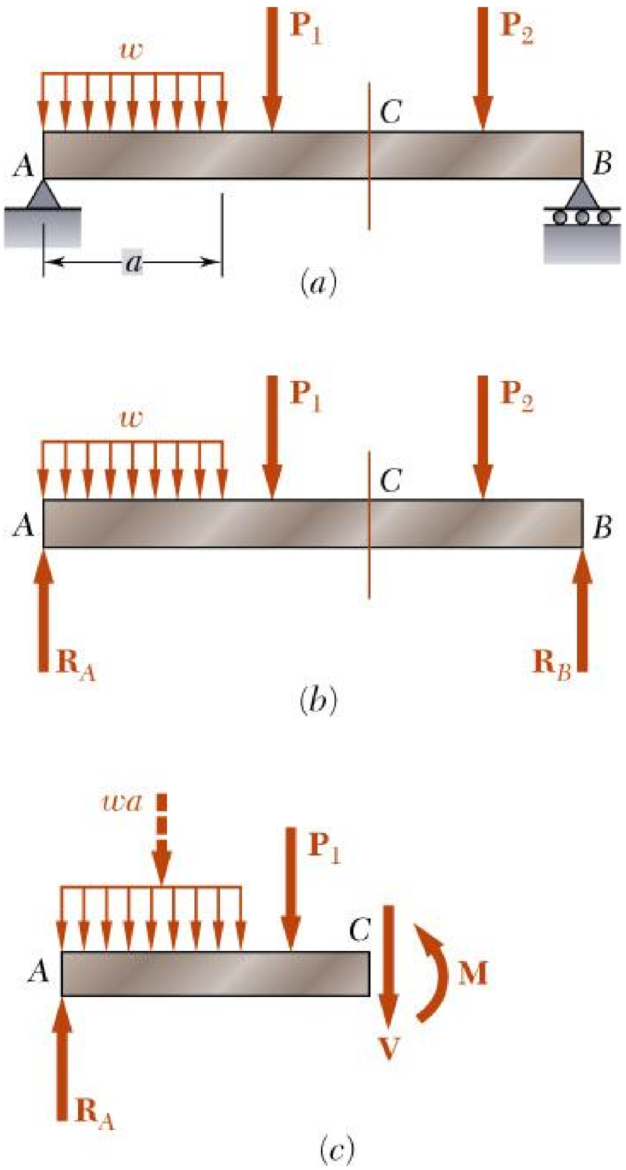
MECHANICS OF MATERIALS

Bending Moment and Shear Force Diagrams

Introduction

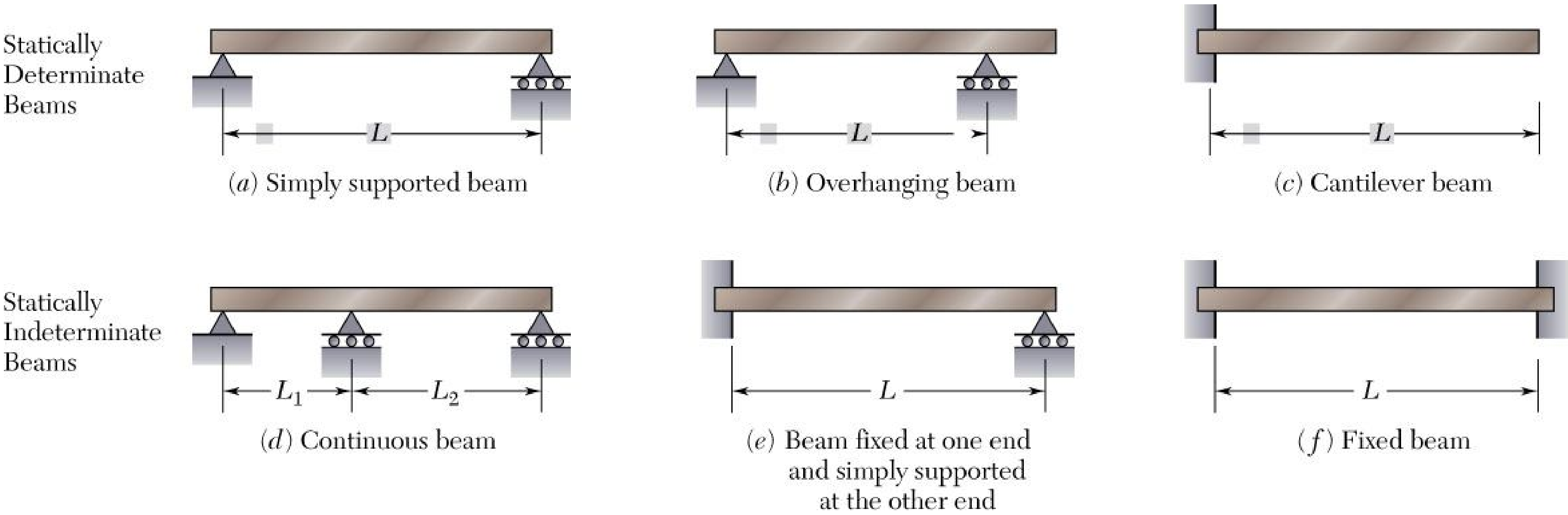


Beams - structural members supporting loads at various points along the member

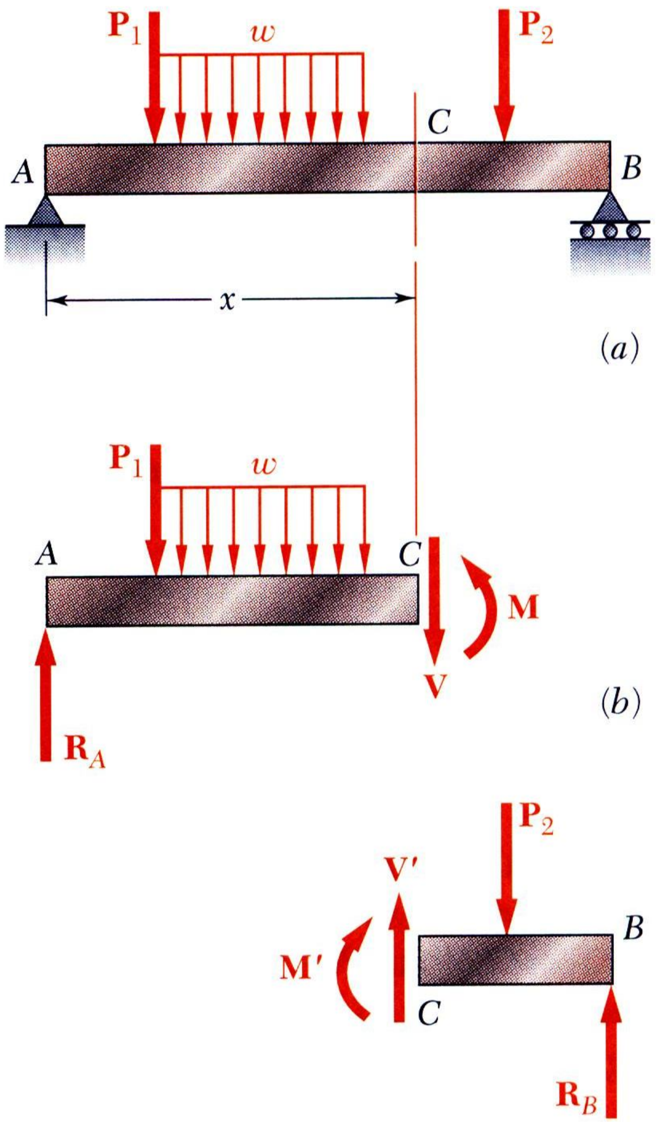
Transverse loadings of beams are classified as *concentrated* loads or *distributed* loads

Applied loads result in internal forces consisting of a shear force (from the shear stress distribution) and a bending couple (from the normal stress distribution)

Types of Beam supports



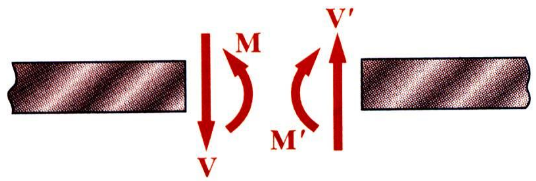
Shear and Bending Moment Diagrams



- We need maximum internal shear force and bending couple in order to find maximum normal and shear stresses in beams.

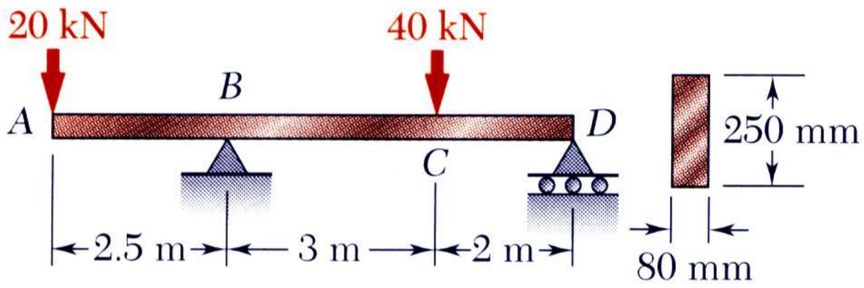
- Shear force and bending couple at a point are determined by passing a section through the beam and applying an equilibrium analysis on the beam portions on either side of the section.

- Sign conventions for shear forces *V* and *V'* and bending couples *M* and *M'*



(*a*) Internal forces
(positive shear and positive bending moment)

Example 1

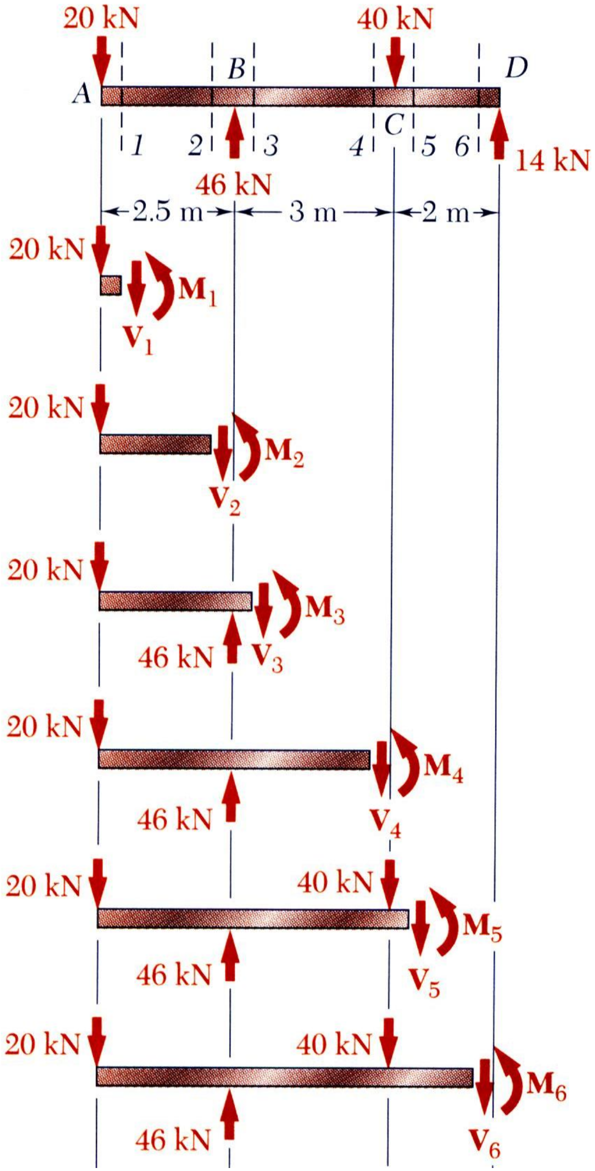


For the timber beam and loading shown, draw the shear and bend-moment diagrams

SOLUTION:

- Treating the entire beam as a rigid body, determine the reaction forces
- Section the beam at points near supports and load application points. Apply equilibrium analyses on resulting free-bodies to determine internal shear forces and bending couples
- Identify the maximum shear and bending-moment from plots of their distributions.

Example 1



SOLUTION:

- Treating the entire beam as a rigid body, determine the reaction forces

  from $\sum F_y = 0 = \sum M_B : \quad R_B = 40\,\text{kN} \quad R_D = 14\,\text{kN}$

- Section the beam and apply equilibrium analyses on resulting free-bodies

$$\sum F_y = 0 \quad -20\text{ kN} - V_1 = 0 \qquad V_1 = -20\,\text{kN}$$

$$\sum M_1 = 0 \quad (20\,\text{kN})(0\,\text{m}) + M_1 = 0 \quad M_1 = 0$$

$$\sum F_y = 0 \quad -20\text{ kN} - V_2 = 0 \qquad V_2 = -20\,\text{kN}$$

$$\sum M_2 = 0 \quad (20\,\text{kN})(2.5\,\text{m}) + M_2 = 0 \quad M_2 = -50\,\text{kN}\cdot\text{m}$$

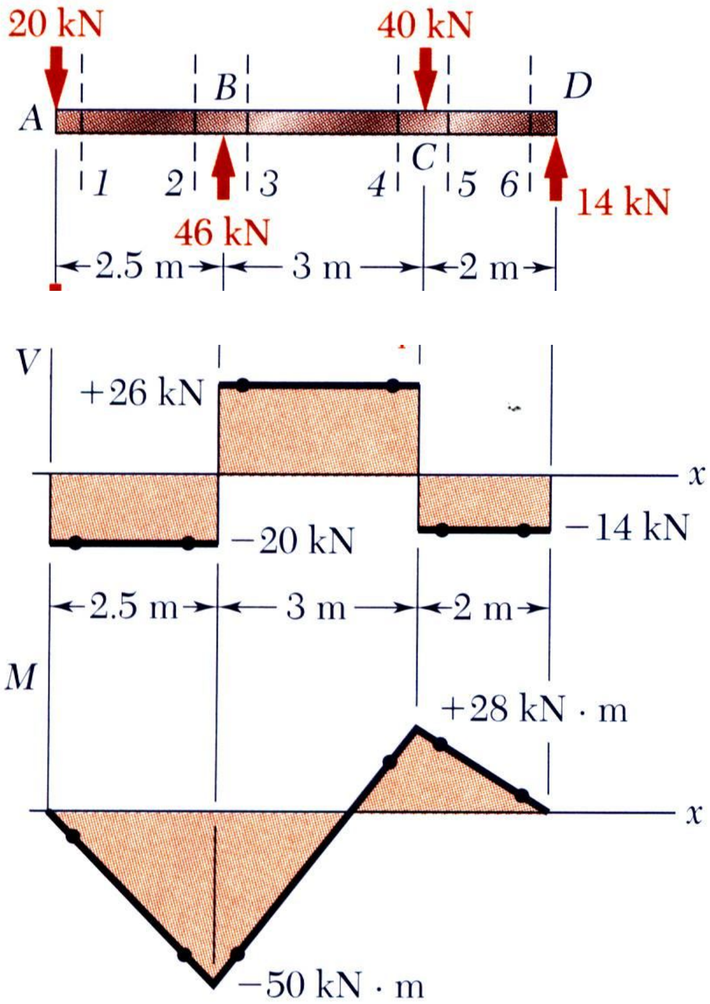
$$V_3 = +26\,\text{kN} \quad M_3 = -50\,\text{kN}\cdot\text{m}$$

$$V_4 = +26\,\text{kN} \quad M_4 = +28\,\text{kN}\cdot\text{m}$$

$$V_5 = -14\,\text{kN} \quad M_5 = +28\,\text{kN}\cdot\text{m}$$

$$V_6 = -14\,\text{kN} \quad M_6 = 0$$

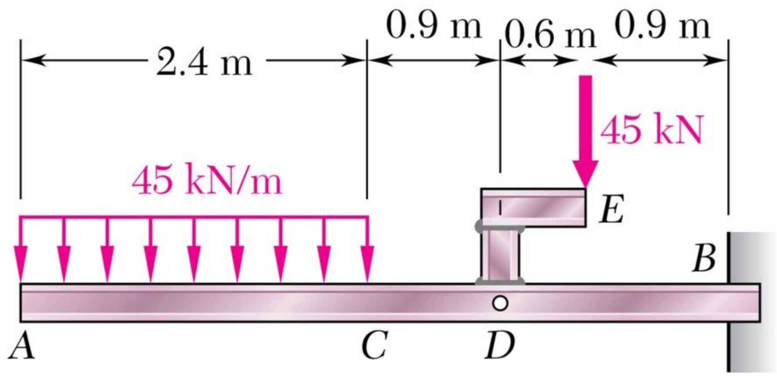
Example 1



- Identify the maximum shear and bending-moment from plots of their distributions.

$$V_m = 26\,\text{kN} \quad M_m = |M_B| = 50\,\text{kN}\cdot\text{m}$$

Example 2



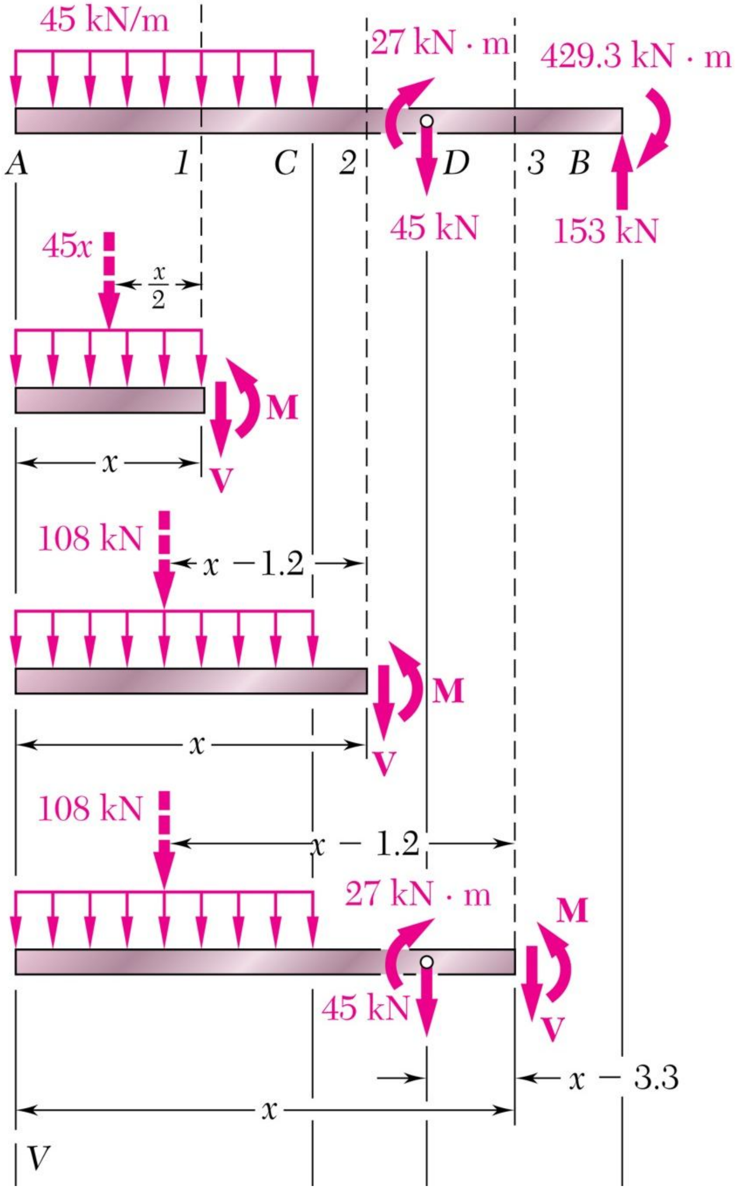
The structure shown is constructed of a W 250x167 rolled-steel beam. Draw the shear and bending-moment diagrams for the beam and the given loading.

SOLUTION:

- Replace the 45 kN load with an equivalent force-couple system at *D*. Find the reactions at *B* by considering the beam as a rigid body.

- Section the beam at points near the support and load application points. Apply equilibrium analyses on resulting free-bodies to determine internal shear forces and bending couples.

Example 2



SOLUTION:

- Replace the 45 kN load with equivalent force-couple system at *D*. Find reactions at *B*.
- Section the beam and apply equilibrium analyses on resulting free-bodies.

From A to C:

$$\sum F_y = 0 \quad -45x - V = 0 \qquad V = -45x \text{ kN}$$

$$\sum M_1 = 0 \quad (45x)\left(\tfrac{1}{2}x\right) + M = 0 \quad M = -22.5x^2 \text{ kNm}$$

From C to D:

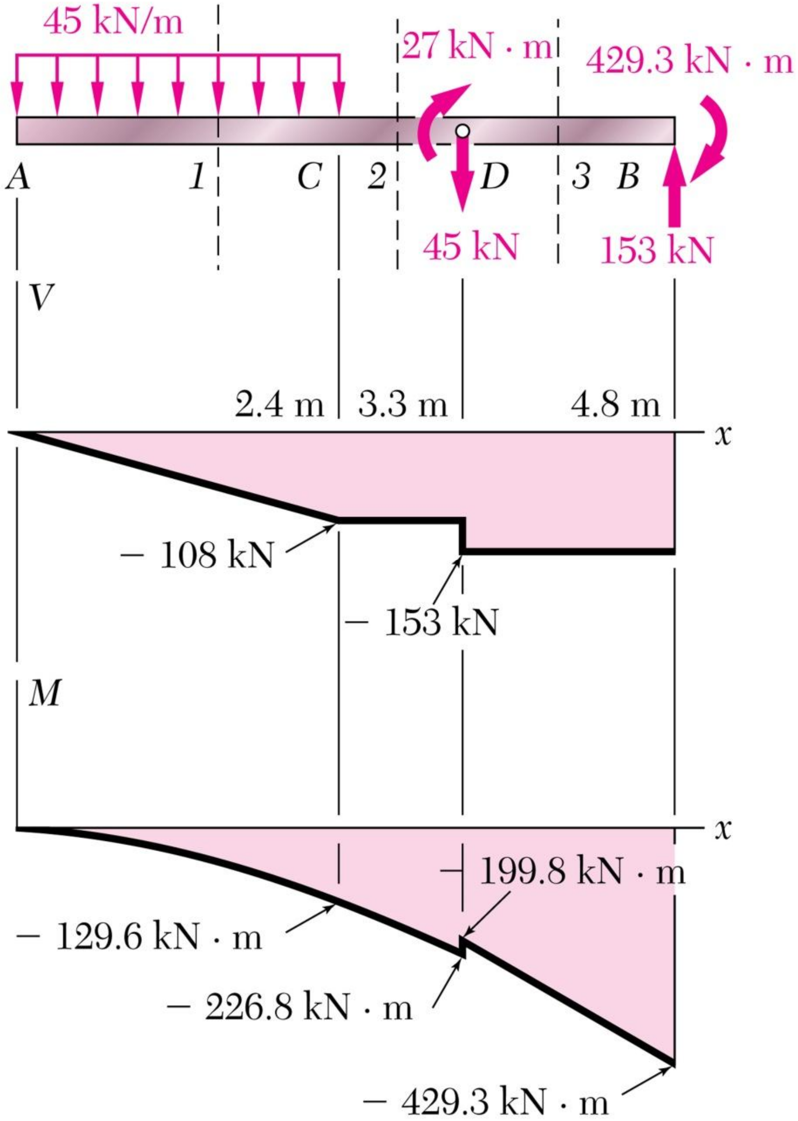
$$\sum F_y = 0 \quad -108 - V = 0 \qquad V = -108 \text{ kN}$$

$$\sum M_2 = 0 \quad 108(x - 1.2) + M = 0 \quad M = (129.6 - 108x) \text{ kNm}$$

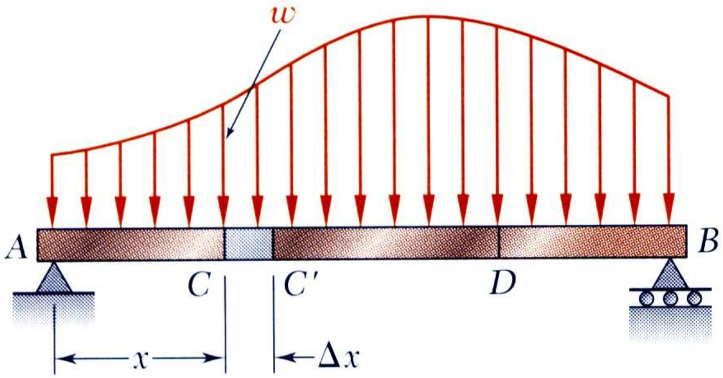
From D to B:

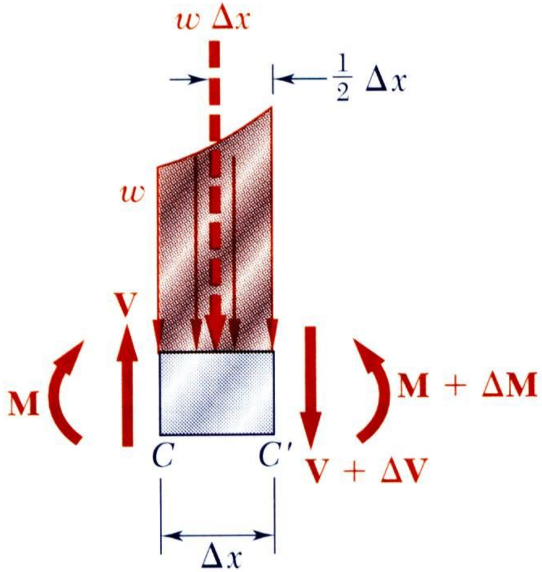
$$V = -153 \text{ kN} \qquad M = (305.1 - 153x) \text{ kNm}$$

Example 2


45 kN/m
27 kN · m
429.3 kN · m
A
1
C
2
D
3
B
45 kN
153 kN
V
2.4 m
3.3 m
4.8 m
x
− 108 kN
− 153 kN
M
x
199.8 kN · m
− 129.6 kN · m
− 226.8 kN · m
− 429.3 kN · m

Relations Among Load, Shear, and Bending Moment





- Relationship between load and shear:

$$\sum F_y = 0: \quad V - (V + \Delta V) - w\Delta x = 0$$

$$\Delta V = -w\Delta x$$

$$\frac{dV}{dx} = -w$$

$$V_D - V_C = -\int_{x_C}^{x_D} w\,dx$$

- Relationship between shear and bending moment:

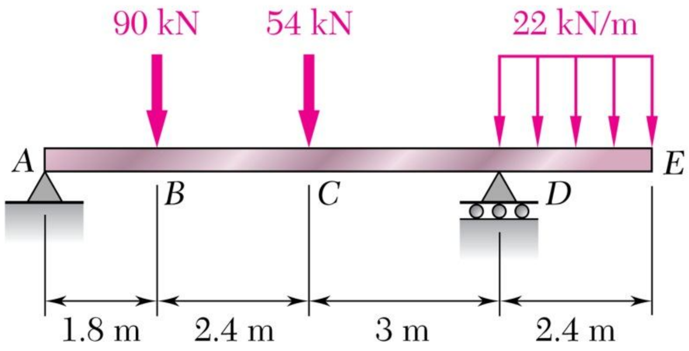
$$\sum M_{C'} = 0: \quad (M + \Delta M) - M - V\,\Delta x + w\Delta x \frac{\Delta x}{2} = 0$$

$$\Delta M = V\,\Delta x - \tfrac{1}{2} w (\Delta x)^2$$

$$\frac{dM}{dx} = V$$

$$M_D - M_C = \int_{x_C}^{x_D} V\,dx$$

Example 3

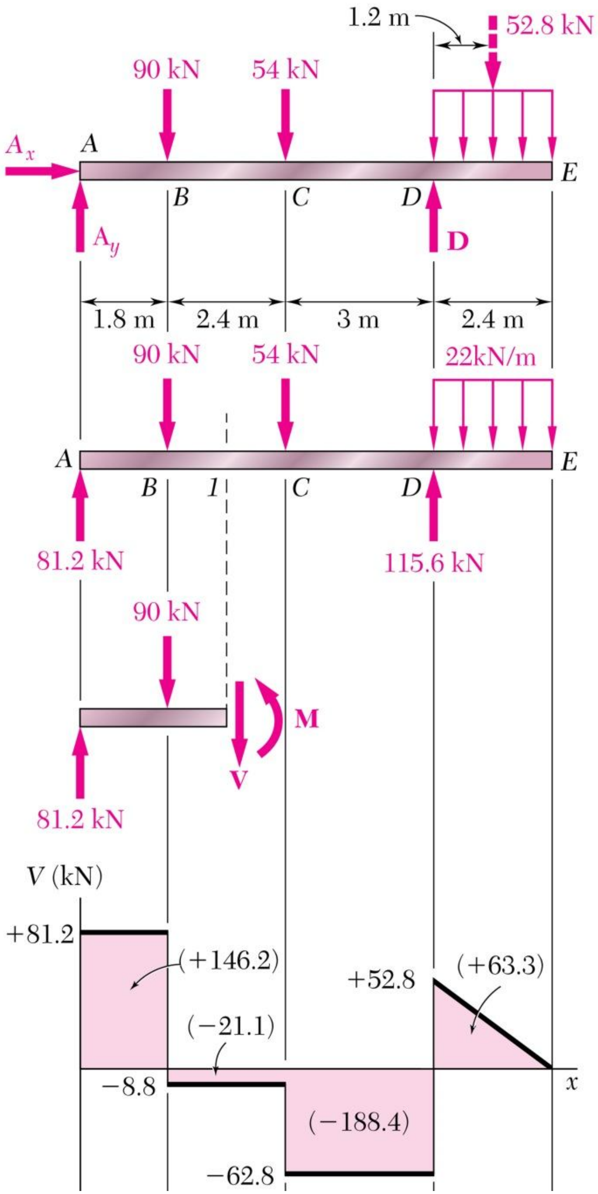


Draw the shear and bending moment diagrams for the beam and loading shown.

SOLUTION:

- Taking the entire beam as a free body, determine the reactions at *A* and *D*.
- Apply the relationship between shear and load to develop the shear diagram.
- Apply the relationship between bending moment and shear to develop the bending moment diagram.

Example 3



SOLUTION:

Taking the entire beam as a free body, determine the reactions at *A* and *D*.

$$\sum M_A = 0$$

$$0 = D(7.2\,\text{m}) - (90\,\text{kN})(1.8\,\text{m}) - (54\,\text{kN})(4.2\,\text{m}) - (52.8\,\text{kN})(8.4\,\text{m})$$

$$D = 115.6\,\text{kN}$$

$$\sum F_y = 0$$

$$0 = A_y - 90\,\text{kN} - 54\,\text{kN} + 115.6\,\text{kN} - 52.8\,\text{kN}$$

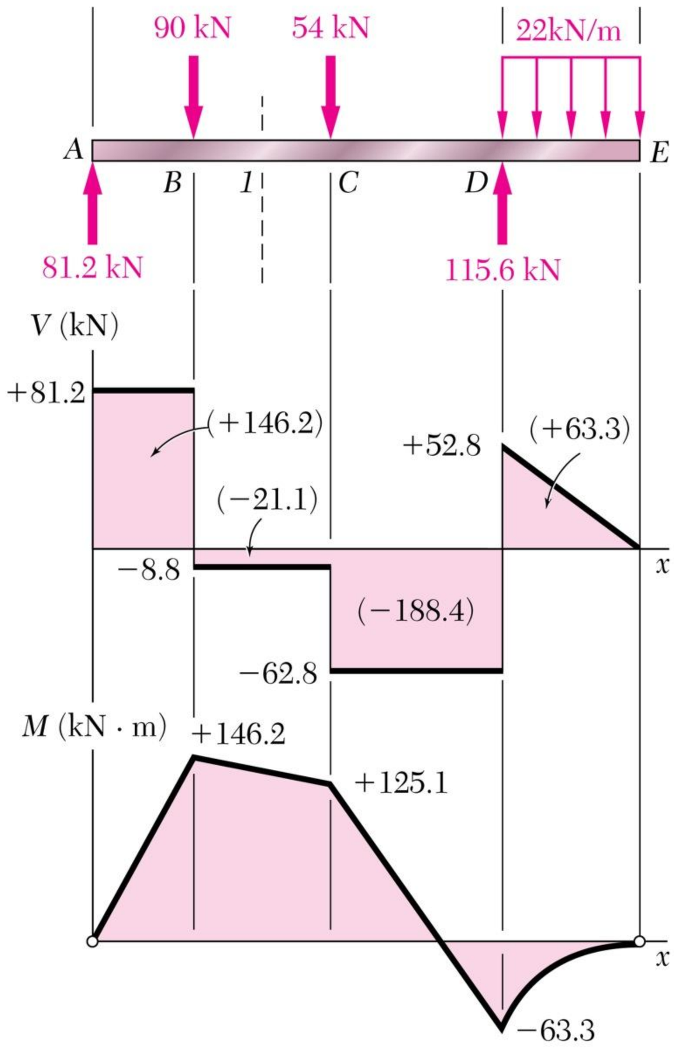
$$A_y = 81.2\,\text{kN}$$

Apply the relationship between shear and load to develop the shear diagram.

$$\frac{dV}{dx} = -w \qquad dV = -w\,dx$$

- zero slope between concentrated loads
- linear variation over uniform load segment

Example 3

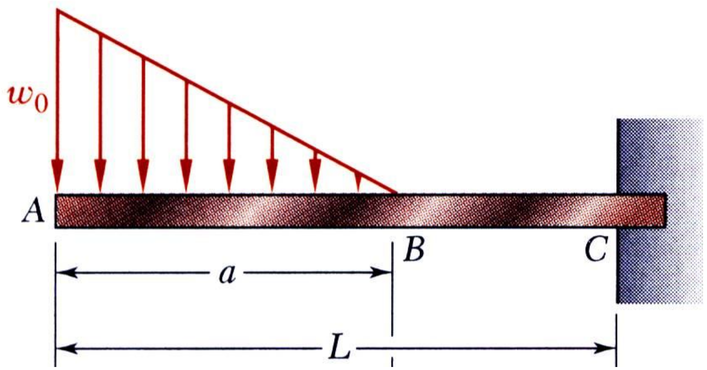


Apply the relationship between bending moment and shear to develop the bending moment diagram.

$$\frac{dM}{dx} = V \qquad dM = V\,dx$$

- bending moment at *A* and *E* is zero
- bending moment variation between *A, B, C* and *D* is linear
- bending moment variation between D and *E* is quadratic
- net change in bending moment is equal to areas under shear distribution segments
- total of all bending moment changes across the beam should be zero

Example 4

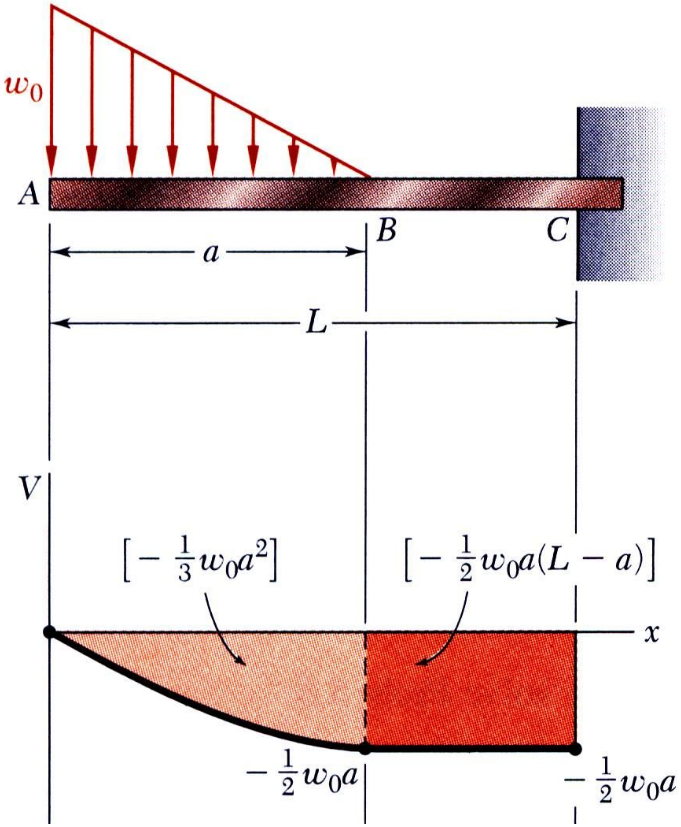


Draw the shear and bending moment diagrams for the beam and loading shown.

SOLUTION:

- Taking the entire beam as a free body, determine the reactions at *C*.
- Apply the relationship between shear and load to develop the shear diagram.
- Apply the relationship between bending moment and shear to develop the bending moment diagram.

Example 4



SOLUTION:

- Taking the entire beam as a free body, determine the reactions at *C*.

$$\sum F_y = 0 = -\tfrac{1}{2}w_0a + R_C \qquad R_C = \tfrac{1}{2}w_0a$$

$$\sum M_C = 0 = \tfrac{1}{2}w_0a\left(L - \frac{a}{3}\right) + M_C \qquad M_C = -\tfrac{1}{2}w_0a\left(L - \frac{a}{3}\right)$$

Results from integration of the load and shear distributions should be equivalent.

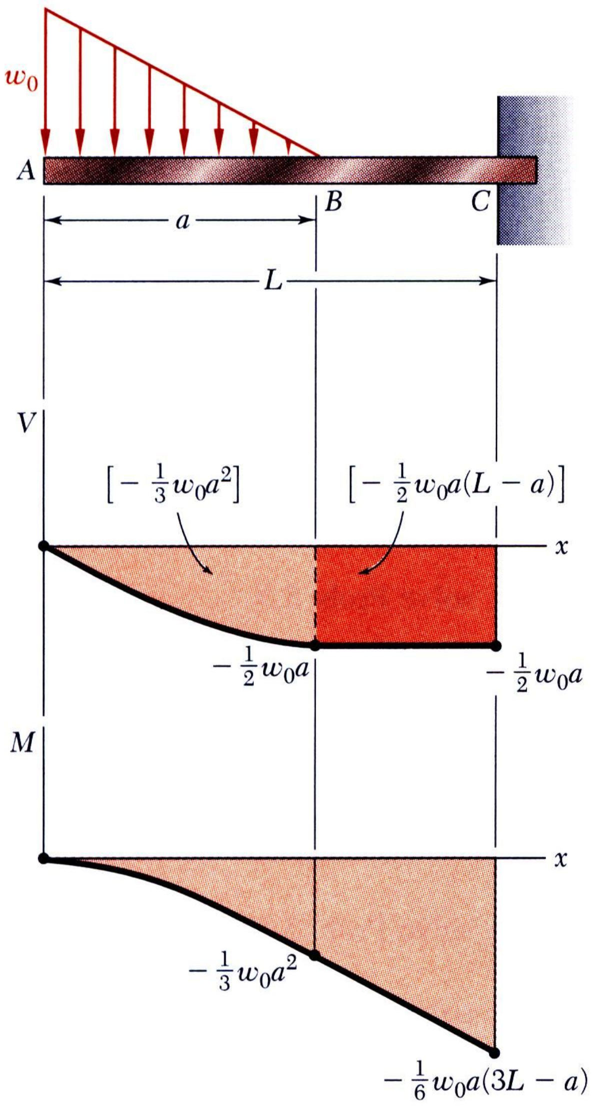
- Apply the relationship between shear and load to develop the shear diagram.

$$V_B - V_A = -\int_0^a w_0\left(1 - \frac{x}{a}\right)dx = -\left[w_0\left(x - \frac{x^2}{2a}\right)\right]_0^a$$

$$V_B = -\tfrac{1}{2}w_0a = -(\mathit{area\ under\ load\ curve})$$

- No change in shear between *B* and *C*.
- Compatible with free body analysis

Example 4



- Apply the relationship between bending moment and shear to develop the bending moment diagram.

$$M_B - M_A = \int_0^a \left(-w_0\left(x - \frac{x^2}{2a}\right)\right)dx = \left[-w_0\left(\frac{x^2}{2} - \frac{x^3}{6a}\right)\right]_0^a$$

$$M_B = -\tfrac{1}{3}w_0a^2$$

$$M_B - M_C = \int_a^L \left(-\tfrac{1}{2}w_0a\right)dx = -\tfrac{1}{2}w_0a(L-a)$$

$$M_C = -\tfrac{1}{6}w_0a(3L-a) = \frac{a\,w_0}{2}\left(L - \frac{a}{3}\right)$$

Results at C are compatible with free-body analysis