478 = 0.3632 \text{ mm}$$

$$\Delta L_{BD} = \delta_{DT} - \delta_{DP} = 0.519 - 0.38222 = 0.1368 \text{ mm}$$

Check: $\Delta L_{AC} + \Delta L_{BD} = \text{gap} = 0.5$

Ex 6 SID – Thermal



CDE rigid, attached to pin support at E, rests on BD.

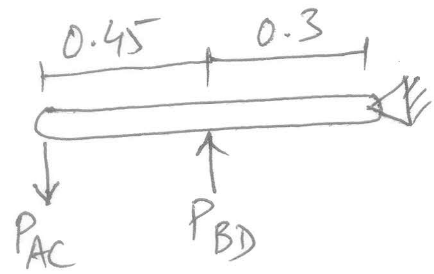
BD → dia = 30mm, E = 105 GPa, $\alpha = 20.9 \times 10^{-6}/°C$

Rod AC → dia = 22mm, E = 200 GPa, $\alpha = 11.7 \times 10^{-6}/°C$

AC passes thru hole in CDE and secured by nut.

Initial T = 20°C. Then temp of BD raised to 50°C, temp AC remains 20°C.

Find σ_{BD}, σ_{AC}.



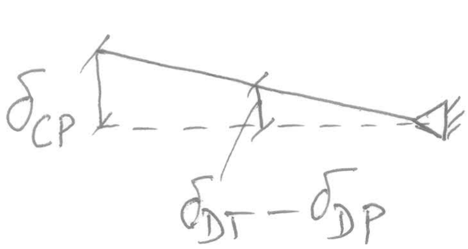
(1) Equilibrium

$$P_{AC}(0.75) = P_{BD}(0.3) \longrightarrow (1)$$

(2) Free expansion (remove constraint at C).

$$\delta_{DT} = \alpha_{BD} \Delta T_{BC} L_{BD} = (20.9 \times 10^{-6})(30)(0.3) = 1.881 \times 10^{-4} \rightarrow (2)$$

(3) Compatibility with constraint applied at C



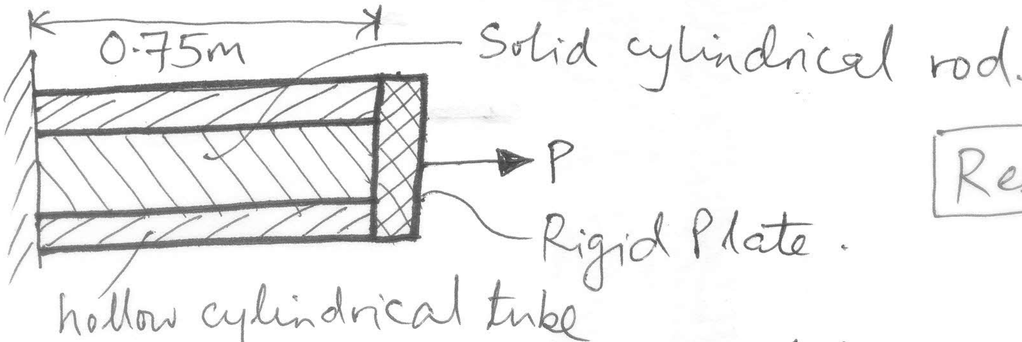
$$\frac{\delta_{DT} - \delta_{DP}}{0.3} = \frac{\delta_{CP}}{0.75}$$

$$\left(1.881 \times 10^{-4} - \frac{P_{BD} L_{BD}}{(AE)_{BD}}\right)(0.75) = (0.3)\frac{P_{AC} L_{AC}}{(AE)_{AB}} \rightarrow (3)$$

(1),(3) ⇒ $P_{AC} = 12675\,N \Rightarrow \sigma_{AC} = 33.34\,N/mm^2$ (MPa)

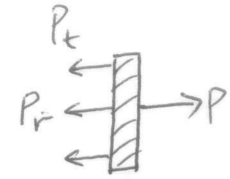
$P_{BC} = 31687.5\,N \Rightarrow \sigma_{BC} = 44.83\,MPa.$

Ex 7



Residual Stresses

Cylindrical rod placed inside tube of same length. Ends of rod & tube attached to rigid support on left end and rigid plate on right end. Load P on assembly increased from 0 to 25 kN & decreased back to 0. Find:
(a) Load-deflection diagram (b) Max elongation (c) permanent set (d) Residual stresses



$\delta_r = \delta_t$ always.

$$\Rightarrow \frac{P_r(0.75)}{(48)(210E3)} = \frac{P_t(0.75)}{(62)(105E3)}$$

in elastic range.

$\sigma_r = 2\,\sigma_t$ in elastic range.

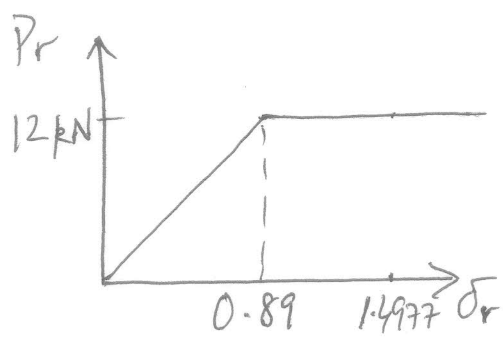
Further $(\sigma_r)_y = 250$ MPa, $(\sigma_t)_y = 310$ MPa so rod yields first. $(\because (\sigma_r)_y < (\sigma_t)_y)$

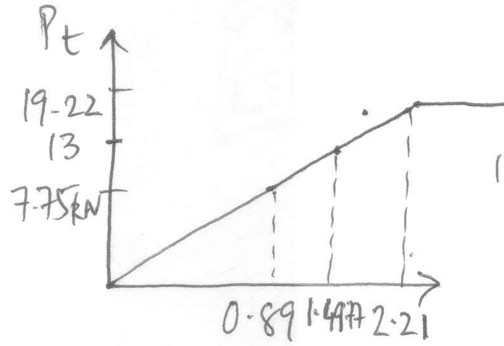
$(P_r)_y = (250)(48) = 12000$ N.

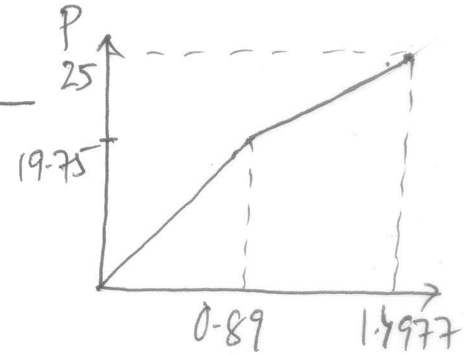
$$(\delta_r)_y = \frac{(250)(750)}{(210E3)} = 0.8929 \text{ mm}$$

When rod yields, $P_t = \dfrac{(0.8929)(62)(105E3)}{(750)} = 7750$ N.

After rod yields, all excess load borne by tube.







$(P_t)_y = (310)(62) = 19220$ kN.

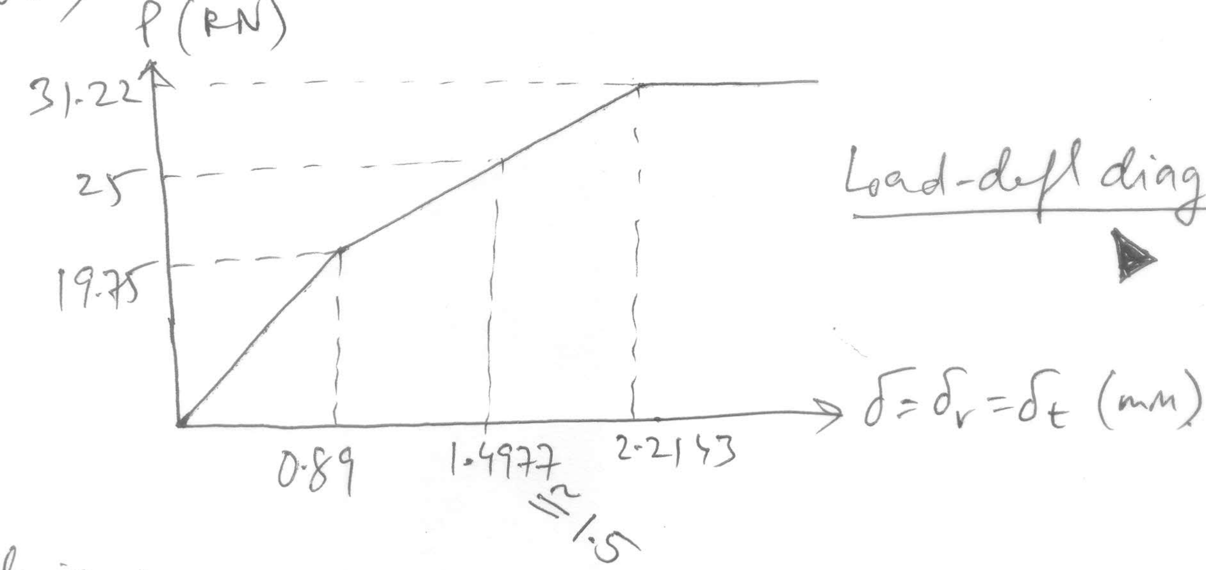
$$(\delta_t)_y = \frac{(310)(750)}{(105E3)} = 2.2143 \text{ mm}$$

At 25 kN,

$$\delta_r = \delta_t = \frac{(13)(750)}{(62)(105)} = 1.4977 \text{ mm}$$

$\blacktriangleright \delta_{max}$

If load increased beyond 25 kN, then at yield point of tube, we have



Load-defl diag. ◀

Unloading:

$$P_r + P_t = P$$

$$\left[12 - (1.5-\delta)\frac{12}{0.89}\right] + \left[13 - (1.5-\delta)\frac{13}{1.5}\right] = P.$$

For $P = 0$, $\delta_p = 0.3713$ (permanent set). ◀

Residual forces

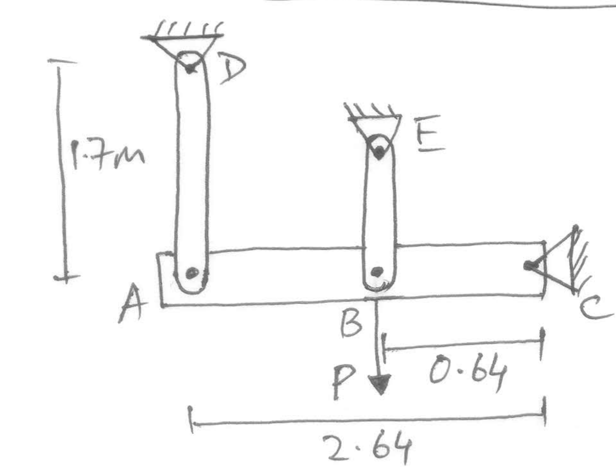
$$(P_r)_p = 12 - (1.5 - 0.3713)\frac{12}{0.89} = -3.22 \text{ kN}.$$

$$(P_t)_p = 13 - (1.5 - 0.3713)\frac{13}{1.5} = 3.22 \text{ kN}.$$

$$(\sigma_r)_p = \frac{-3.22}{48} * 10^9 = 67 \text{ MPa (C)}$$ ◀

$$(\sigma_t)_p = \frac{3.22}{62} * 10^9 = 51.9 \text{ MPa (T)}$$ ◀

Ex8 — RESIDUAL STRESSES



ABC → rigid.

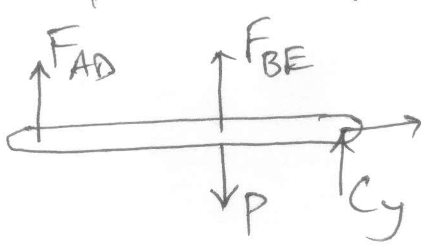
AD, BE → 37.5 mm × 6mm rectangular section

$E = 200\,GPa$, $\sigma_y = 250\,MPa$

P increased from 0 to 260 kN & decreased back to 0.

Find: (a) Max deflection of B
(b) Residual stresses in AD, BE
(c) Final deflection of B.

Equilibrium:



$\Sigma M_C = 0$, ⇒ $F_{AD}(2.64) + (F_{BE} - P)(0.64) = 0$ ← ① (always true).

Compatibility: ② ← $\delta_A = \frac{2.64}{0.64}\delta_B$ → always true. ~~when both rods within elastic limits.~~

When both rods in elastic limit,

$$\frac{F_{AD}(1.7)}{AE} = \frac{F_{BE}(1)}{AE}(4.125) \rightarrow ③$$

- So $F_{AD} > F_{BE}$ when both rods within elastic limit.
- So F_{AD} yields first, since areas of both rods are same

At yield, $(F_{AD})_y = (250)(37.5 \times 6) = 56250\,N$.

$F_{BE} = 23182\,N$ (from ③).

$P = 255213.25\,N$.

$\delta_A = (56250)(1.7)/AE = 2.125$, $\delta_B = \frac{0.64}{2.64}\delta_A = 0.5152$

Beyond yield, as P increases, F_{AD} remains constant at $(F_{AD})_y$. Eqn (3) becomes invalid. But Eqns (1), (2) always valid. So F_{BE} takes up all the excess load applied in order to maintain equilibrium.

At P = 260 kN, $F_{AD} = (F_{AD})_y = 56250$.

$$F_{BE} = 23182 + (260000 - 255213)$$

$$= 27969\ \text{N} < (F_{BE})_y = 56250$$

$$(\delta_B)_{max} = \frac{F_{BE}(1)}{AE} \quad (\because \text{BE not yielded})$$

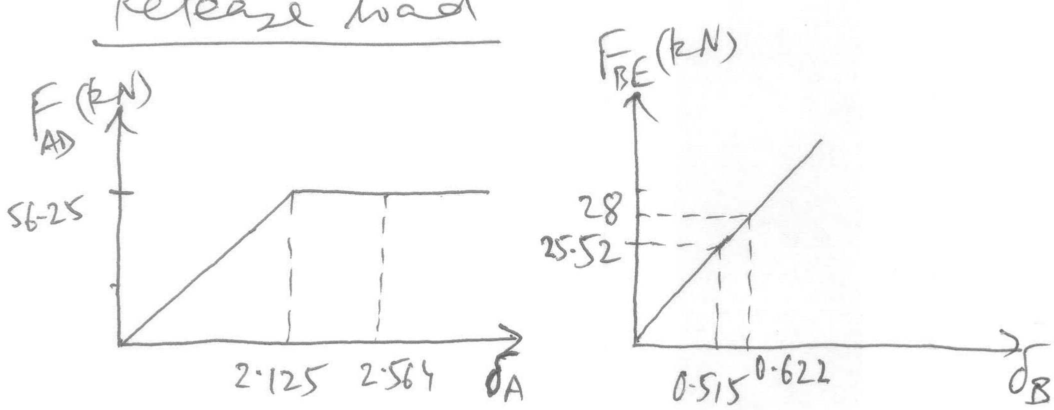
$$= \frac{(27969)(1000)}{(37.5 * 6)(200E3)} = 0.6215\ \text{mm} \downarrow$$

$$(\delta_A)_{max} = \frac{2.64}{0.64}(\delta_B)_{max} = 2.5637\ \text{mm}$$

$$\sigma_{AD} = \sigma_y = 250\ \text{MPa}$$

$$\sigma_{BE} = \frac{27969}{(37.5)(6)} = 124.3\ \text{MPa}$$

Release load



$$F_{AD} = 56.25 - (2.564 - \delta_A)\left(\frac{56.25}{2.125}\right) \longrightarrow (4)$$

$$F_{BE} = 27.969 - (0.6215 - \delta_B)\,\frac{27.969}{0.6215} \longrightarrow (5)$$

(1), (2), (4), (5) with P = 0 gives, $(\delta_B)_{P=0,res} = 0.09679\ \text{mm}$ ◄

From (4), (5) with $(\delta_B)_{P=0,res} = 0.09676$ and $(\delta_A)_{P=0,res} = \left(\frac{2.64}{0.64}\right)(\delta_B)_{P=0}$

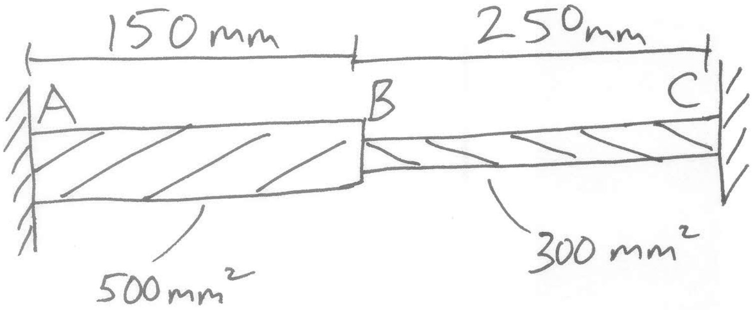
we get,

$(F_{AD})_{res} = -1.05525$ kN $\Rightarrow (\sigma_{AD})_{res} = -4.69$ MPa ◄

$(F_{BE})_{res} = 4.35443$ kN $\Rightarrow (\sigma_{BE})_{res} = 19.35$ MPa ◄

——————— X ———————

Ex9 →



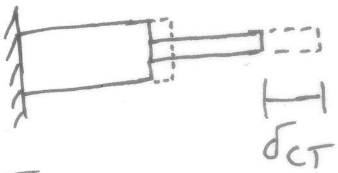
$E = 200$ GPa YIELD

$\sigma_y = 250$ MPa

$\Delta T = 125°C$

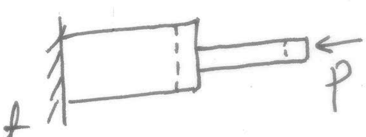
$\alpha = 11.7 \times 10^{-6}/°C$

Find a) Stresses b) δ_B



Free deflection due to ΔT :

$\delta_{CT} = (11.7 * 10^{-6})(125)(400) = 0.585$ mm



Fitting it back :

$A_{AB} > A_{BC}$, so BC yields first.

At yield, $P = P_y = P_{BC} = (\sigma_y)(300) = (250)(300)$
$= 75000$ N. $= P_{AB}$

$$\delta_{C,P_y} = \frac{75000}{200E3}\left(\frac{150}{500} + \frac{250}{300}\right) = 0.425 \text{ mm}$$

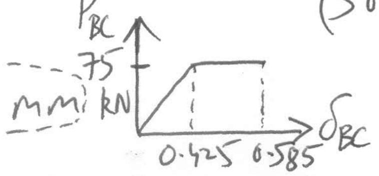
At P_y, portion BC strains further, by any amount, without further stress. So slack of $(0.585 - 0.425) = 0.16$ taken up by BC. So B remains stationary, ie no further displacement, and entire 0.16mm displ is at C.

$$\delta_B = (11.7 \times 10^{-6})(125)(150) + \frac{(75000)(150)}{(500)(200E3)}$$

$(\longrightarrow)$

(Consider AB portion, superpose $\delta_{B,T}$ & δ_{B,P_y})

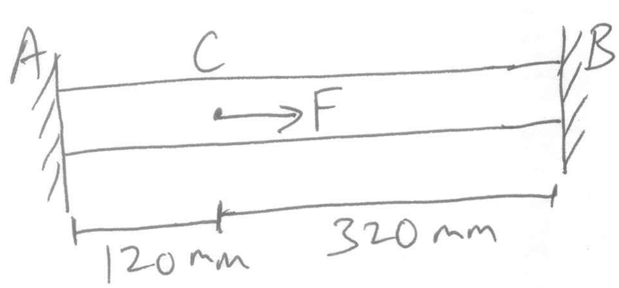
$= 0.1069 (\longrightarrow)$ mm



$$\sigma_{AB} = \frac{75000}{500} = 150 \text{ MPa}, \quad \sigma_{BC} = \frac{75000}{300} = 250 \text{ MPa}$$

———— X ————

Ex 10



Area = 1200 mm²
E = 200 GPa
σ_y = 250 MPa

RESIDUAL STRESS

'F' increases from 0 to 520 kN and decreases back to 0.

Find a) Permanent deflection of C
b) Residual stresses.

<u>Upto yield</u>:

Remove right constraint $\rightarrow \delta_F = \frac{F(120)}{1200 * 200E3}$

Apply constraint P at right $\rightarrow \delta_P = \frac{P(440)}{(1200)(200E3)}$

Compatibility $\rightarrow \delta_F = \delta_P \Rightarrow P = \frac{6}{22} F = F_{CB}$

$$F_{AC} = F - P = \frac{16}{22} F$$

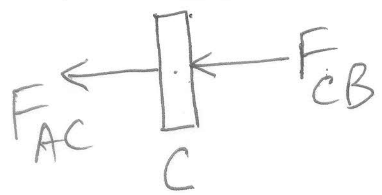
<u>Note</u>: This uses superposition; which is valid only upto linear elastic limit, ie upto yield.

$\because F_{AC} > F_{CB}$ & areas are same, AC yields first.

So $\frac{16}{22} F_y = (F_{AC})_y = (250)(1200) = 300000 \text{ N} \Rightarrow 412500 \text{ N} = F_y$.

F_y = F value when AC yields

Beyond yield



$F = F_{AC} + F_{CB}$, always true.
(T) (C)

F_{AC} saturates at yield, ie $(F_{AC})_{max} = (F_{AC})_y$
$= 300000N$.

So any excess F applied is countered (balanced) by F_{CB}. So, at F = 520 kN,

$$F_{CB} = \frac{6}{22} F_y + (520 - F_y) = 220000N = (F_{CB})_{max}$$

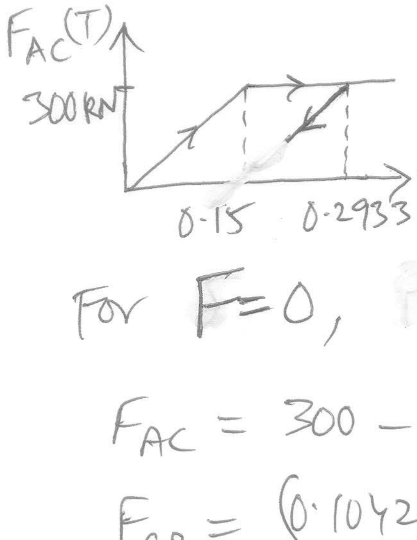
At yield, $\delta_{C,y} = \frac{(250)(120)}{(250E3)} = 0.15$ mm.

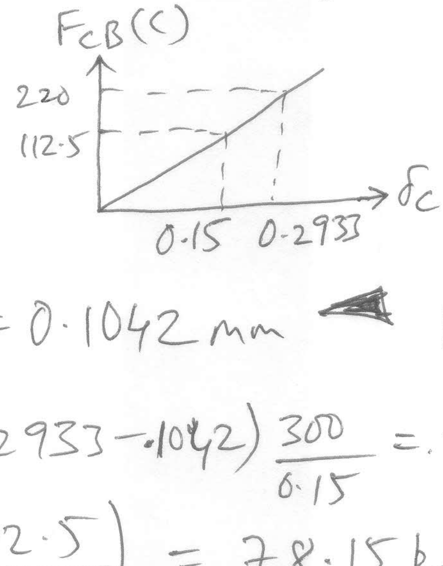
Beyond yield, $\delta_{C,max} = \frac{(F_{CB})_{max} * L_{CB}}{AE} = \frac{(220000) \times (320)}{(1280)(200E3)}$

$= 0.2933$ mm.

Unloading

$$F = 300 - (0.2933 - \delta)\frac{300}{0.15} + \delta\left(\frac{112.5}{0.15}\right)$$





For F = 0, $\delta = 0.1042$ mm ◄ permanent set.

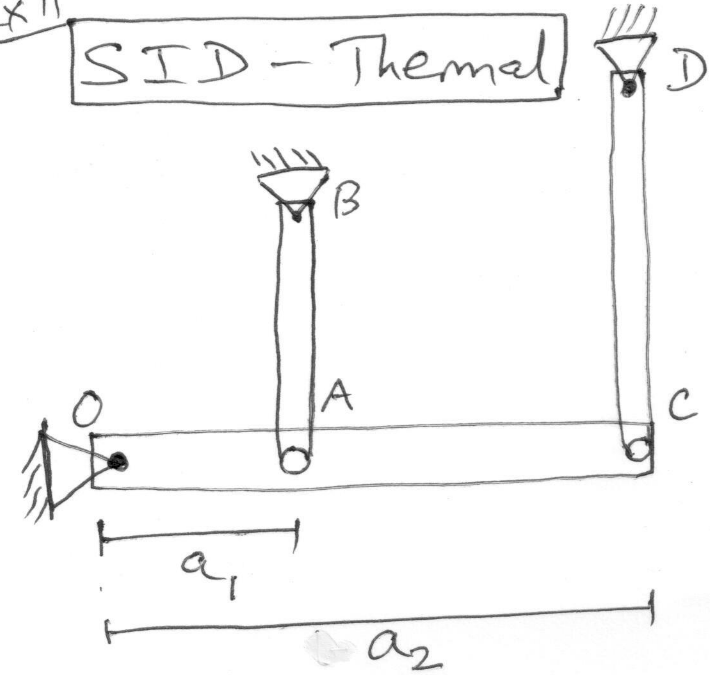
$$F_{AC} = 300 - (0.2933 - .1042)\frac{300}{0.15} = -78.2 \text{ kN} = 78.2 \text{ kN (C)}$$

$$F_{CB} = (0.1042)\left(\frac{112.5}{0.15}\right) = 78.15 \text{ kN (C)} \simeq 78.2 \text{ (C)}$$

$$\Rightarrow \sigma_{AC} = \frac{78.2 * 1000}{1200} = 65.2 \text{ MPa} = \sigma_{CB}$$, both compressive

Ex 11

SID – Thermal



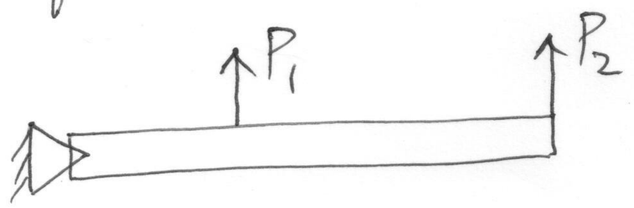
OAC → Rigid.

AB → L_1, E_1, A_1, α_1, ΔT_1

CD → L_2, E_2, A_2, α_2, ΔT_2

Solve for δ_A, δ_C, σ_{AB}, σ_{CD}.

Equilibrium:



$$\sum M_0 = 0$$

$$\Rightarrow P_1 a_1 + P_2 a_2 = 0 \rightarrow (1)$$

Displacements:

Let, $W_1 = \alpha_1 \Delta T_1 L_1$ and $W_2 = \alpha_2 \Delta T_2 L_2$

ie, W_1, W_2 are free thermal displacements of points A and C, respectively, ie free thermal elongations of AB & CD, respectively.

$$\delta_1 = \delta_{AB} = W_1 + \frac{P_1 L_1}{A_1 E_1} \; ; \; \delta_2 = \delta_{CD} = W_2 + \frac{P_2 L_2}{A_2 E_2}$$

Compatibility:

$$\frac{\delta_1}{a_1} = \frac{\delta_2}{a_2}$$

$$\Rightarrow \left(\frac{L_1}{A_1 E_1 a_1}\right) P_1 - \left(\frac{L_2}{A_2 E_2 a_2}\right) P_2 = \frac{W_2}{a_2} - \frac{W_1}{a_1} \rightarrow (2)$$

$$①,② \rightarrow P_1 = \frac{\frac{W_2}{a_2} - \frac{W_1}{a_1}}{\frac{L_1}{A_1E_1a_1} + \frac{L_2}{A_2E_2a_2}\left(\frac{a_2}{a_1}\right)}$$

$$P_2 = -P_1\left(\frac{a_2}{a_1}\right)$$

Put P_1, P_2 in δ_{AB}, δ_{CD}, and $\sigma_{AB} = \frac{P_1}{A_1}$, $\sigma_{CD} = \frac{P_2}{A_2}$

$\quad\quad\quad\quad\quad\quad\quad \overset{\shortparallel}{\delta_A} \quad\quad \overset{\shortparallel}{\delta_C}$

Moral of story:

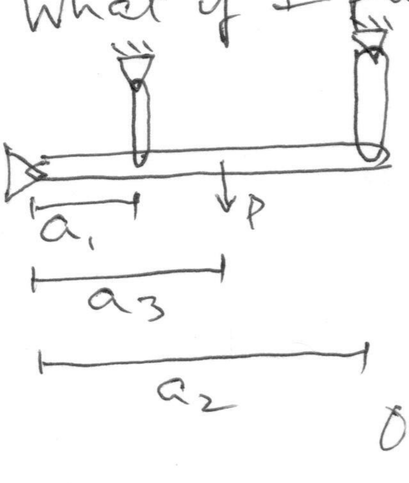
No need to decide physically which rod in tension & which in compression. Assume both in tension and let the signs work themselves out.

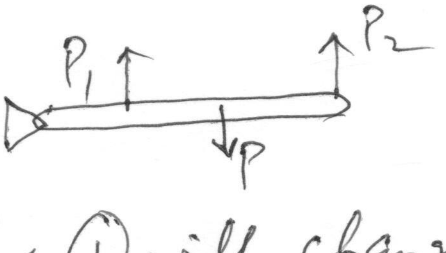
If $\frac{W_2}{a_2} > \frac{W_1}{a_1}$, $P_1 > 0$, $P_2 < 0$

$P_1 > 0 \downarrow$ AB tension, $P_2 < 0 \downarrow$ CD compression

& vice-versa if $\frac{W_2}{a_2} < \frac{W_1}{a_1}$.

What if I put mechanical load 'P' at a_3, ie, in addittion to given thermal load.





Only ① will change to $P_1a_1 + P_2a_2 = Pa_3$

② remains same → Solve P_1, P_2.