DEPARTMENT OF CIVIL ENGINEERING

CE-221 SOLID MECHANICS

End-Sem Exam **07/11/16** **PAPER CODE: D**

Note: Write your name & roll no. on answerbook and on summary-answer-sheet provided with the question paper.
You must submit the summary-answer-sheet along with the answerbook.
Closed book, closed notes exam. No formula sheet allowed. No mobile phones allowed in the exam hall.
All questions carry equal marks. Assume suitable data if required and state the same clearly.
Use formulae from provided table, **if applicable**.

Problem 1

A hollow circular steel tube ***A*** fits over the end of a solid circular steel rod ***B*** as shown in **Fig. 1**. The far ends of the tube and the rod are fixed. A hole through rod ***B*** makes an angle $\beta = \pi/6$ **radians** with a line through two holes in tube ***A*** as shown. Rod ***B*** is twisted until the holes in tube ***A*** and rod ***B*** are aligned and then a pin is placed through the holes and the system is left to deform. **Determine** the **final angles of twist of** the **tube *A* and** the **rod *B***. For tube ***A*** and rod ***B*** use data: polar moment of inertia $J_a = 5 \times 10^{-4}$ m^4 , $J_b = 2.5 \times 10^{-4}$ **m;** lengths L_a **= 2000 mm,** L_b **= 1500 mm.**

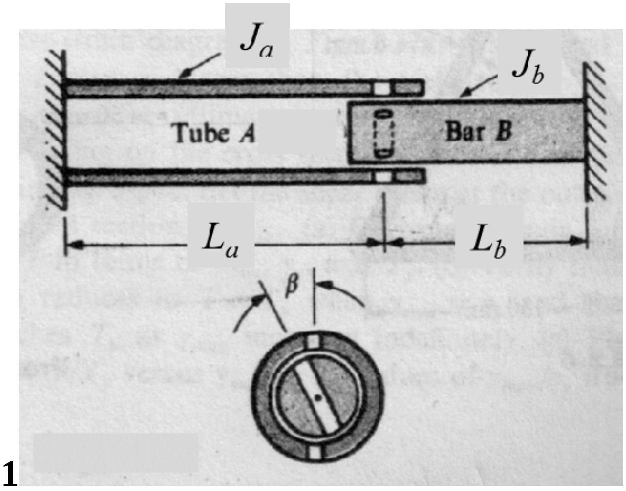
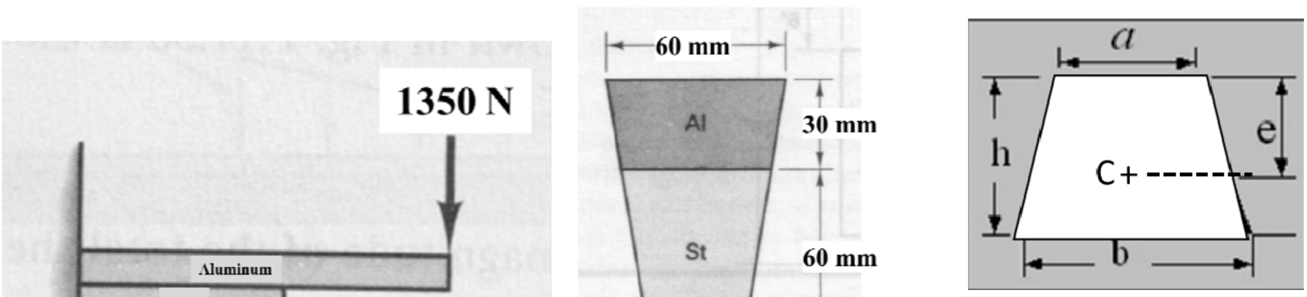


Fig. 1

Problem 2

The cross-section of a perfectly bonded composite beam is made up of aluminum and steel as shown in **Fig.2**. Young’s modulus of aluminum and steel are **35 GPa** and **70 GPa** respectively. **Calculate** the **maximum tensile** bending **stresses in steel and aluminum**.



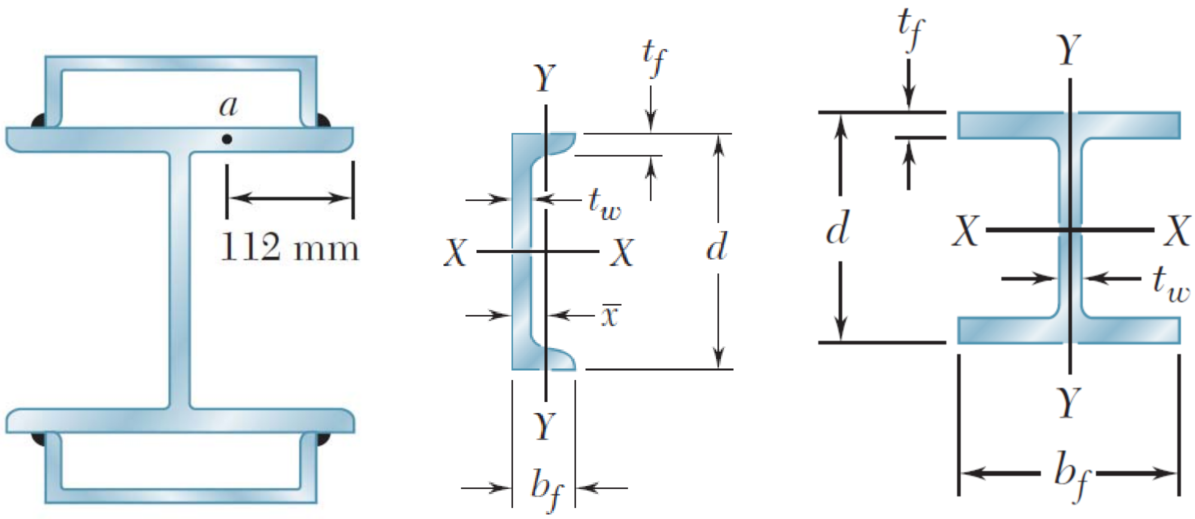
$$e = \frac{h(a+2b)}{3(a+b)}$$

$$I = \frac{h^3(a^2+4ab+b^2)}{36(a+b)}$$

Fig. 2

Problem 3

The composite beam shown in **Fig. 3** is made by welding **C200 x 17.1** rolled steel channels (C-sections) to the flanges of a **W 250 x 80** wide-flange rolled-steel I-section. The beam is subjected to a vertical shear of **200 kN**. **Determine** (a) the **horizontal shear force per meter at each weld**, (b) the **horizontal shearing stress at** point ***a*** on the flange of the wide-flange section.



Section	d (mm)	b_f (mm)	t_f (mm)	t_w (mm)	Area (mm^2)	$\bar{x}$ (mm)	I_{XX} ($x10^6$ mm^4)	I_{YY} ($x10^6$ mm^4)
C 200 x 17.1	203	57.4	9.91	5.59	2170	14.5	13.5	0.545
W 250 X 80	257	254	15.6	9.40	10200	-	126.0	42.9

Fig. 3

Problem 4

A semicircular bar ***AB*** lying in a horizontal plane is supported at ***B*** as shown in **Fig. 4**. Thus the self-weight of the bar acts into the plane of the paper. The bar has centerline radius ***R* = 500 mm** and self-weight ***q* = 40 N/m**. The cross-section of the bar is circular with diameter ***d* = 25 mm**. **Determine** the **maximum tensile stress** σ_t , the **maximum compressive stress** σ_c , and the **maximum shear stress** τ_{max} at point ***P*** lying on the top of the bar at the support as shown in the cross-section figure. Also **show** the **original stress element (block)**, the **principal stress element** and the **maximum shear stress element**, **with proper relative orientations**. The center of gravity of the bar is at point ***C*** at a distance ***c*** = **2R/π.**

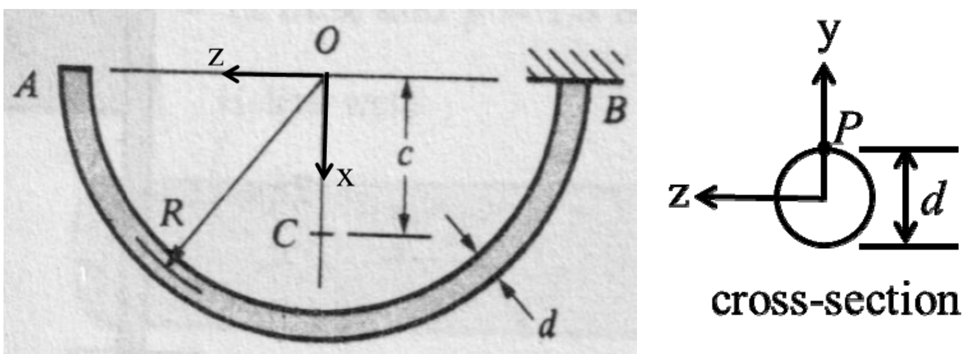


Fig. 4

Problem 5

Two cantilever beams are interconnected by a linear spring having a spring constant ***K*=9x10^5 N/m** as shown in **Fig. 5**. **Determine** the **force in** the **spring** due to an applied load of ***P* = 45,000 N**. Moment of inertia is ***I* = 1.4 x 10^{-4} m^4** , length is ***L* = 3m,** modulus of elasticity is ***E* = 2x10^{11} Pa,** for each beam.

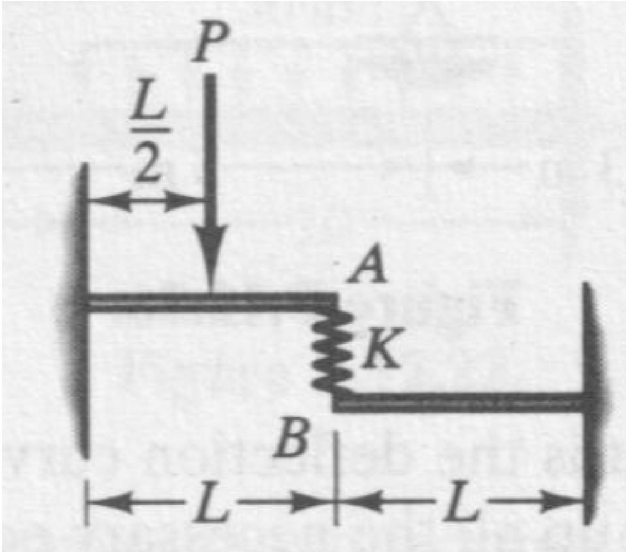


Fig. 5

Problem 6

Determine the **characteristic equation** whose solution gives the critical buckling load for the system shown in **Fig.6**. Bar ***AB*** is flexible with constant ***EI*** and bar ***BC*** is rigid**.** You do not have to solve the characteristic equation. Use ***L=2a*.**

(Note that the characteristic equation is the transcendental algebraic-trignometric equation which is obtained as the condition for a non-trivial (non-zero) solution of the second order differential equation of moment equilibrium).

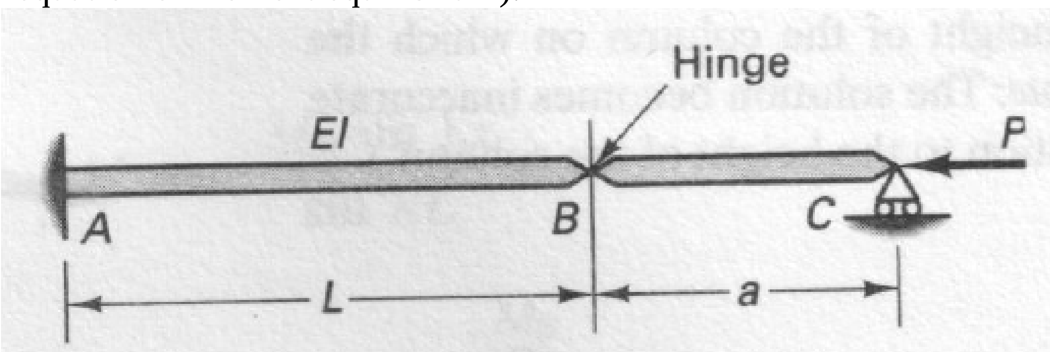


Fig. 6

SUMMARY ANSWER SHEET

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Name:

Roll no:

Problem 1

ϕ_A =

ϕ_B =

Problem 2

Max bending tensile stress in steel =

Max bending tensile stress in aluminum =

Problem 3

(a) **Horizontal shear force per meter at each weld =**

(b) **Horizontal shear stress at point *a* =**

Problem 4

σ_t =

σ_c =

τ_{max} =

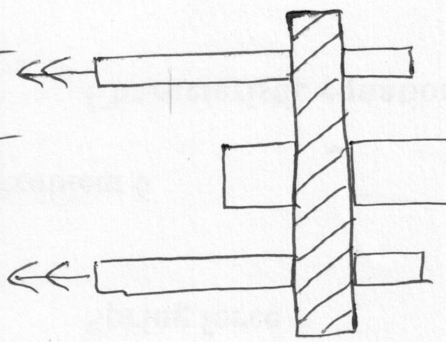
Problem 5

Spring force =

Problem 6

Characteristic equation is:

P1



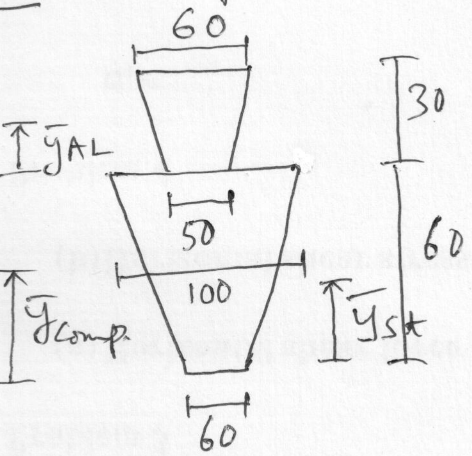
$$\frac{TL_A}{GJ_A} = \theta_A \; ; \; \frac{TL_B}{GJ_B} = \frac{\pi}{6} - \theta_A$$

eliminate T/G

$$\Rightarrow \theta_A = \left[\frac{J_B}{L_B} \Big/ \left(\frac{J_A}{L_A} + \frac{J_B}{L_B} \right) \right] \frac{\pi}{6} = \frac{\pi}{15}$$

$$\theta_B = \frac{\pi}{6} - \frac{\pi}{15} = \frac{\pi}{10}$$

P2 Transform to AL section.



$$\bar{y}_{comp} = \frac{\overbrace{\tfrac{1}{2}(160)(60)\frac{(60+200)(60)}{3(60+100)}}^{\text{for } \bar{y}_{st}} + \overbrace{\tfrac{1}{2}(110)(30)\left\{\frac{(50+120)(30)}{3(50+60)} + 60\right\}}^{\text{for } \bar{y}_{AL}}}{\tfrac{1}{2}\left[\underbrace{(160)(60)}_{\text{for } \bar{y}_{st}} + \underbrace{(110)(30)}_{\text{for } \bar{y}_{AL}}\right]}$$

$$= 43.488 \simeq 43.5$$

$$\bar{y}_{AL} = 15.4545 \quad ; \quad \bar{y}_{st} = 32.5$$

$$I_{comp} = \frac{(60)^3(60^2 + 100^2 + 4.60.100)}{36(60+100)} + \frac{1}{2}(160)(60)(43.488 - 32.5)^2$$

$$+ \frac{(30)^3(50^2 + 60^2 + 4.50.60)}{36(50+60)} + \frac{1}{2}(110)(30)(60 - 43.488 + 15.4545)^2$$

$$= 379.901 * 10^4 \text{ mm}^4$$

$$n = \frac{E_{st}}{E_{AL}} = 2$$

$$I_{AL} = 12.341 * 10^4 \text{ mm}^4$$

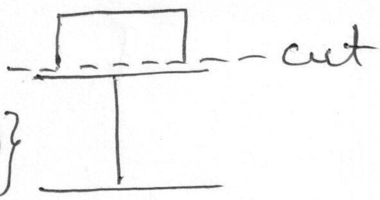
$$M_1 = 1350 * 5.4 \quad ; \quad M_2 = 1350 * 2.4 \text{ N.m}$$

Fixed end : $(\sigma_{AL})_{max\,T} = \frac{M_1(90 - 43.5)}{I_{comp}} = 89.23$ MPa.

$(\sigma_{st})_{max\,T} = \frac{M_1(60 - 43.5)}{I_{comp}} * n^{=2} = 63.32$ MPa ◀

At Junction : $(\sigma_{AL})_{max\,T} = \frac{M_2(30 - 15.45)}{I_{AL}} = 381.99$ MPa ◀

P3 (i) $Q_1 = (2170)\left(\frac{257}{2} + 57.4 - 14.5\right) = 371938 \text{ mm}^3$

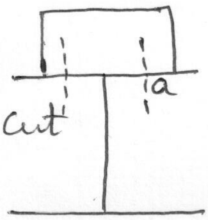


$I = \left\{126E6 + 2\left(0.545E6 + 2170\left[\frac{257}{2} + 57.4 - 14.5\right]^2\right)\right\}$

$= 254590346.4 \text{ mm}^4$

$\Delta H = \frac{1}{2}(200)\frac{Q_1}{I} = 0.1461 \text{ kN/mm} = 146.1 \text{ kN/m}$

(ii) $Q_2 = 371938 + 2(112)(15.6)\left(\frac{257}{2} - \frac{15.6}{2}\right) = 793712.08 \text{ mm}^3$

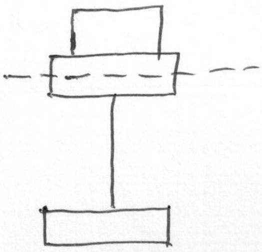


$\tau_{xz} = \frac{200E3}{2}\frac{Q_2}{I t_f} = 19.985 \text{ MPa}$ ◀ → Horizontal shear at 'a'.

(t_f = 15.6)

Not required [(iii) Extra – not required

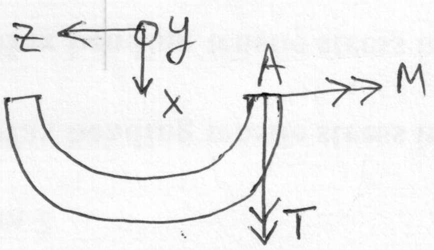
$Q_3 = 371938 + (254)\left(\frac{15.6}{2}\right)\left(\frac{257}{2} - \frac{15.6}{4}\right) = 618795.52 \text{ mm}^3$

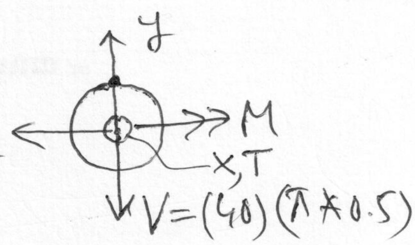


$\tau_{yz} = \frac{200E3\, Q_3}{I\, b_f} = 1.914 \text{ MPa}$

↳ negligible, as expected.
↳ Vertical shear at 'a']

P4

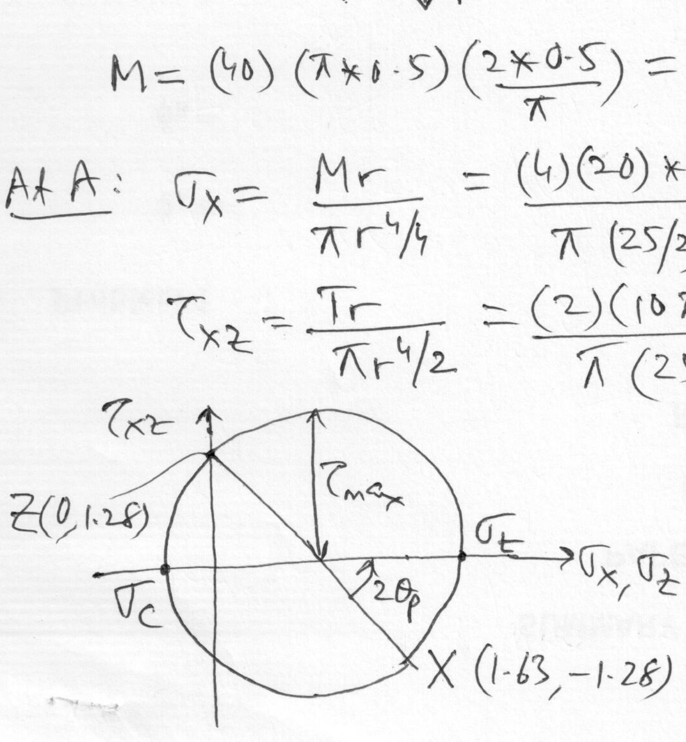




$M = (40)(\pi * 0.5)\left(\frac{2*0.5}{\pi}\right) = 20 \text{ N.m}$; $T = (40)(\pi * 0.5)(0.5) = 10\pi \text{ N.m}$

At A: $\sigma_x = \frac{Mr}{\pi r^4/4} = \frac{(4)(20)*10^3}{\pi(25/2)^3} = 13.04 \text{ MPa}$, $\sigma_z = 0$, $\underbrace{\sigma_y = \tau_{yz} = \tau_{yx} = 0}_{\text{traction free surface}}$

$\tau_{xz} = \frac{Tr}{\pi r^4/2} = \frac{(2)(10\pi)*10^3}{\pi(25/2)^3} = 10.24 \text{ MPa}$

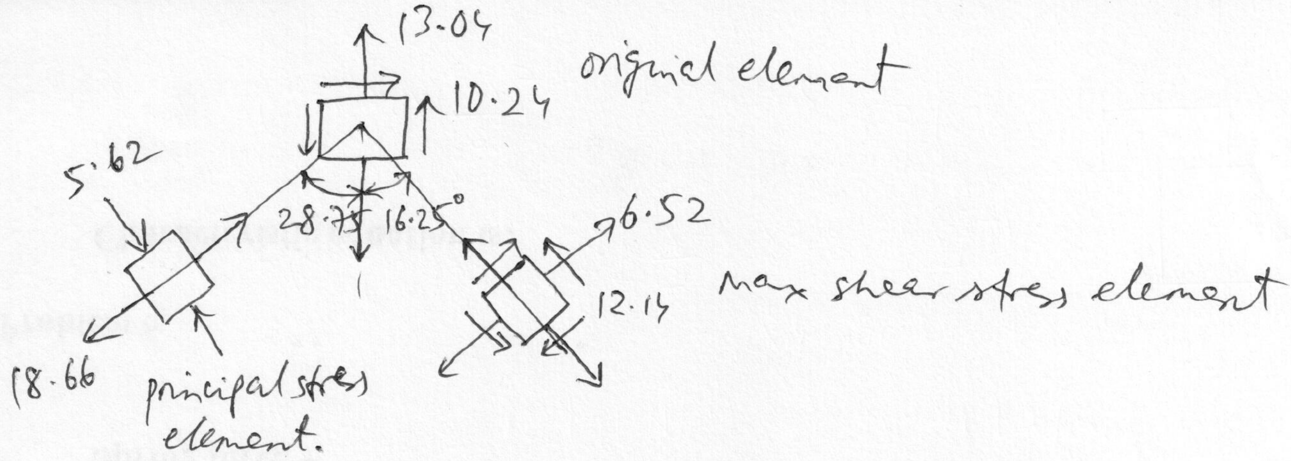


$\sigma_t, \sigma_c = \frac{13.04}{2} \pm \frac{1}{2}\sqrt{13.04^2 + 20.48^2}$

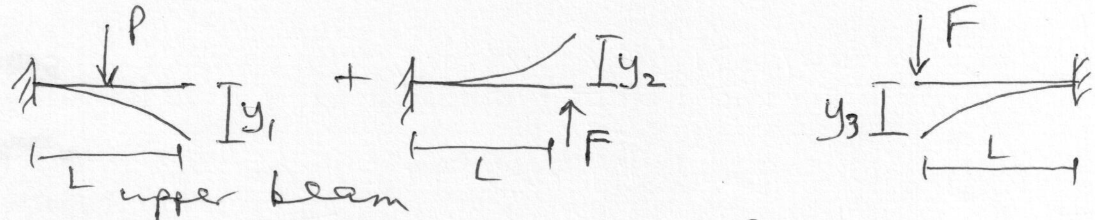
$= 18.66, -5.62 \text{ MPa}$

$\tau_{max} = 12.14 \text{ MPa}$

$\theta_p = \frac{1}{2}\cos^{-1}\left(\frac{13.04/2}{12.14}\right) = 28.75°$



P5

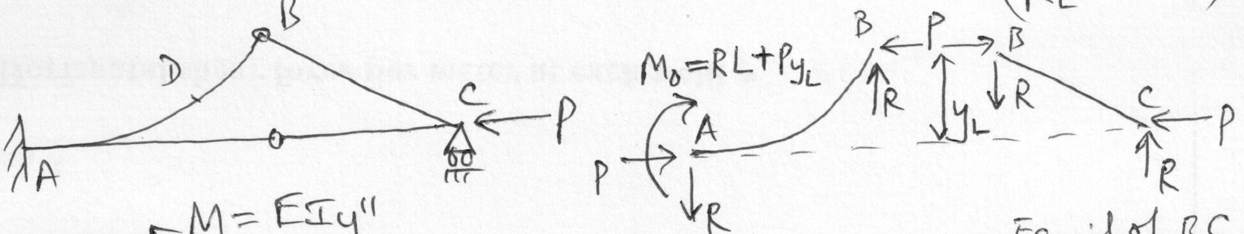


$$y_1 = \frac{PL^3}{24EI} + \frac{L}{2}\frac{PL^2}{8EI} = \frac{5}{48}\frac{PL^3}{EI} \; ; \quad y_2 = \frac{FL^3}{3EI} = y_3$$

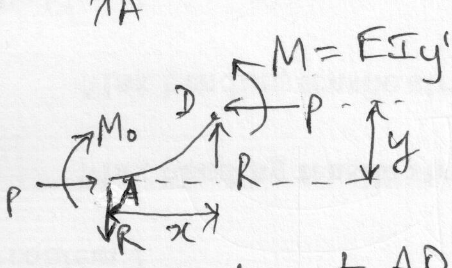
$$F = k(y_1 - y_2 - y_3) = \frac{kL^3}{EI}\left(\frac{5}{48}P - \frac{F}{3} - \frac{F}{3}\right) \Rightarrow F = \frac{(5/48)P}{\left(\frac{EI}{kL^3} + \frac{2}{3}\right)} = 2577 \text{ N}$$

for given data.

P6



Equil of BC $\Rightarrow R = Py_L/a$



FBD of element AD.

Equil of AD $\Rightarrow$ $EIy'' + Py + Rx - M_0 = 0$

$$y'' + k^2y + \frac{k^2 y_L}{a}(x-L) - k^2 y_L = 0$$

$$y'' + k^2y + k^2\frac{y_L}{a}(x-L-a) = 0$$

solution is $\rightarrow$ $y = A_1\cos kx + A_2\sin kx + \frac{y_L}{a}(L+a-x)$

BC's

$y(0)=0 \rightarrow$

$y'(0)=0 \rightarrow$

$y(L)=y_L \rightarrow$

$$\begin{bmatrix} 1 & 0 & (L+a) \\ 0 & k & -1 \\ \cos kL & \sin kL & 0 \end{bmatrix} \begin{Bmatrix} A_1 \\ A_2 \\ y_L/a \end{Bmatrix} = 0$$

det $=0 \Rightarrow \tan kL = k(L+a) = \frac{3}{2}kL$ ◀