DEPARTMENT OF CIVIL ENGINEERING

CE-221 SOLID MECHANICS

End-Sem Exam **17/11/2017** **PAPER CODE: A**

Note: Write your name & roll no. on answerbook and on summary answer sheet provided with the question paper.
You must submit the question-paper-cum-summary-answer-sheet along with the answerbook.
Closed book, closed notes test. No formula sheet allowed. No mobile phones allowed in the exam hall.
All questions carry equal marks. Assume suitable data if required and state the same clearly.
Use formula from provided tables, **if applicable**.

Problem 1

High strength steel wires are stretched by applying a force ***Q*** which produces an initial stress σ_0**= 820 MPa** as shown in **Fig. 1 (a)**. Concrete is then poured around the stretched wires to form a beam as shown in **Fig. 1 (b)**. After the concrete sets properly, the force ***Q*** is removed as shown in **Fig. 1 (c)**. Thus the beam is left in a prestressed condition with the wires in tension and concrete in compression. The moduli of elasticity of steel and concrete are in the ratio of **8:1**. The areas of concrete and steel are in the ratio **30:1**. Calculate the final stresses in steel (σ_s) and concrete (σ_c).

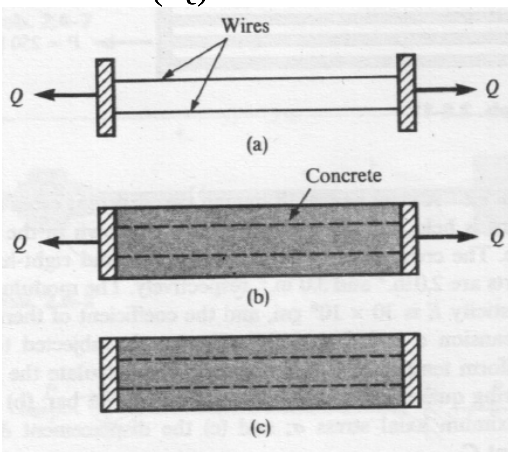


Fig. 1

Problem 2

A wooden beam is reinforced by steel plates as shown in **Fig. 2**. The ratio of moduli of elasticity for steel and wood E_s/E_w **= 7**. Calculate the maximum tensile and maximum compressive stresses in steel due to the moment applied as shown in **Fig. 2**.

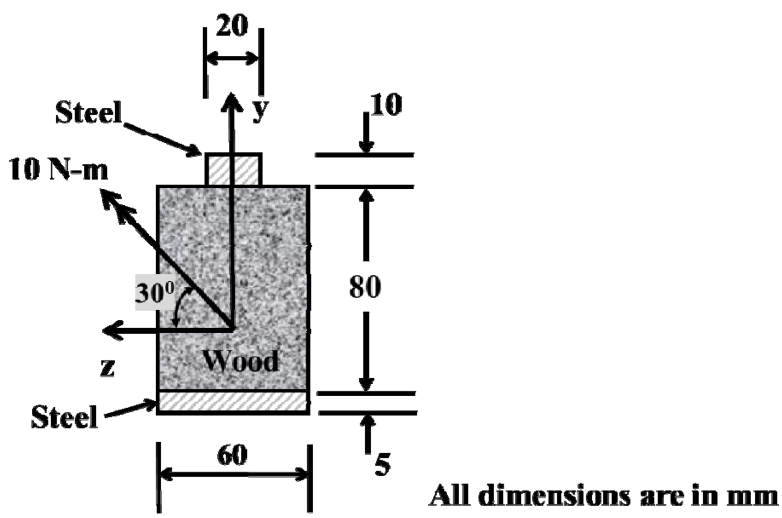
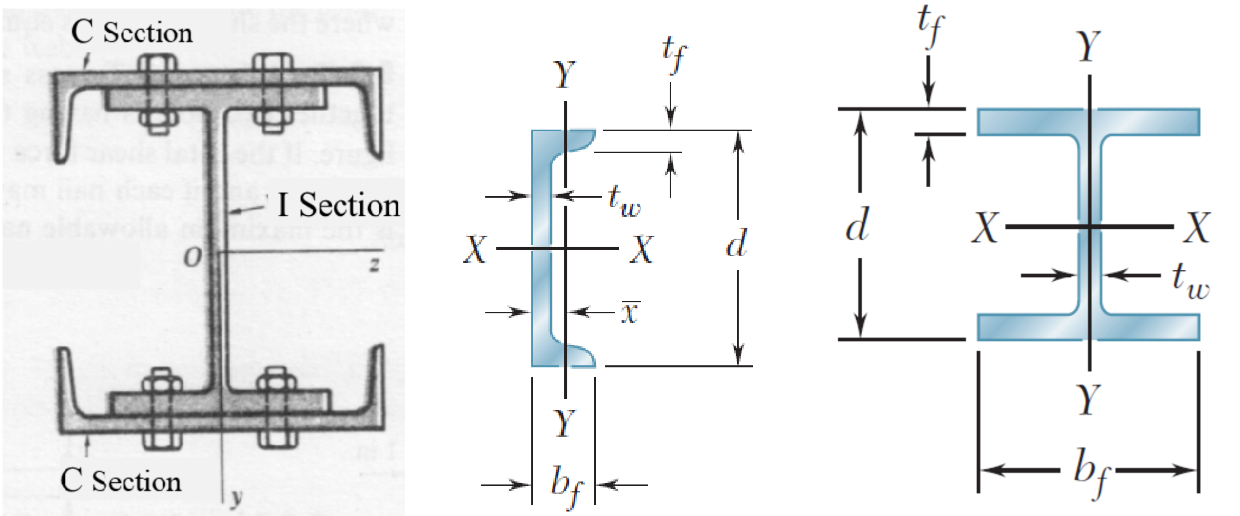


Fig. 2

Problem 3

A built-up beam is fabricated using a steel I-section and two aluminium channel sections as shown in **Fig. 3**. The sections are joined by bolts spaced at **60 mm** longitudinally. Calculate the maximum allowable shear force that can be resisted by the cross-section if each bolt can carry a shear force of **2.4 kN**. The ratio of moduli of elasticity for steel and aluminium is $\boldsymbol{E_s / E_a = 7}$.



Section Properties	d (mm)	b_f (mm)	t_f (mm)	t_w (mm)	Area (mm^2)	$\bar{x}$ (mm)	I_{XX} ($x10^6\ mm^4$)	I_{YY} ($x10^6\ mm^4$)
C Section	120.0	31.7	5.0	5.1	882	6.74	1.62	0.0514
I Section	121.9	80.8	6.4	3.7	1470	-	3.94	0.56

Fig. 3

Problem 4

A shaft of diameter **25 mm** supports a **300 mm** diameter pulley weighing **500 N** as shown in **Fig. 4**. The horizontal forces in the belt going over the pulley are **1750 N** and **250 N** as shown. Calculate the maximum tensile stress (σ_{max}) and the maximum shear stress (τ_{max}) in the shaft at the first bearing located **60 mm** from the pulley. Neglect shear stresses due to transverse loads i.e., due to bending shear.

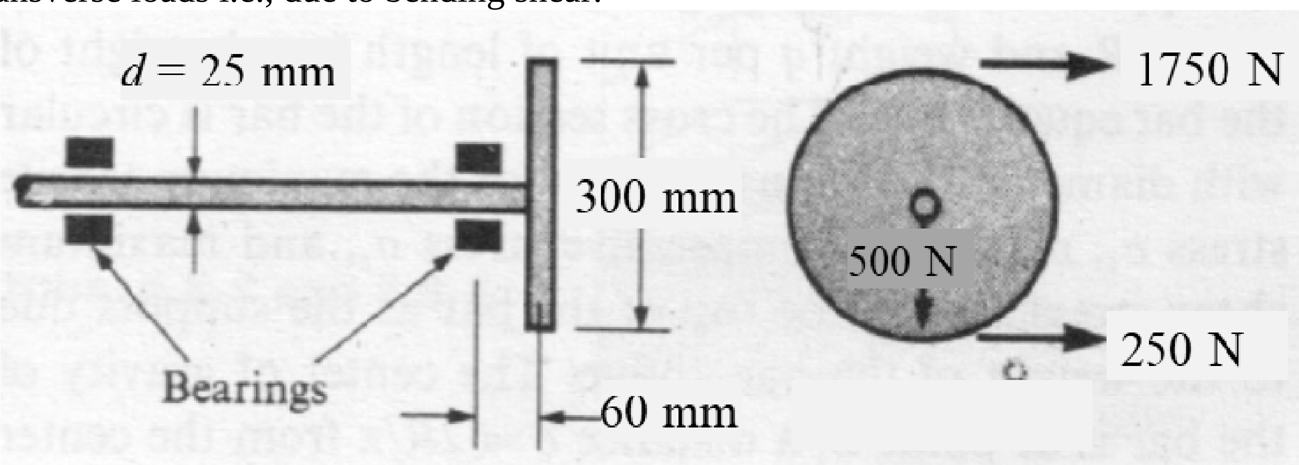


Fig. 4

Problem 5

The beam ABC rests on supports at A and C before the load is applied as shown in **Fig. 5**. There is a small gap Δ between the beam and the support at B. When the uniform load is applied to the beam, the gap closes and reactions develop at all three supports**.** Calculate the magnitude of the gap (Δ) for which the bending moment at point B of the beam ABC would be zero. Use the table provided on the next page to the extent possible.

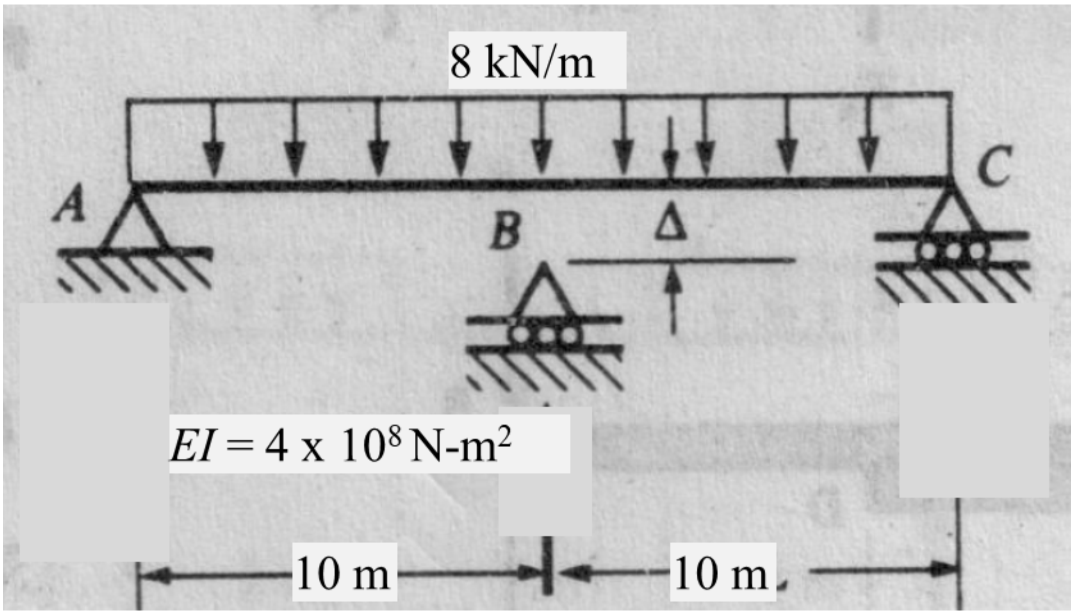


Fig. 5

Problem 6

The horizontal bar is supported by columns AB and CD as shown in **Fig. 6**. Each column is pinned at the top to the horizontal bar, and support A is fixed and support D is pinned. Both columns are solid steel bars (**E_s = 200 GPa**) of square cross-section of side **15 mm**. Use the table provided on the next page to the extent possible.

(a) If the distance a **= 0.4 m**, calculate the critical value of the load Q_{cr}.

(b) Calculate the maximum value of Q_{cr} if the distance a can be varied between 0 and 1 m.

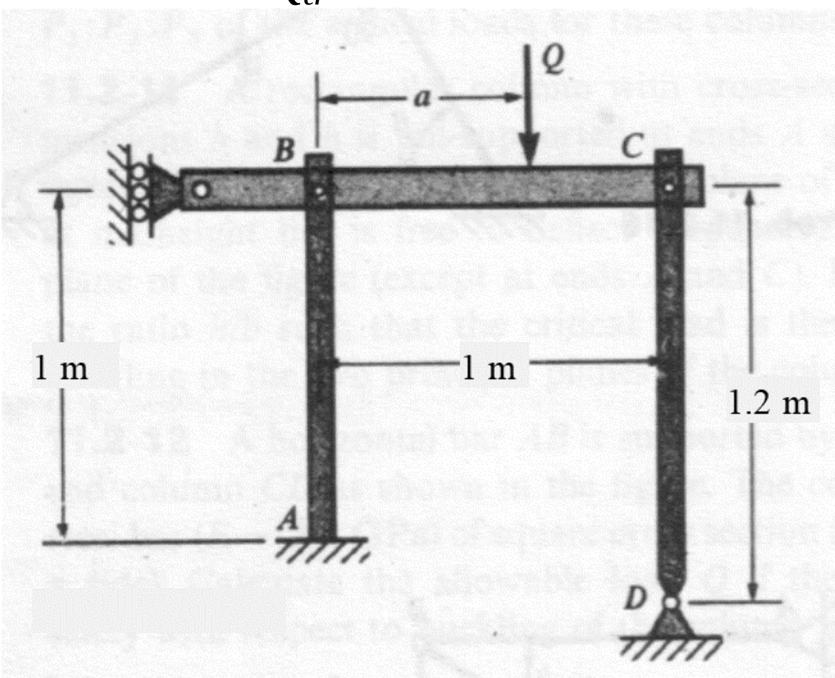


Fig. 6

DEPARTMENT OF CIVIL ENGINEERING

CE-221 SOLID MECHANICS

End-Sem Exam **17/11/2017** **PAPER CODE: B**

Note: Write your name & roll no. on answerbook and on summary answer sheet provided with the question paper.
You must submit the question-paper-cum-summary-answer-sheet along with the answerbook.
Closed book, closed notes test. No formula sheet allowed. No mobile phones allowed in the exam hall.
All questions carry equal marks. Assume suitable data if required and state the same clearly.
Use formula from provided tables, **if applicable**.

Problem 1

High strength steel wires are stretched by applying a force Q which produces an initial stress σ_0= **720 MPa** as shown in **Fig. 1 (a)**. Concrete is then poured around the stretched wires to form a beam as shown in **Fig. 1 (b)**. After the concrete sets properly, the force Q is removed as shown in **Fig. 1 (c)**. Thus the beam is left in a prestressed condition with the wires in tension and concrete in compression. The moduli of elasticity of steel and concrete are in the ratio of **9:1**. The areas of concrete and steel are in the ratio **40:1**. Calculate the final stresses in steel (σ_s) and concrete (σ_c).

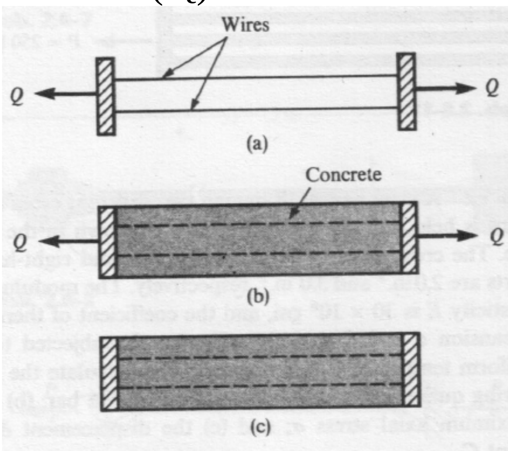


Fig. 1

Problem 2

A wooden beam is reinforced by steel plates as shown in **Fig. 2**. The ratio of moduli of elasticity for steel and wood E_s/E_w = **8**. Calculate the maximum tensile and maximum compressive stresses in steel due to the moment applied as shown in **Fig. 2**.

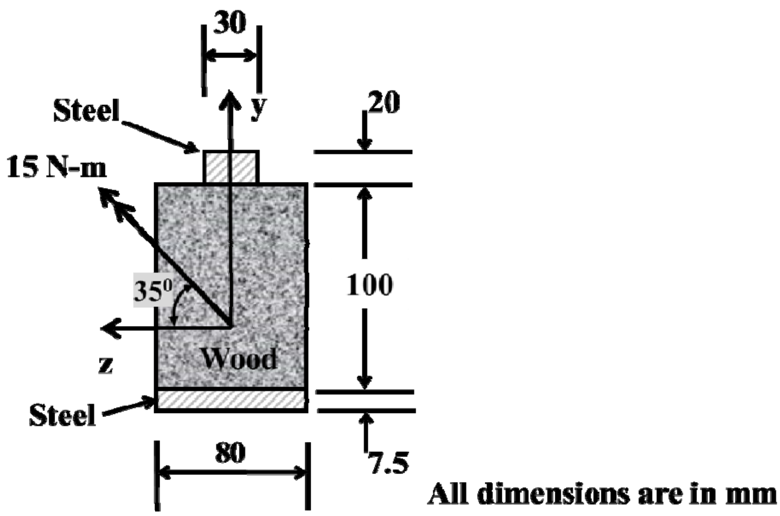
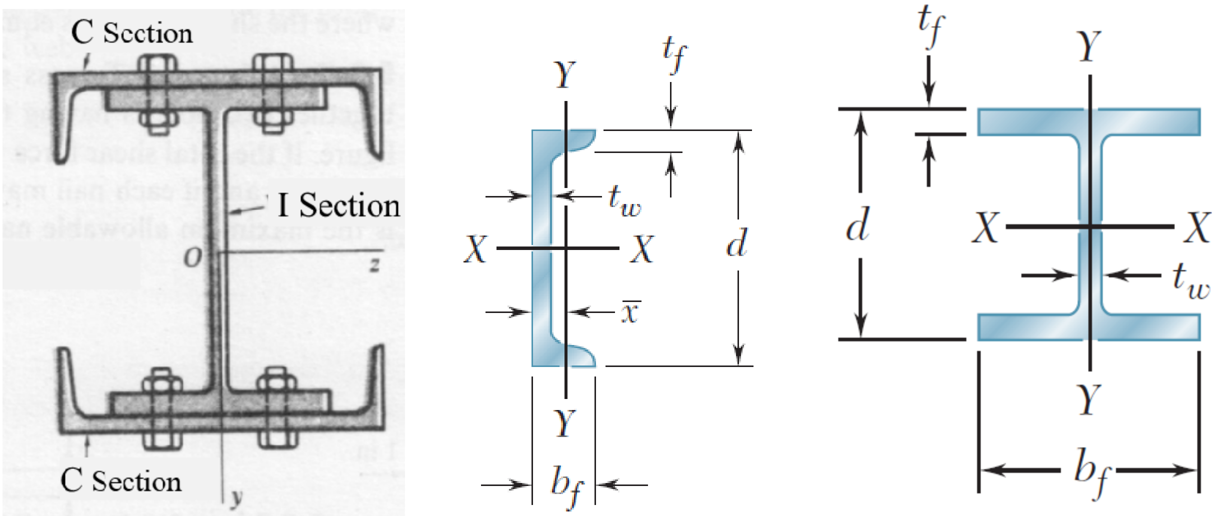


Fig. 2

Problem 3

A built-up beam is fabricated using a steel I-section and two aluminium channel sections as shown in **Fig. 3**. The sections are joined by bolts spaced at **120 mm** longitudinally. Calculate the maximum allowable shear force that can be resisted by the cross-section if each bolt can carry a shear force of **2.8 kN**. The ratio of moduli of elasticity for steel and aluminium is $\boldsymbol{E_s / E_a = 8}$.



Section Properties	d (mm)	b_f (mm)	t_f (mm)	t_w (mm)	Area (mm^2)	$\bar{x}$ (mm)	I_{XX} ($x10^6$ mm^4)	I_{YY} ($x10^6$ mm^4)
C Section	120.0	31.7	5.0	5.1	882	6.74	1.62	0.0514
I Section	121.9	80.8	6.4	3.7	1470	-	3.94	0.56

Fig. 3

Problem 4

A shaft of diameter **30 mm** supports a **350 mm** diameter pulley weighing **600 N** as shown in **Fig. 4**. The horizontal forces in the belt going over the pulley are **2000 N** and **350 N** as shown. Calculate the maximum tensile stress (σ_{max}) and the maximum shear stress (τ_{max}) in the shaft at the first bearing located **80 mm** from the pulley. Neglect shear stresses due to transverse loads i.e., due to bending shear.

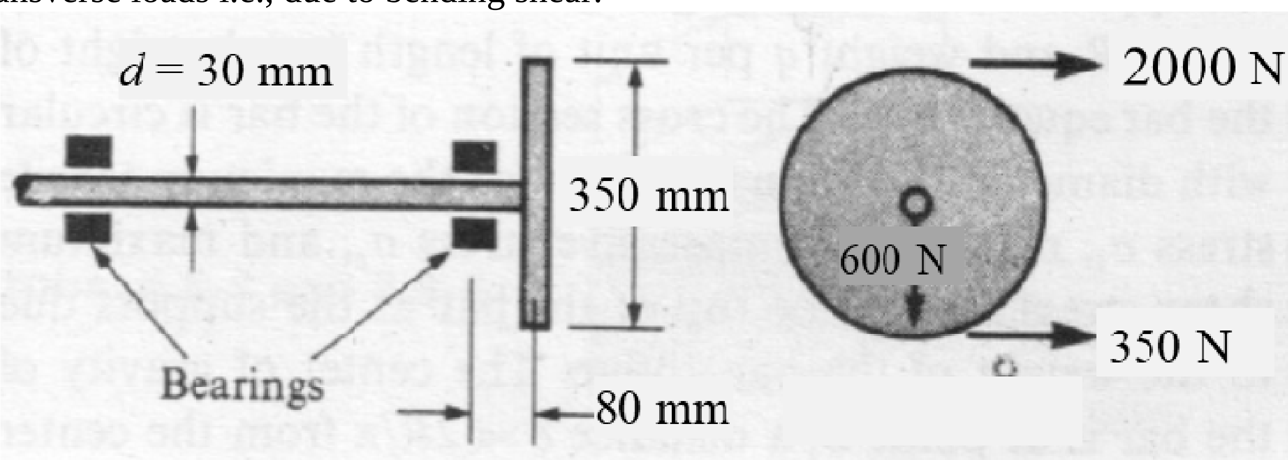


Fig. 4

Problem 5

The beam ***ABC*** rests on supports at ***A*** and ***C*** before the load is applied as shown in **Fig. 5**. There is a small gap Δ between the beam and the support at ***B***. When the uniform load is applied to the beam, the gap closes and reactions develop at all three supports**.** Calculate the magnitude of the gap (Δ) for which the bending moment at point ***B*** of the beam ***ABC*** would be zero. Use the table provided on the next page to the extent possible.

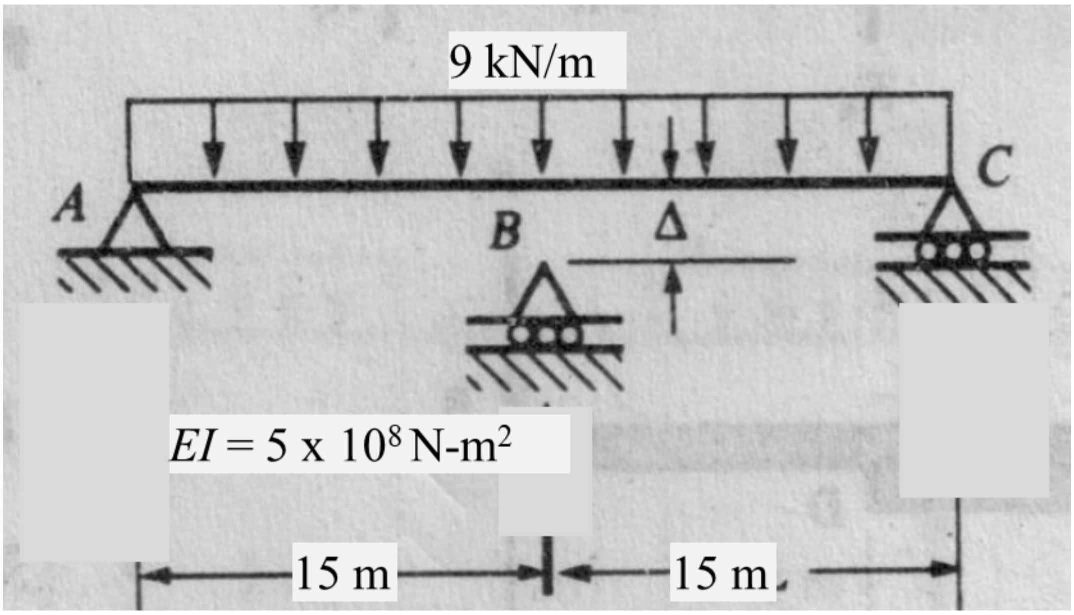


Fig. 5

Problem 6

The horizontal bar is supported by columns ***AB*** and ***CD*** as shown in **Fig. 6**. Each column is pinned at the top to the horizontal bar, and support ***A*** is fixed and support ***D*** is pinned. Both columns are solid steel bars (**E_s = 250 GPa**) of square cross-section of side **20 mm**. Use the table provided on the next page to the extent possible.

(a) If the distance ***a*** **= 0.3 m**, calculate the critical value of the load Q_{cr}.

(b) Calculate the maximum value of Q_{cr} if the distance ***a*** can be varied between 0 and 1 m.

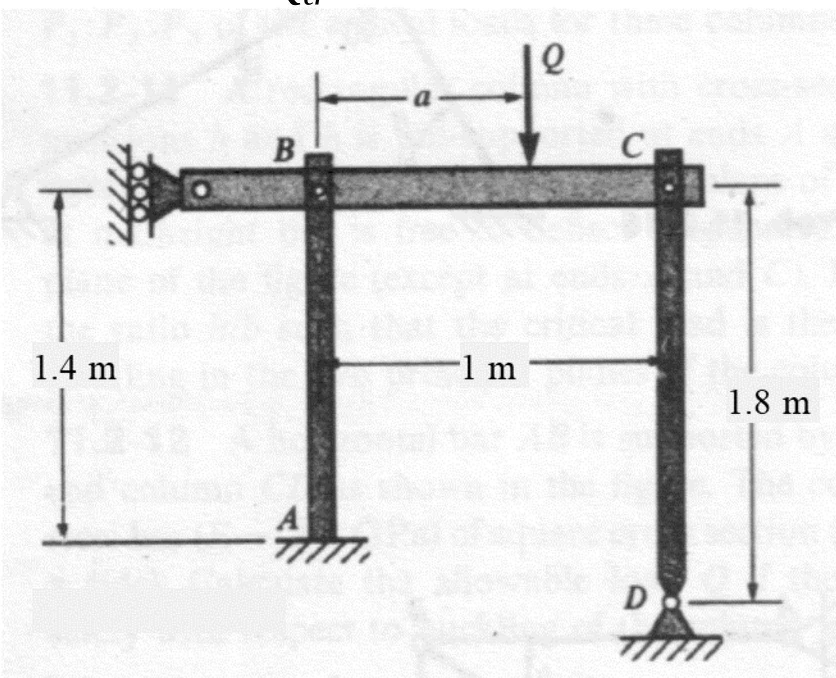


Fig. 6

DEPARTMENT OF CIVIL ENGINEERING

CE-221 SOLID MECHANICS

End-Sem Exam **17/11/2017** **PAPER CODE: C**

Note: Write your name & roll no. on answerbook and on summary answer sheet provided with the question paper.
You must submit the question-paper-cum-summary-answer-sheet along with the answerbook.
Closed book, closed notes test. No formula sheet allowed. No mobile phones allowed in the exam hall.
All questions carry equal marks. Assume suitable data if required and state the same clearly.
Use formula from provided tables, **if applicable**.

Problem 1

High strength steel wires are stretched by applying a force $\boldsymbol{Q}$ which produces an initial stress σ_o= **920 MPa** as shown in **Fig. 1 (a)**. Concrete is then poured around the stretched wires to form a beam as shown in **Fig. 1 (b)**. After the concrete sets properly, the force $\boldsymbol{Q}$ is removed as shown in **Fig. 1 (c)**. Thus the beam is left in a prestressed condition with the wires in tension and concrete in compression. The moduli of elasticity of steel and concrete are in the ratio of **7:1**. The areas of concrete and steel are in the ratio **20:1**. Calculate the final stresses in steel (σ_s) and concrete (σ_c).

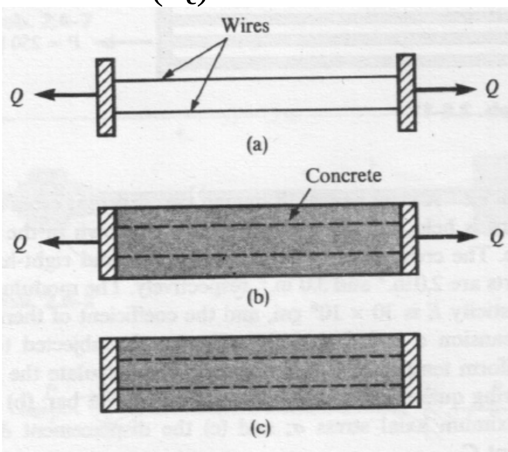


Fig. 1

Problem 2

A wooden beam is reinforced by steel plates as shown in **Fig. 2**. The ratio of moduli of elasticity for steel and wood $\boldsymbol{E_s / E_w}$ = **9**. Calculate the maximum tensile and maximum compressive stresses in steel due to the moment applied as shown in **Fig. 2**.

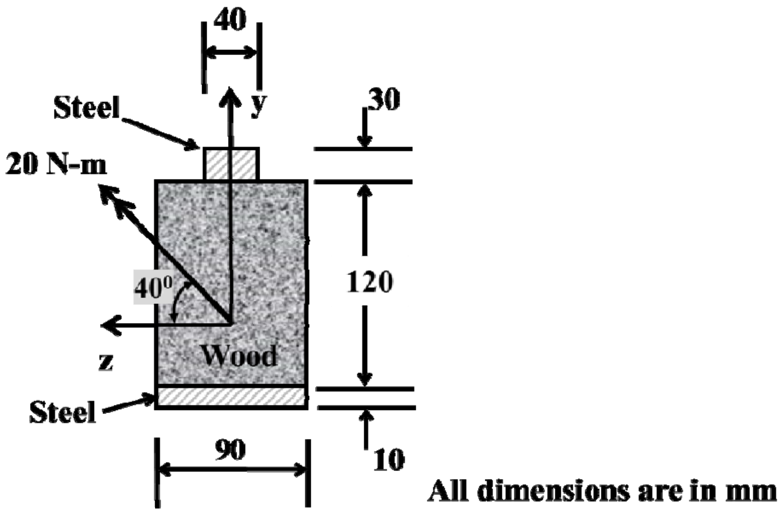
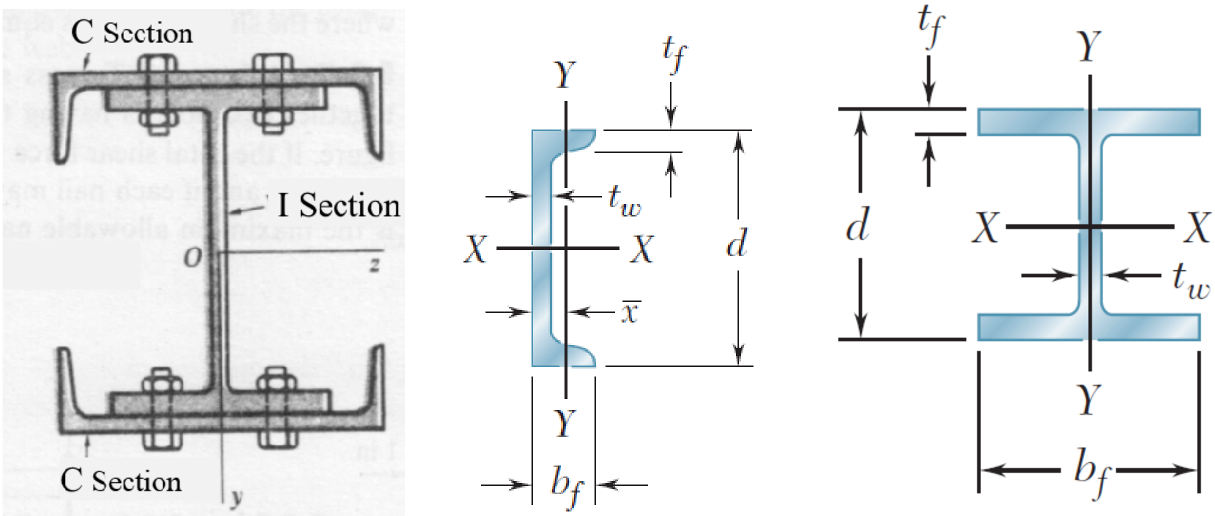


Fig. 2

Problem 3

A built-up beam is fabricated using a steel I-section and two aluminium channel sections as shown in **Fig. 3**. The sections are joined by bolts spaced at **80 mm** longitudinally. Calculate the maximum allowable shear force that can be resisted by the cross-section if each bolt can carry a shear force of **3.8 kN**. The ratio of moduli of elasticity for steel and aluminium is $\boldsymbol{E_s / E_a = 9}$.



Section Properties	d (mm)	b_f (mm)	t_f (mm)	t_w (mm)	Area (mm^2)	$\bar{x}$ (mm)	I_{XX} ($\times 10^6\ mm^4$)	I_{YY} ($\times 10^6\ mm^4$)
C Section	120.0	31.7	5.0	5.1	882	6.74	1.62	0.0514
I Section	121.9	80.8	6.4	3.7	1470	-	3.94	0.56

Fig. 3

Problem 4

A shaft of diameter **35 mm** supports a **400 mm** diameter pulley weighing **700 N** as shown in **Fig. 4**. The horizontal forces in the belt going over the pulley are **2250 N** and **450 N** as shown. Calculate the maximum tensile stress (σ_{max}) and the maximum shear stress (τ_{max}) in the shaft at the first bearing located **100 mm** from the pulley. Neglect shear stresses due to transverse loads i.e., due to bending shear.

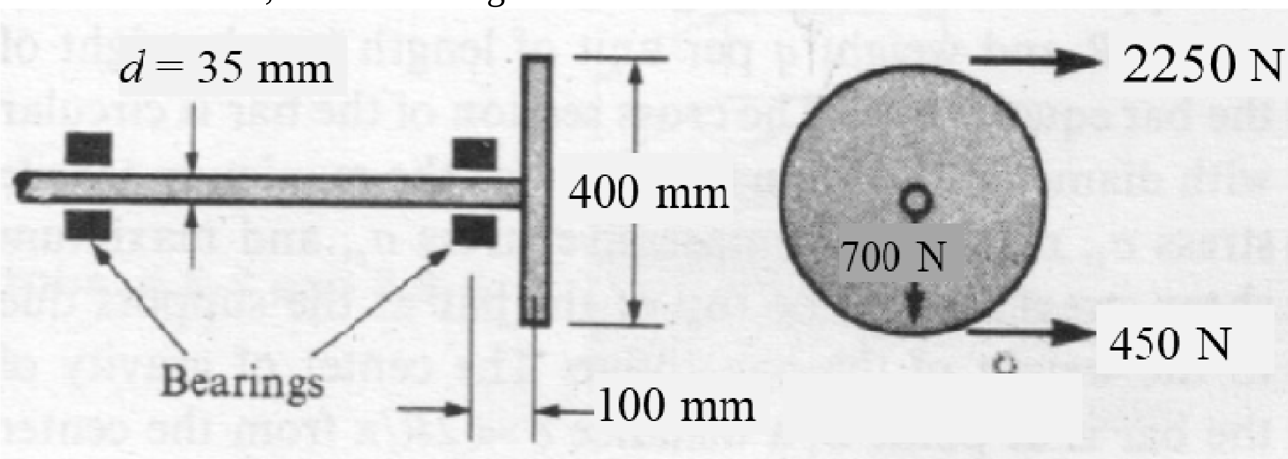


Fig. 4

Problem 5

The beam ***ABC*** rests on supports at ***A*** and ***C*** before the load is applied as shown in **Fig. 5**. There is a small gap Δ between the beam and the support at ***B***. When the uniform load is applied to the beam, the gap closes and reactions develop at all three supports. Calculate the magnitude of the gap (Δ) for which the bending moment at point ***B*** of the beam ***ABC*** would be zero. Use the table provided on the next page to the extent possible.

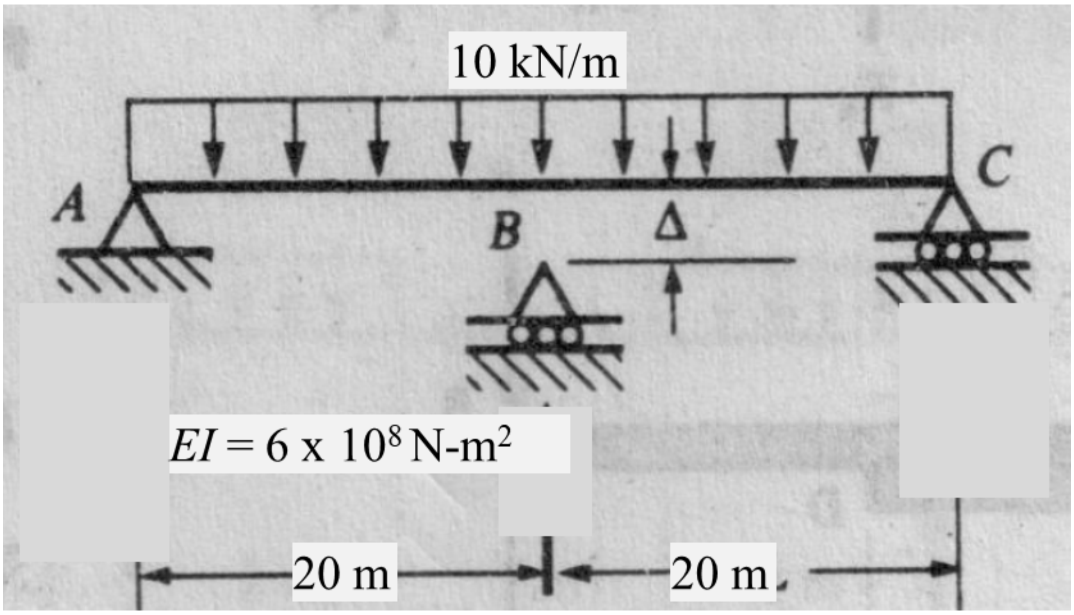


Fig. 5

Problem 6

The horizontal bar is supported by columns ***AB*** and ***CD*** as shown in **Fig. 6**. Each column is pinned at the top to the horizontal bar, and support ***A*** is fixed and support ***D*** is pinned. Both columns are solid steel bars (E_s **= 150 GPa**) of square cross-section of side **30 mm**. Use the table provided on the next page to the extent possible.

(a) If the distance ***a*** **= 0.2 m**, calculate the critical value of the load Q_{cr}.

(b) Calculate the maximum value of Q_{cr} if the distance ***a*** can be varied between 0 and 1 m.

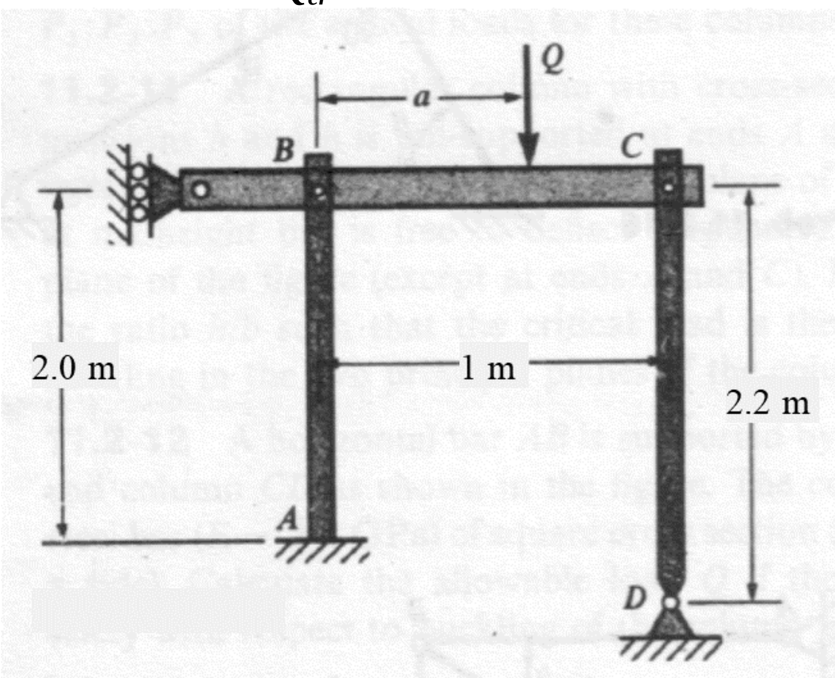


Fig. 6

DEPARTMENT OF CIVIL ENGINEERING

CE-221 SOLID MECHANICS

End-Sem Exam **17/11/2017** **PAPER CODE: D**

Note: Write your name & roll no. on answerbook and on summary answer sheet provided with the question paper.
You must submit the question-paper-cum-summary-answer-sheet along with the answerbook.
Closed book, closed notes test. No formula sheet allowed. No mobile phones allowed in the exam hall.
All questions carry equal marks. Assume suitable data if required and state the same clearly.
Use formula from provided tables, **if applicable**.

Problem 1

High strength steel wires are stretched by applying a force ***Q*** which produces an initial stress **σ_0= 1020 MPa** as shown in **Fig. 1 (a)**. Concrete is then poured around the stretched wires to form a beam as shown in **Fig. 1 (b)**. After the concrete sets properly, the force ***Q*** is removed as shown in **Fig. 1 (c)**. Thus the beam is left in a prestressed condition with the wires in tension and concrete in compression. The moduli of elasticity of steel and concrete are in the ratio of **10:1**. The areas of concrete and steel are in the ratio **50:1**. Calculate the final stresses in steel (σ_s) and concrete (σ_c).

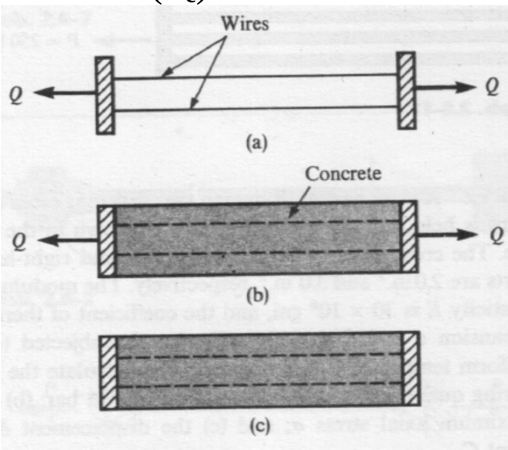


Fig. 1

Problem 2

A wooden beam is reinforced by steel plates as shown in **Fig. 2**. The ratio of moduli of elasticity for steel and wood E_s/E_w **= 10**. Calculate the maximum tensile and maximum compressive stresses in steel due to the moment applied as shown in **Fig. 2**.

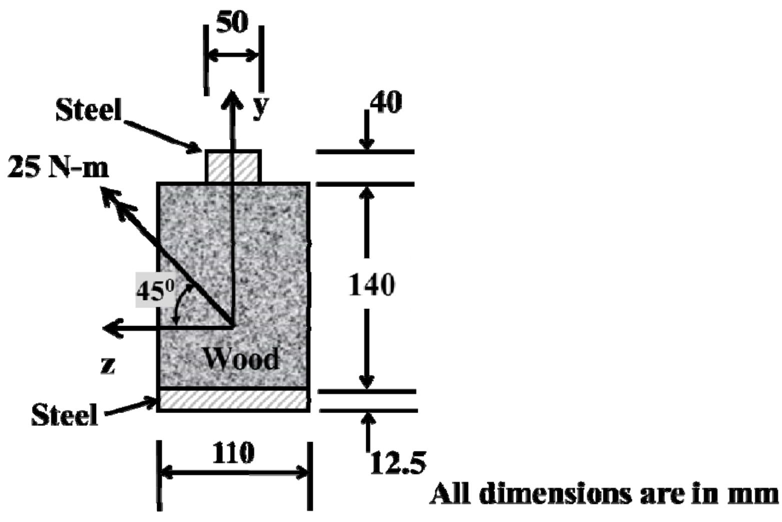
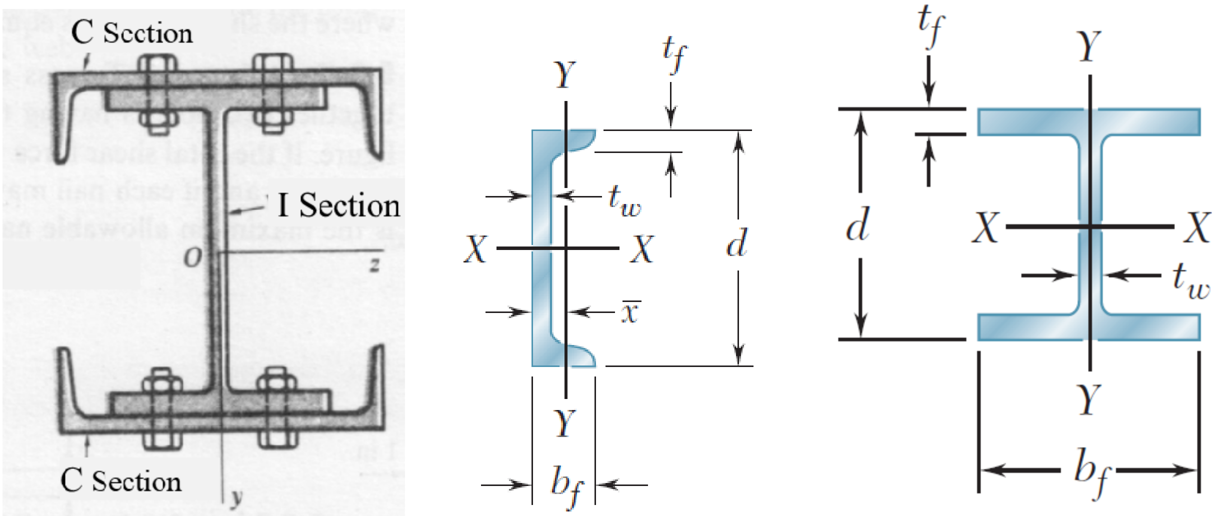


Fig. 2

Problem 3

A built-up beam is fabricated using a steel I-section and two aluminium channel sections as shown in **Fig. 3**. The sections are joined by bolts spaced at **100 mm** longitudinally. Calculate the maximum allowable shear force that can be resisted by the cross-section if each bolt can carry a shear force of **5.2 kN**. The ratio of moduli of elasticity for steel and aluminium is $\boldsymbol{E_s / E_a = 10}$.



Section Properties	d (mm)	b_f (mm)	t_f (mm)	t_w (mm)	Area (mm^2)	$\bar{x}$ (mm)	I_{XX} ($x10^6$ mm^4)	I_{YY} ($x10^6$ mm^4)
C Section	120.0	31.7	5.0	5.1	882	6.74	1.62	0.0514
I Section	121.9	80.8	6.4	3.7	1470	-	3.94	0.56

Fig. 3

Problem 4

A shaft of diameter **40 mm** supports a **450 mm** diameter pulley weighing **800 N** as shown in **Fig. 4**. The horizontal forces in the belt going over the pulley are **2500 N** and **550 N** as shown. Calculate the maximum tensile stress (σ_{max}) and the maximum shear stress (τ_{max}) in the shaft at the first bearing located **120 mm** from the pulley. Neglect shear stresses due to transverse loads i.e., due to bending shear.

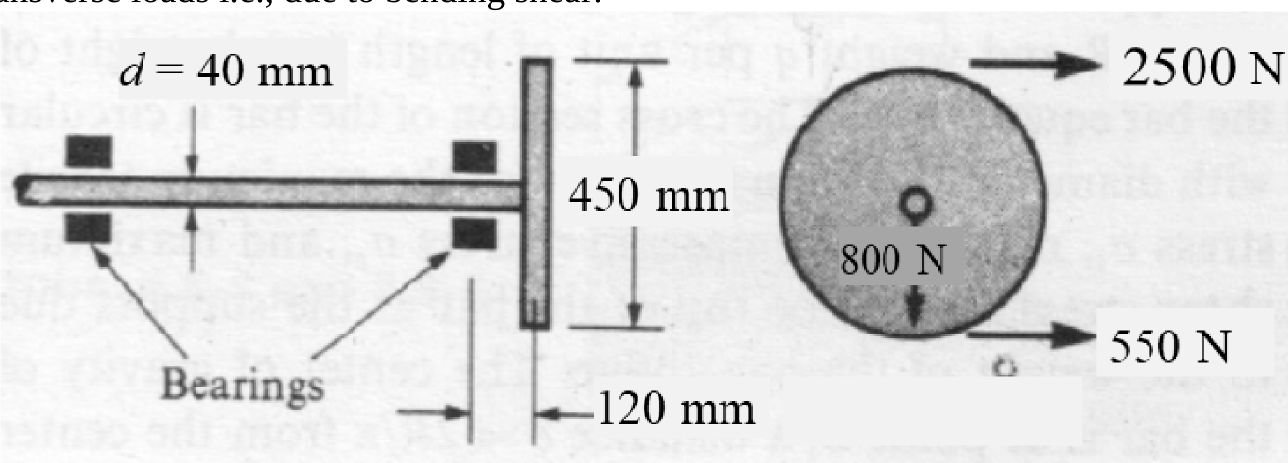


Fig. 4

Problem 5

The beam ***ABC*** rests on supports at ***A*** and ***C*** before the load is applied as shown in **Fig. 5**. There is a small gap Δ between the beam and the support at ***B***. When the uniform load is applied to the beam, the gap closes and reactions develop at all three supports**.** Calculate the magnitude of the gap (Δ) for which the bending moment at point ***B*** of the beam ***ABC*** would be zero. Use the table provided on the next page to the extent possible.

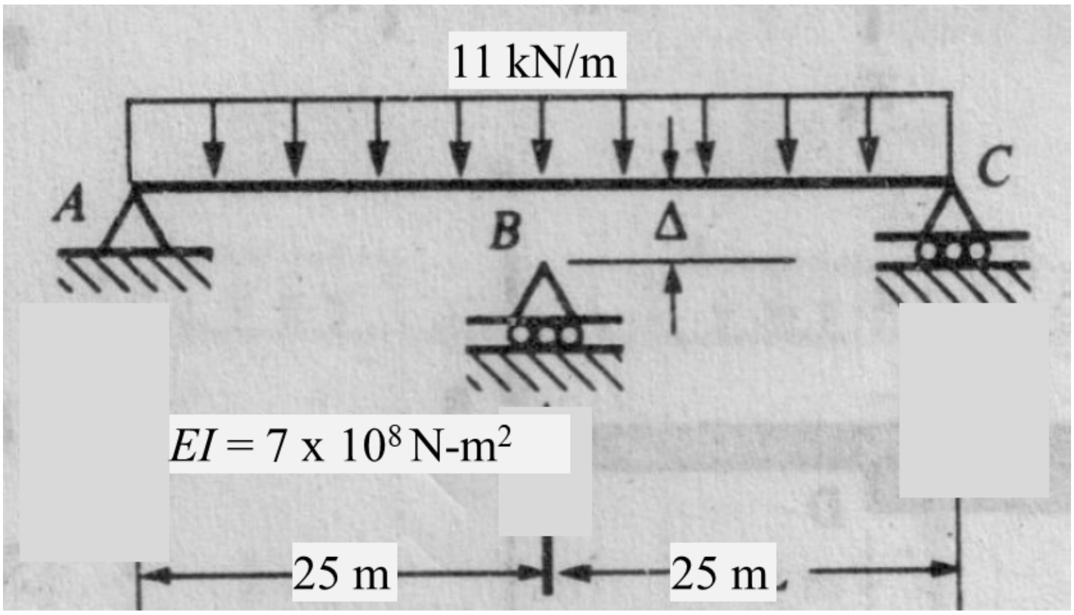


Fig. 5

Problem 6

The horizontal bar is supported by columns ***AB*** and ***CD*** as shown in **Fig. 6**. Each column is pinned at the top to the horizontal bar, and support ***A*** is fixed and support ***D*** is pinned. Both columns are solid steel bars ($\boldsymbol{E_s}$ **= 300 GPa**) of square cross-section of side **25 mm**. Use the table provided on the next page to the extent possible.

(a) If the distance $\boldsymbol{a}$ **= 0.1 m**, calculate the critical value of the load $\boldsymbol{Q_{cr}}$.

(b) Calculate the maximum value of $\boldsymbol{Q_{cr}}$ if the distance $\boldsymbol{a}$ can be varied between 0 and 1 m.

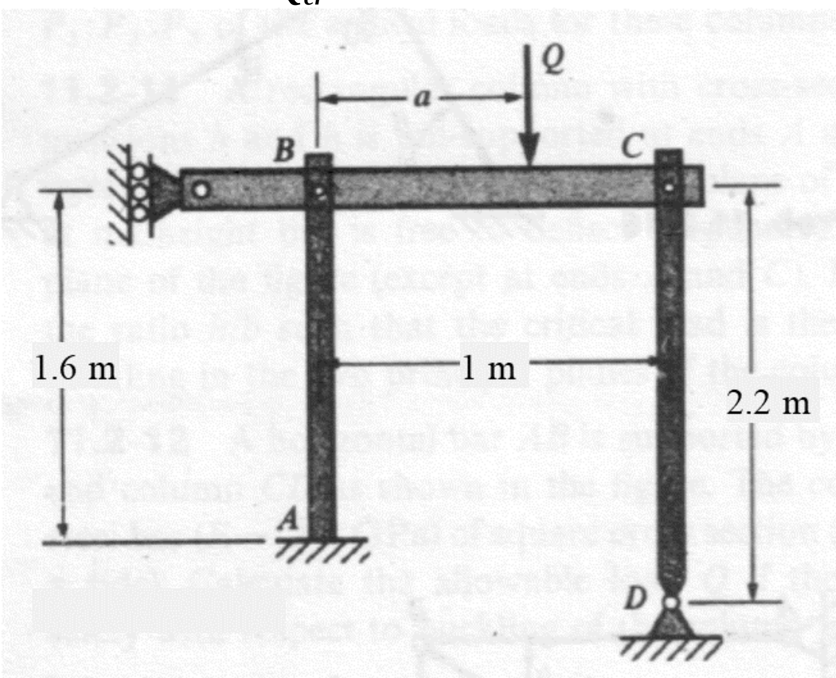


Fig. 6

CE221 Endsem 2017 ①

P1 $F_s = \sigma_s A_s = E_s(\varepsilon_0 - \varepsilon) A_s = F_c = \sigma_c A_c = E_c \varepsilon A_c$

(used $F_s = F_c$ and $\varepsilon_s = \varepsilon_c = \varepsilon$).

$$\Rightarrow \varepsilon = \frac{\varepsilon_0}{1 + \frac{A_c}{A_s}\frac{E_c}{E_s}} \Rightarrow E_s \varepsilon = \frac{E_s \varepsilon_0}{1 + \frac{A_c}{A_s}\frac{E_c}{E_s}}$$

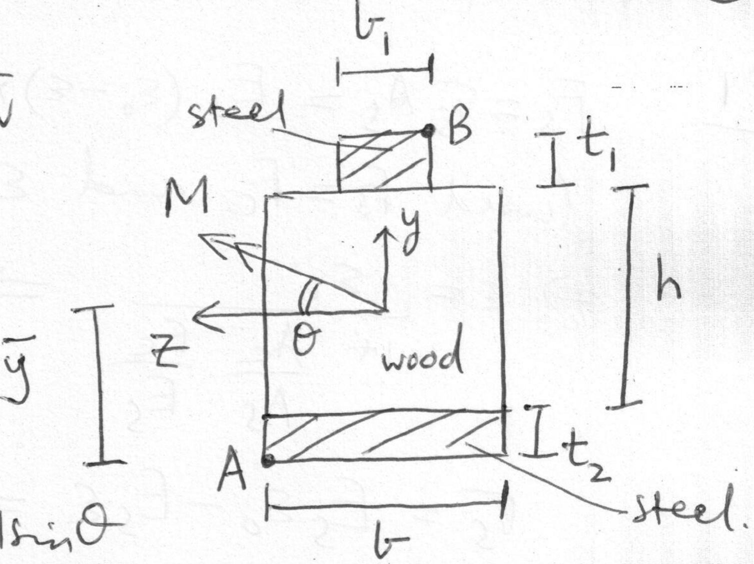
$$\sigma_s = E_s \varepsilon_0 - E_s \varepsilon = E_s \varepsilon_0 \left(1 - \frac{1}{1 + \frac{A_c}{A_s}\frac{E_c}{E_s}}\right) =$$

$$\sigma_c = \sigma_s \frac{A_s}{A_c} =$$

	Code – A	B	C	D
σ_s (MPa)	647.37	587.76	681.48	850
σ_c (MPa)	21.58	14.69	34.07	17

IIT Bombay

$$n = \frac{E_s}{E_w}$$



P2

$$M_z = M\cos\theta, \quad M_y = M\sin\theta$$

$$\bar{y} = \frac{n b t_2 \frac{t_2}{2} + bh\left(\frac{h}{2} + t_2\right) + n b_1 t_1 \left(t_2 + h + t_1/2\right)}{n b t_2 + bh + n b_1 t_1}$$

$$I_y = \frac{1}{12}\left(h b^3 + n t_1 b_1^3 + n t_2 b^3\right)$$

$$I_z = \frac{1}{12}\left(b h^3 + n b t_2^3 + n b_1 t_1^3\right) + bh\left(\frac{h}{2} + t_2 - \bar{y}\right)^2 + n b t_2\left(\frac{t_2}{2} - \bar{y}\right)^2 + n b_1 t_1\left(h + t_2 + \frac{t_1}{2} - \bar{y}\right)^2$$

$$\sigma_x = -\frac{M\cos\theta}{I_z} y + \frac{M\sin\theta}{I_y} z$$

max tensile stress in steel at A $= \left\{-\frac{M\cos\theta}{I_z}(-\bar{y}) + \frac{M\sin\theta}{I_y}\left(\frac{b}{2}\right)\right\} \cdot n$

max compr " " " at B $\Rightarrow \left\{-\frac{M\cos\theta}{I_z}\left(h + t_1 + t_2 - \bar{y}\right) + \frac{M\sin\theta}{I_y}\left(-\frac{b_1}{2}\right)\right\} \cdot n$

	code A	B	C	D
max tensile stress in steel (MPA) =	0.774	0.536	0.470	0.356
max compr stress in steel (MPa) =	0.579	0.321	0.268	0.196

IIT Bombay

INDIAN INSTITUTE OF TECHNOLOGY BOMBAY

P.3

$$n = \frac{E_{st}}{E_{Al}}$$

$$\Delta H = -\int(\sigma_R - \sigma_L)\,dA = -\int(\sigma_R' - \sigma_L')\frac{1}{n}dA$$

σ_R: right, σ_L: left. $dA \to Al$; $\sigma_R' - \sigma_L' \to st$; $\frac{1}{n}dA \to dA'$, $dA' \to st$

$$= \frac{\Delta M}{I_{T,st}}\frac{1}{n}\int y\,dA = \frac{V\Delta x}{I_S + \frac{1}{n}I_A}\frac{1}{n}Q$$

} Transform to Steel

same

or

$$\Delta H = -\int(\sigma_R - \sigma_L)\,dA = \frac{\Delta M}{I_{T,Al}}\int y\,dA = \frac{V\Delta x}{nI_S + I_A}Q$$

($\sigma_R - \sigma_L \to Al$)

} Transform to Al.

Each bolt can carry ΔH_{max}, bolt spacing is Δx,

$$\Rightarrow V_{max} = 2\left(\frac{\Delta H_{max}}{\Delta x}\right)\left(\frac{nI_S + I_A}{Q}\right)$$ → can write this directly based on transformed section

$I_S \to$ as given, for I-section

$$I_A = 2(\bar{I}_A + A_{Al}\bar{d}^2)$$

$$\bar{d} = \frac{d}{2} + t_w - \bar{x}$$

for I-section ($\frac{d}{2}$); for channel section ($t_w - \bar{x}$)

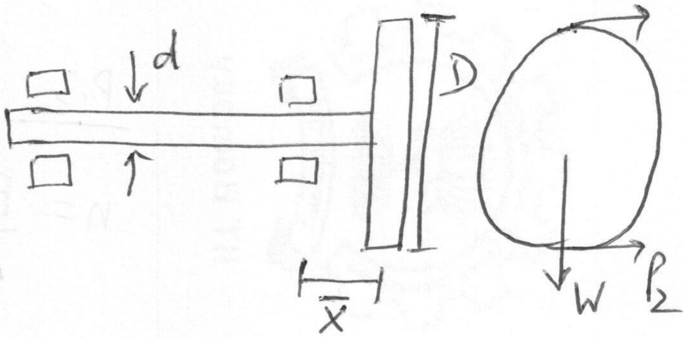
$$Q = A_{Al}\cdot\bar{d}$$

Given: $\Delta H_{max}, \Delta x, n, I_S, \bar{I}_A, A_{Al}, d, t_w, \bar{x}$

	Code-A	B	C	D
V_{max} (kN)	51.82	33.74	75.85	90.87

P4 $M_y = (P_1+P_2)\bar{x}$, $M_z = -W\bar{x}$, $T = (P_1-P_2)D/2$

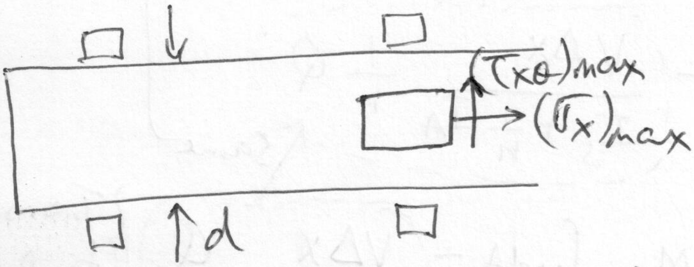
$$M = \sqrt{M_y^2 + M_z^2}$$



$$(\sigma_x)_{max} = \frac{Md/2}{\frac{\pi}{64}d^4}$$

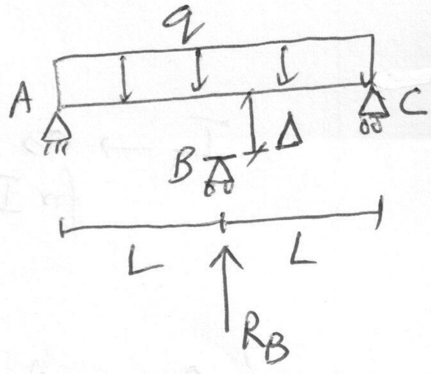
$$(\tau_{x\theta})_{max} = \frac{Td/2}{\frac{\pi}{32}d^4}$$

$$(\sigma_{tensile})_{max} = \frac{(\sigma_x)_{max}}{2} + \underbrace{\sqrt{\left(\frac{(\sigma_x)_{max}}{2}\right)^2 + (\tau_{x\theta})_{max}^2}}_{\tau_{max}}$$



	code-A	B	C	D
$(\sigma_t)_{max}$ (MPa)	124.01	102.22	87.23	76.22
τ_{max} (MPa)	83.69	65.62	54.09	46.11

P.5 Use Tables & superposition.



$$y_1 = \frac{5q(2L)^4}{384EI}; \quad y_2 = \frac{R_B(2L)^3}{48EI}$$

only q acting w/o R_B — only R_B acting w/o q

BM=0 ⇒ (ie $M_B=0$) $\quad R_A L - \frac{qL^2}{2} \longrightarrow R_A = R_C = \frac{qL}{2}$, $R_B = qL$

compatibility $\longrightarrow \Delta = y_1 - y_2 = \left(\frac{5\times16}{384} - \frac{8}{48}\right)\frac{qL^4}{EI} = \frac{1}{24}\frac{qL^4}{EI}$

	code-A	B	C	D
Δ (mm)	8.33	37.97	111.11	255.77

IIT Bombay

P6

Equil : $P_{AB}(1-a)Q$; $P_{CD} = aQ$

$$X = \frac{(P_{cr})_{\text{pinned-pinned}}}{(P_{cr})_{\text{pinned-fixed}}} = \frac{(L_e)^2_{AB}}{(L_e)^2_{CD}} = \frac{0.7^2 L^2_{AB}}{L^2_{CD}}$$

$$Y = \frac{P_{CD}}{P_{AB}} = \frac{a}{1-a}$$

If $Y > X$ column CD buckles first $\Rightarrow Q_{cr} = \dfrac{(P_{CD})_{cr}}{a} = \dfrac{\pi^2 EI/L^2_{CD}}{a}$

$Y < X$ " AB "

$$Q_{cr} = (P_{AB})_{cr}/(1-a)$$

$$= \frac{\pi^2 EI}{(0.7 L_{AB})^2} \cdot \frac{1}{(1-a)}$$

Max Q_{cr} when both AB & CD buckle simultaneously,

ie, $Y = X$

$$\frac{a}{1-a} = \frac{0.7^2 L^2_{AB}}{L^2_{CD}}$$

$$(Q_{cr})_{max} = \frac{(P_{CD})_{cr}}{a} = \frac{(P_{AB})_{cr}}{1-a} = \frac{\pi^2 EI/L^2_{CD}}{a} = \frac{\pi^2 EI/(0.7 L_{AB})^2}{1-a}$$

	code-A	B	C	D
Q_{cr} (kN)	14.457	33.846	63.731	85.373
$(Q_{cr})_{max}$ (kN)	22.778	44.409	71.631	96.749