DEPARTMENT OF CIVIL ENGINEERING, IIT BOMBAY

CE 221 Solid Mechanics: Midsem

Note: **Assume suitable data if not given.**

13/11/2014 Total Marks:50 Duration: 3 hrs Instructors S. Banerjee / N.K. Chandiramani

Problem 1 (8 marks): See Fig. 1

A single strain guage forming an angle $\beta = 18^0$ with a horizontal plane is used to determine the gage pressure in the cylindrical steel tank shown. The cylindrical wall of the tank is 6-mm thick, has a 600-mm inside diameter, and is made of steel with $E = 200$ GPa and $\nu = 0.30$. Determine the pressure in the tank corresponding to a strain gage reading of 280×10^{-6}.

Problem 2 (9 marks): See Fig. 2

For the beam and loading shown, determine the spring constant *k* for which the force in the spring is equal to one-third of the total load on the beam. Note: Show complete working. You are not allowed to use any formulae from deflection tables.

Problem 3 (8 marks): See Fig. 3

Consider a column fixed at the base and supported by a linear elastic spring of stiffness β, as shown. Determine the characteristic equation for the critical buckling load (i.e., the equation whose solution gives the critical buckling load). The characteristic equation should be strictly in terms of the following parameters: k, L, EI, β, where $k^2 = P/EI$. Note that you are not required to solve the characteristic equation.

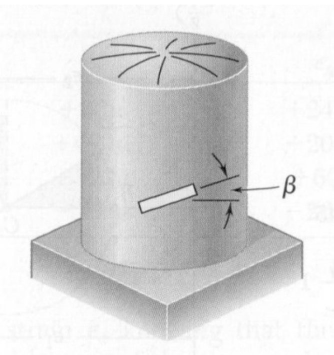


Fig. 1

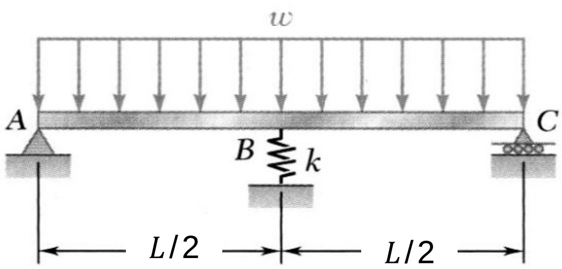


Fig. 2

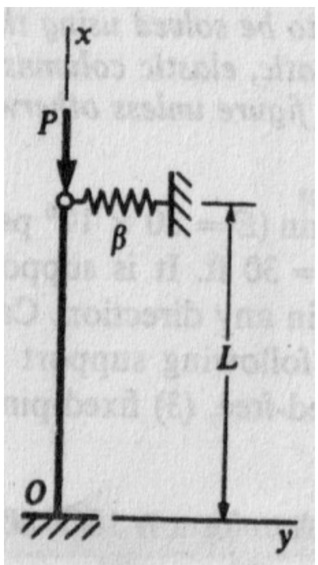


Fig. 3

Problem 4 (9 marks): See Fig. 4

A horizontal bracket *ABC* consists of two perpendicular arms *AB* and *BC*. Arm *AB* has a solid circular cross-section with diameter **60mm**. At point *C* a load **P_1=2kN** acts vertically and a load **P_2=30kN** acts horizontally and parallel to arm *AB*. Point *P* is located on the y_o axis at the circumference of the cross-section at the fixed end *A* as shown.

Neglecting selfweight, determine

(i) the principal stresses and their orientations at point *P*.

(ii) the maximum shear stress and its orientation at point *P*.

Problem 5 (8 marks): See Fig. 5

A steel beam having T section has been strengthened by bolting to it the two rectangular wood sections as shown. The modulii of elasticity are **12.5 GPa** for wood and **200 GPa** for steel. Find the normal stress σ_s in steel, and the normal stress σ_w in wood, at the point P due to a bending moment **50 kNm** as shown. The point P is located at the wood-steel interface as shown.

Problem 6 (8 marks): See Fig. 6

A U-shaped *thin walled* cross-section of constant thickness is shown. Find the location of the shear center with respect to *O* (the center of the semi-circle). Use ***b*=200mm, *r* = 100mm**

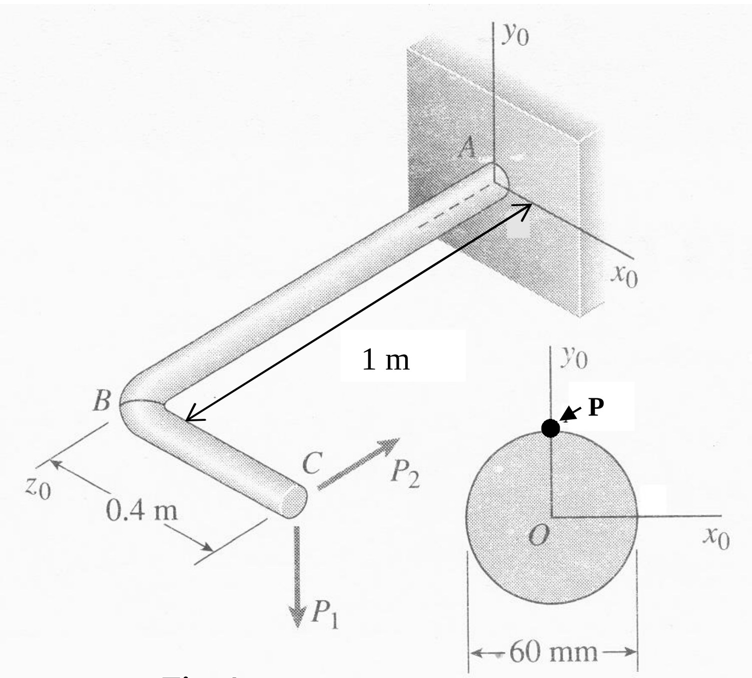


Fig. 4

Cross section at *A*

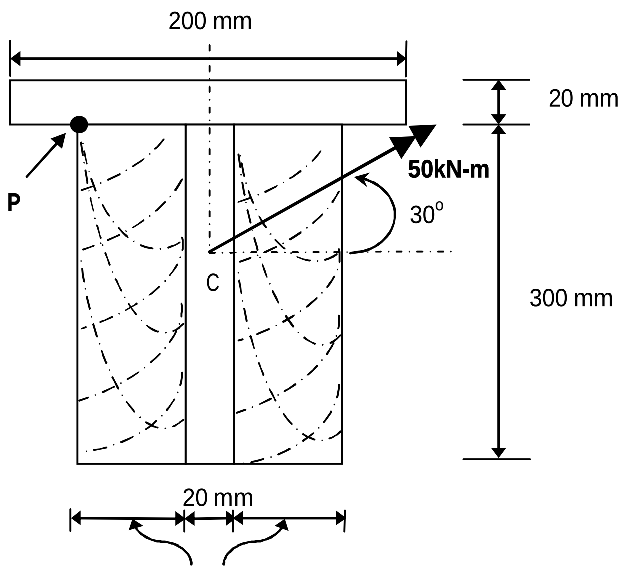


Fig. 5

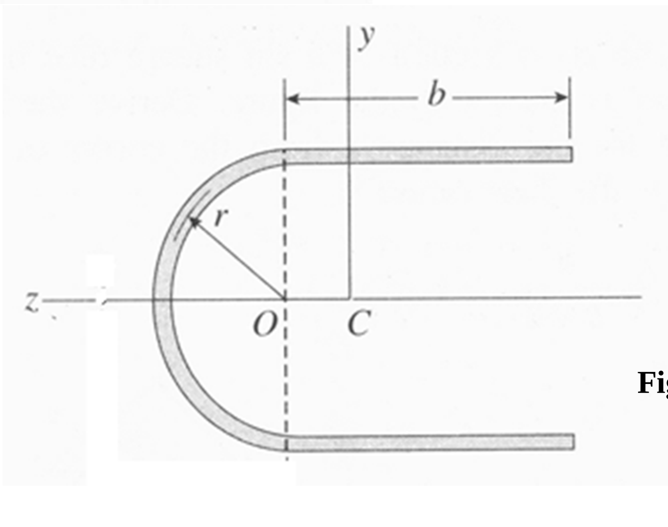


Fig. 6

P1 $\sigma_\theta = \dfrac{pr}{t}$, $\sigma_x = \dfrac{pr}{2t}$, $\tau_{x\theta} = 0$, $\sigma_r = \sigma_{r\theta} = \sigma_{rx} = 0$

$$\varepsilon_\theta = \frac{\sigma_\theta}{E} - \frac{\nu}{E}\sigma_x = \frac{pr}{t}\left(\frac{1-\nu/2}{E}\right)$$

$$\varepsilon_x = \frac{\sigma_x}{E} - \frac{\nu}{E}\sigma_\theta = \frac{pr}{t}\left(\frac{1/2-\nu}{E}\right)$$

$$\gamma_{x\theta} = 0$$

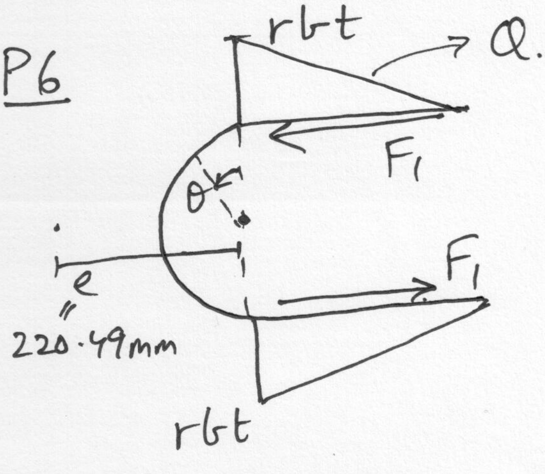
$$\varepsilon_{18^\circ} = \varepsilon_\theta \cos^2 18 + \varepsilon_x \sin^2 18 + \gamma_{x\theta} \sin 18 \cos 18 = 280 * 10^{-6}$$

$$p = 280 * 10^{-6}\left(\frac{tE}{r}\right)\frac{1}{\left((1-\nu/2)\cos^2 18 + (1/2-\nu)\sin^2 18\right)}$$

$t = 6\text{mm}$, $r = 300\text{ mm}$, $\nu = 0.3$, $E = 200 * 10^3\text{ MPa}$

$p = 1.421\text{ MPa}$.

P6



$$\tau = \frac{VQ}{It}$$

$$I = 2btr^2 + \pi r t \frac{r^2}{2}$$

Q in straight horizontal legs is as shown. (linear variation).

$$\Rightarrow F_1 = \int \tau\, dA = \int_0^b \frac{VQ}{It} t\, dz = \frac{V}{I}\frac{1}{2} b\, rbt$$

$$Q \text{ in semicircle} = rbt + \int_0^\theta r\, d\phi\, t\, r\cos\theta = r^2 t \sin\theta.$$

$$M_0 = Ve = \left[F_1 r + \int_0^{\pi/2} \frac{V}{It}(rbt + r^2 t\sin\theta)\, r\, d\theta\, t\, r\right] \cdot 2.$$

$$\Rightarrow e = \frac{1}{I}\left(\frac{b^2 r^2 t}{2} + b r^3 t \frac{\pi}{2} + r^4 t\left[-\cos\theta\right]_0^{\pi/2}\right) \cdot 2 = \frac{2(b^2 + br\pi + 2r^2)}{(4b + \pi r)}$$

Put $b = 200$, $r = 100$, $e = 220.49\text{ mm}$

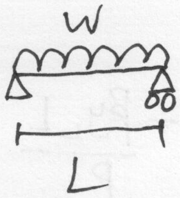
P2. Spring force $\equiv$ concentrated load $= P = \frac{wL}{3}$

Δ_1 = midspan deflection due to w

Δ_2 = midspan deflection due to $P = \frac{wL}{3}$

compatibility $\Rightarrow$ $\Delta_1 - \Delta_2 = \frac{P}{k} = \frac{wL}{3}\cdot\frac{1}{k} \longrightarrow (1)$

Δ_1 : $M(x) = \frac{wL}{2}x - wx\cdot\frac{x}{2}$; $EIy'' = M(x)$

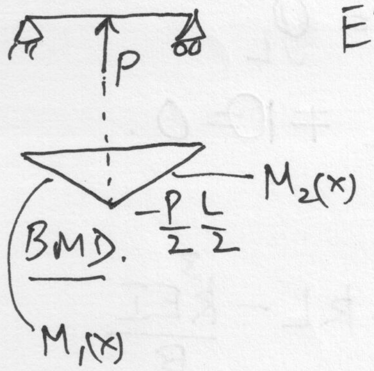


$$EIy = \frac{wL}{2}\frac{x^3}{6} - \frac{w}{2}\frac{x^4}{12} + C_1x + C_2$$

$$y(0) = 0 \Rightarrow C_2 = 0,\quad y(L) = 0 \Rightarrow C_1 = -\frac{1}{24}wL^3$$

$$y\left(\frac{L}{2}\right) = \Delta_1 = \frac{wL^4}{EI}\left(-\frac{5}{384}\right), \text{ ie } \downarrow \longrightarrow (2)$$

Δ_2 : $M_1(x) = -\frac{P}{2}x = EIy_1''$, $0 \le x \le \frac{L}{2}$



$$EIy_1 = -\frac{P}{2}\frac{x^3}{6} + \bar{C}_1x + \bar{C}_2$$

$$y_1(0) = 0 \Rightarrow \bar{C}_2 = 0$$

$$y_1'\left(\frac{L}{2}\right) = 0 \Rightarrow \bar{C}_1 = \frac{PL^2}{16} \text{ (from symmetry)}$$

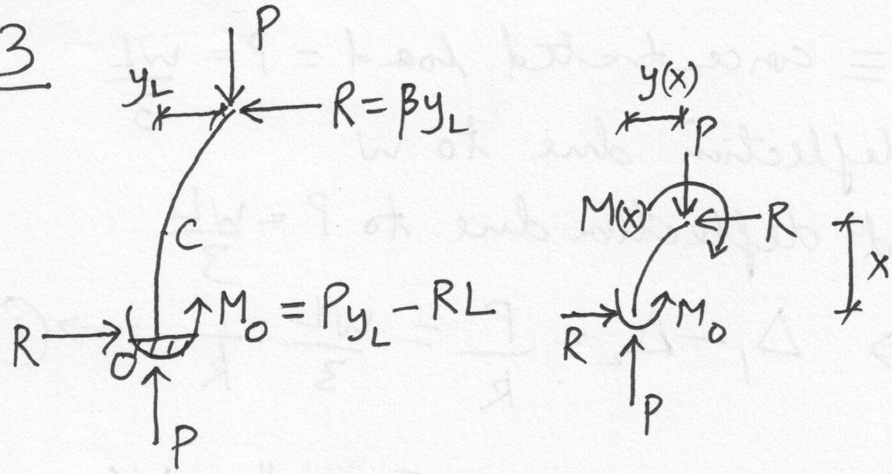
$$y\left(\frac{L}{2}\right) = \Delta_2 = \frac{1}{EI}\left[-\frac{P}{2}\frac{L^3}{48} + \frac{PL^2}{16}\cdot\frac{L}{2}\right] = \frac{PL^3}{EI}\cdot\left(\frac{1}{48}\right)\uparrow \rightarrow (3)$$

Put (2), (3) in (1), with $P = \frac{wL}{3}$,

$$\frac{wL^4}{EI}\left(\frac{5}{384} - \frac{1}{48}\cdot\frac{1}{3}\right) = \frac{wL}{3}\cdot\frac{1}{k}$$

$$\Rightarrow k = \frac{EI}{L^3}\left(\frac{384}{7}\right)$$

P3



$$\Sigma M_c = 0 = M(x) + Py - Rx - M_o = \underline{EIy'' + Py - Rx + RL - Py_L}$$
$$EI$$

Solution : $y = y_h + y_p$

$$= C_1 \cos kx + C_2 \sin kx + \frac{\beta y_L}{P} x + y_L - \frac{\beta y_L L}{P}$$

$$y\big|_{x=0} = 0 \Rightarrow C_1 = y_L\left(\frac{\beta L}{P} - 1\right) = y_L\left(\frac{\beta L}{k^2 EI} - 1\right)$$

$$y'\big|_{x=0} = 0 \Rightarrow C_2 k = -\frac{\beta y_L}{k^2 EI}$$

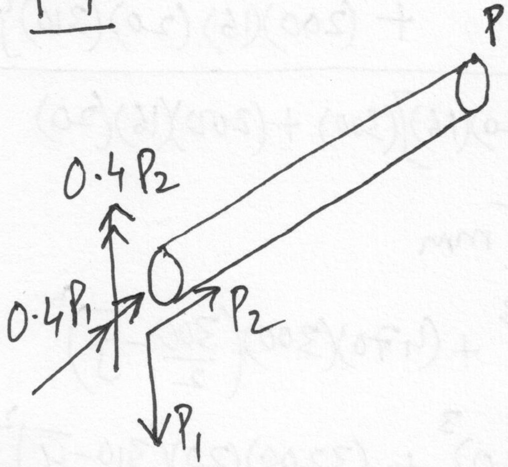
$$y\big|_{x=L} = y_L \Rightarrow C_1 \cos kL + C_2 \sin kL + y_L = y_L$$

$$\left(\frac{\beta L}{k^2 EI} - 1\right)\cos kL - \frac{\beta}{k^3 EI}\sin kL = 0$$

$$\tan kL = \left(\frac{\beta L}{k^2 EI} - 1\right)\frac{k^3 EI}{\beta} = kL - \frac{k^3 EI}{\beta}$$

$$\boxed{\tan kL = kL - \frac{k^3 EI}{\beta}}$$

P4



$P_1 = 2\,kN$

$P_2 = 30\,kN$

At P,

$$\sigma_z = \frac{(P_1 * 10^3)\left(\frac{60}{2}\right)}{\frac{\pi}{4}\left(\frac{60}{2}\right)^4} - \frac{P_2}{\pi\left(\frac{60}{2}\right)^2} = 83.7037\ \text{MPa}$$

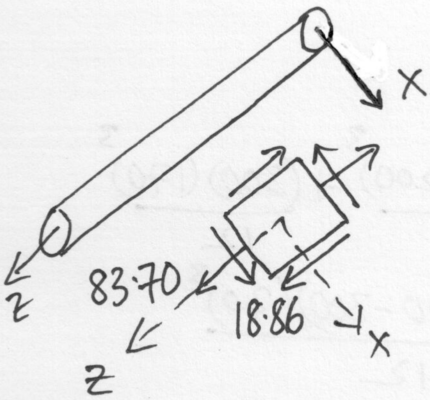
(Note: $0.4P_2$ gives no σ_z ∵ P on NA)

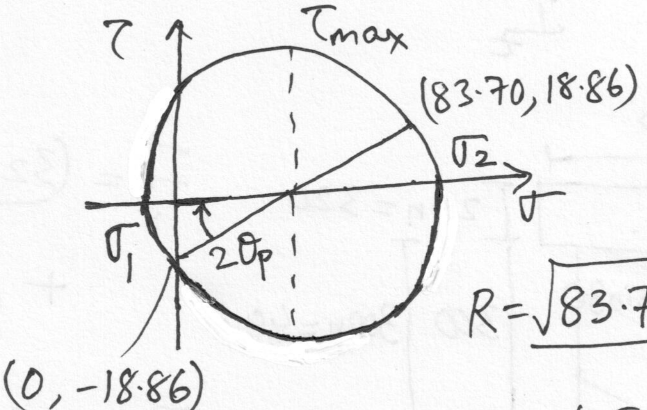
$$\tau_{zx} = \frac{(400\ P_1)\left(\frac{60}{2}\right)}{\frac{\pi}{2}\left(\frac{60}{2}\right)^4}$$

Note: P_1 gives no τ_{zx} ∵ P on top boundary ie $Q=0$ in VQ/IT

$$\tau_{zx} = 18.8628\,\text{MPa}$$

$$\sigma_x = 0$$





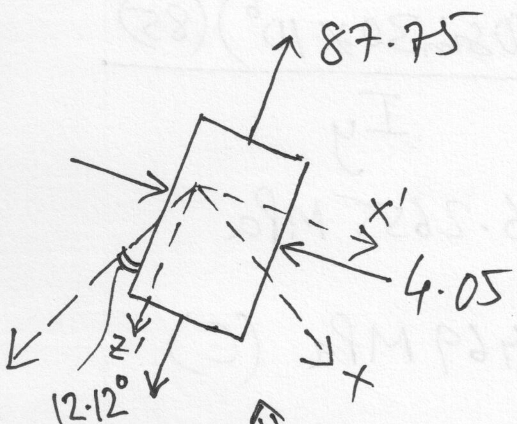
$$R = \frac{\sqrt{83.70^2 + (2*18.86)^2}}{2}$$

$$= 45.90\,\text{MPa} = \tau_{max}$$

$$\sigma_1 = \frac{83.70+0}{2} - R = -4.05\ \text{MPa}$$

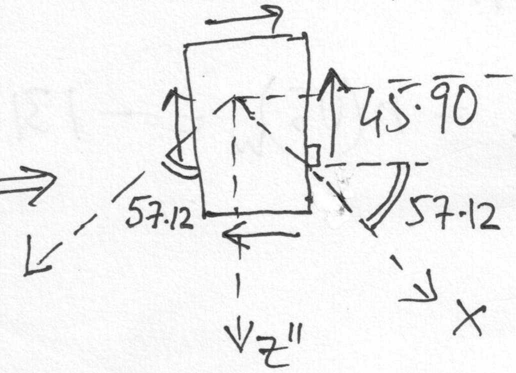
$$\sigma_2 = \frac{83.70+0}{2} + R = 87.75\ \text{MPa}$$

$$\theta_p = \frac{1}{2}\cos^{-1}\left(\frac{83.7/2}{R}\right) = 12.12^\circ$$

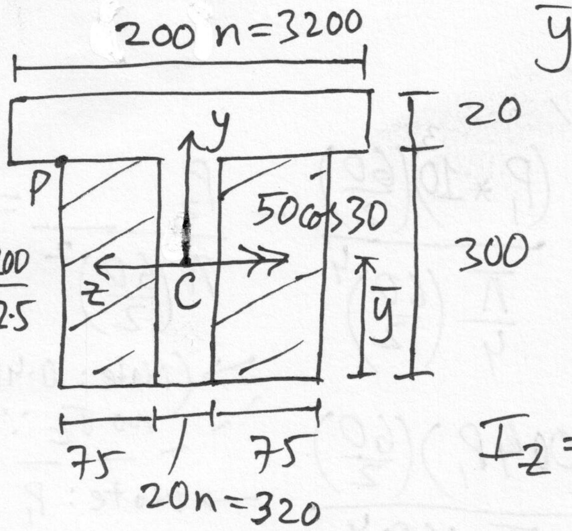


Principal stresses & planes.

Max shear stres & planes ⇒



P5



$$n = \frac{E_s}{E_w} = \frac{200}{12.5}$$

$$n = 16$$

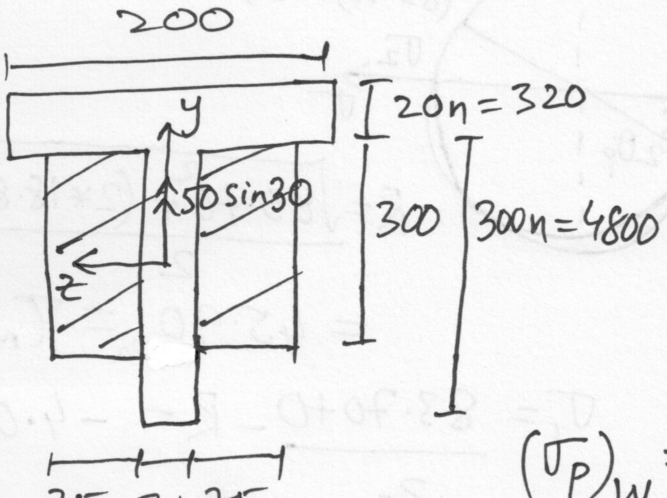
$$\bar{y} = \frac{\left\{\left[(75)(2)+(20)(16)\right](300)(150) + (200)(16)(20)(310)\right\}}{\left[(75)(2)+(20)(16)\right](300) + (200)(16)(20)}$$

$$\bar{y} = 199.95 \text{ mm}$$

$$I_z = \frac{(470)(300)^3}{12} + (470)(300)\left(\frac{300}{2} - \bar{y}\right)^2 + \frac{(3200)(20)^3}{12} + (3200)(20)(310-\bar{y})^2$$

$$= 2186532846 \text{ mm}^4$$

$$(\sigma_P)_w = -\frac{(50\cos 30 \times 10^6)(300-\bar{y})}{I_z} = -1.9814 \text{ MPa}$$



$$I_y = \frac{(320)(200)^3}{12} + \frac{(300)(170)^3}{12} + \frac{(4800-300)(20)^3}{12}$$

$$I_y = 339158333.33 \text{ mm}^4$$

$$(\sigma_P)_W = -\frac{(50\sin 30 \times 10^6)(85)}{I_y}$$

$$= -6.2655 \text{ MPa}$$

$$(\sigma_P)_W = -1.9814 - 6.2655 = -8.2469 \text{ MPa (C)}$$

$$(\sigma_P)_S = n(\sigma_P)_W = -131.9504 \text{ MPa (C)}$$