DEPARTMENT OF CIVIL ENGINEERING, IIT BOMBAY

CE 221 Solid Mechanics: Midsem

Note: **Assume suitable data if not given.**

10/09/2014 Total Marks:30 Duration: 2 hrs Instructors S. Banerjee / N.K. Chandiramani

Problem 1

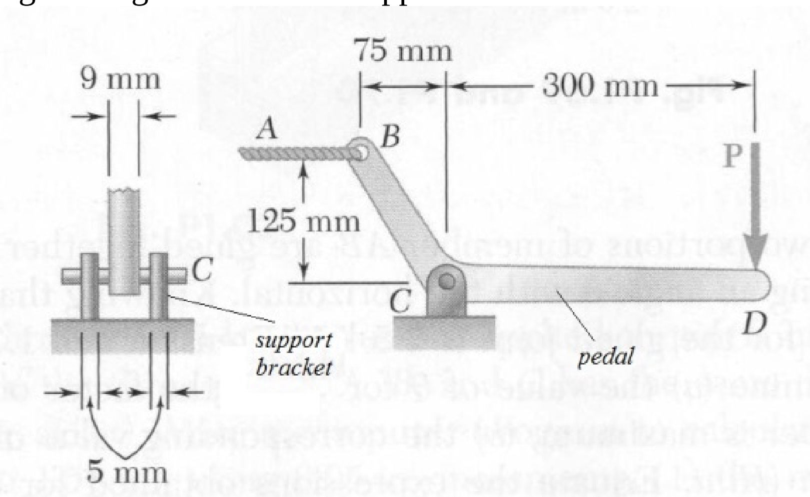
(a) **(3 marks)**

A force ***P* = 750N** is applied to the pedal ***BCD*** as shown. Find:

(i) the diameter of the pin at ***C*** for which the shearing stress in the pin is 40 MPa.

(ii) the corresponding bearing stress in the pedal at ***C***.

(iii) the corresponding bearing stress in each support bracket at ***C***.

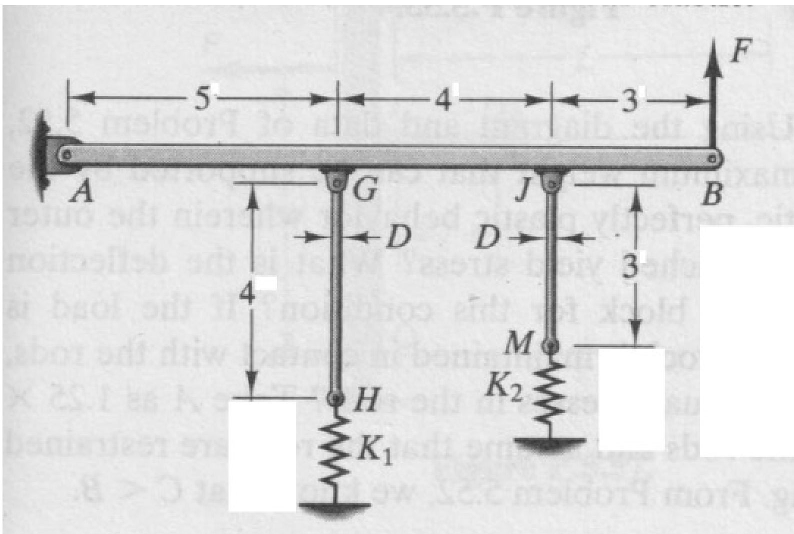


(b) **(4 marks):**

A steel shaft consists of a hollow shaft 2m long, with an outside diameter of 100 mm and an inside diameter of 70 mm, rigidly attached to a solid shaft of 1.5 m length and 70 mm diameter. Determine the maximum power that can be transmitted by the shaft at a speed of 100 rpm without exceeding a shear stress of 50 MPa and a twist of 5^0 in the 3.5 m length. Use ***G*** = 80 GPa.

Problem 2 (8 marks):

A rigid bar ***AB*** is connected to flexible cylindrical rods ***GH*** and ***JM*** (diameter ***D***) which are connected to linear springs with stiffness K_1 and K_2, respectively. The bar ***AB*** is horizontal when the system is unstressed and at room temperature. A force ***F*** = 100kN is applied and simultaneously the rod ***JM*** is heated by 50°C and the rod ***GH*** is cooled by 50°C with respect to room temperature. Find the angle of rotation of bar ***AB***. Use data $K_1 = 0.15 \times 10^9$ N/m, $K_2 = 0.2 \times 10^9$ N/m, ***E*** = 200 GPa, ***D*** = 100 mm, $\alpha = 12 \times 10^{-6}$ / ^{0}C. All dimensions shown are in meters.

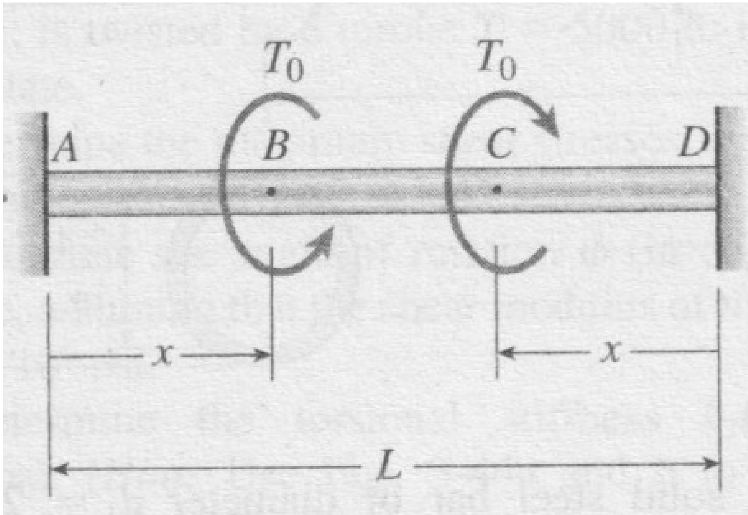


Problem 3 (7 marks):

A solid circular shaft ***ABCD*** with **fixed supports** at ends ***A*** and ***D*** is acted upon by two equal and oppositely directed torques $\boldsymbol{T_0}$ applied at points ***B*** and ***C*** as shown. Find:

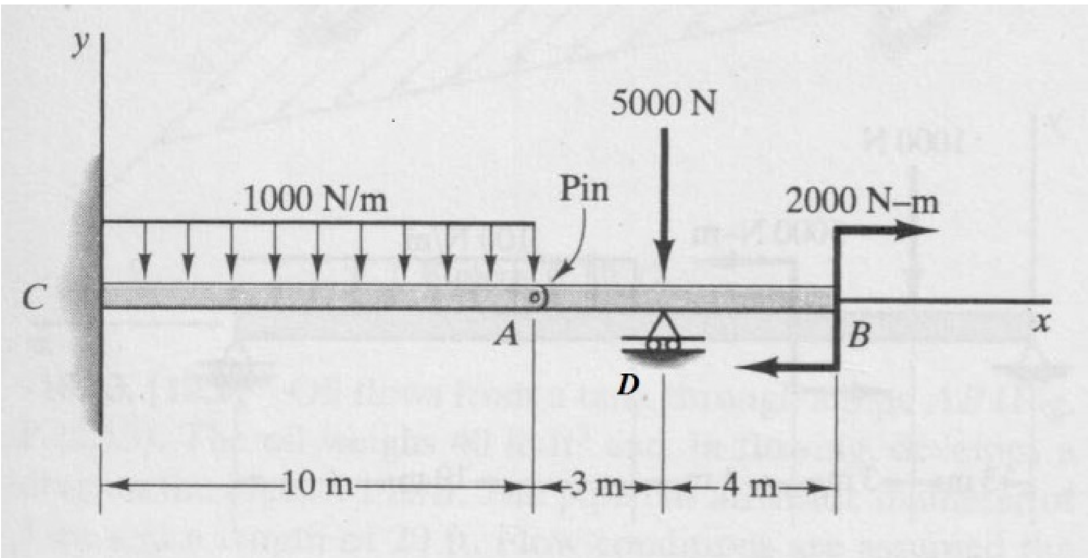
(i) the distance ***x*** for which the maximum angle of twist occurs at points ***B*** and ***C***.

(ii) the corresponding maximum angle of twist at ***B*** and ***C***.



Problem 4 (8 marks):

The structure shown comprises beams ***CA*** and ***AB*** connected together by a pin at ***A***, with **fixed support** at ***C***, and **roller support** at ***D***, and loading as shown. The load at ***B*** is a clockwise couple moment. **Draw** the **shear force** and **bending moment diagrams**. You **<u>must show</u>** all the key values, proper shapes (indicate whether linear, quadratic, cubic, etc), location of zero shear force, maximum shear force, zero bending moment, maximum bending moment, etc.



P1

(a) $C_y = P = 750, \quad C_x = F_{AB} = P.\frac{300}{125} = 1800$

$$C = \sqrt{C_x^2 + C_y^2} = 1950\,N$$

$$\underset{N/mm^2}{40} = \frac{1950}{2 * \frac{\pi}{4}(d^2)} \Rightarrow d = 5.571\,mm$$ ◄

$$\sigma_{\substack{bearing\\pedal}} = \frac{1950}{9 * 5.571} = 38.89\,MPa$$ ◄

$$\sigma_{\substack{bearing\\support\\bracket}} = \frac{1950}{2 * 5 * 5.571} = 35.003\,MPa$$ ◄

(b) $P_{max} = T_{max}\left(\frac{100 * 2\pi}{60}\right)$

$50\,Mpa = \tau_{max} = \frac{Tr}{J}$ so shaft with higher $\frac{r}{J}$ is critical for τ_{max} criteria

$\left(\frac{r}{J}\right)_{solid} = \frac{70/2}{\frac{\pi}{32}.70^4}$

$\left(\frac{r}{J}\right)_{hollow} = \frac{100/2}{\frac{\pi}{32}(100^4 - 70^4)}$

so $\left(\frac{r}{J}\right)_{solid}$ dominates

$$T = \frac{50\left(\frac{\pi}{32}\cdot 70^4\right)}{(70/2)} = 3367395\,N.mm = 3367.4\,N.m$$ from τ_{max} criteria

$$\frac{5\pi}{180} = \theta_{max} = \sum \frac{TL}{GJ} = \frac{T}{80E9}\left(\frac{2}{(0.1^4 - 0.07^4)} + \frac{1.5}{0.07^4}\right)\frac{32}{\pi}$$

$\Rightarrow T = 7719\,N.m$ from θ_{max} criteria

Choose lower $\Rightarrow T_{max} = 3367.4\,N.m.$ ◄

P2 Equilibrium → $5F_1 + 9F_2 = 12 * 100E3$ → (1)

Tensile, expansion, positive. $F_1 = F_{GH}$, $F_2 = F_{JM}$

Compatibility → $$\frac{\frac{F_1}{K_1} + \frac{F_1 L_1}{AE} + \alpha \Delta T_1 L_1}{5} = \frac{\frac{F_2}{K_2} + \frac{F_2 L_2}{AE} + \alpha \Delta T_2 L_2}{9}$$ ← (2)

Solve (1), (2),

$$\Rightarrow F_1 = \frac{\frac{5}{9}\left[F_2\left(\frac{1}{K_2} + \frac{L_2}{AE}\right) + \alpha \Delta T_2 L_2\right] - \alpha \Delta T_1 L_1}{\left(\frac{1}{K_1} + \frac{L_1}{AE}\right)}$$

$$F_2 = \frac{\left[12 * 100E3 - \frac{\left(\frac{25}{9} \alpha \Delta T_2 L_2 - 5 \alpha \Delta T_1 L_1\right)}{\left(\frac{1}{K_1} + \frac{L_1}{AE}\right)}\right]}{9 + \frac{\frac{25}{9}\left(\frac{1}{K_2} + \frac{L_2}{AE}\right)}{\left(\frac{1}{K_1} + \frac{L_1}{AE}\right)}}$$

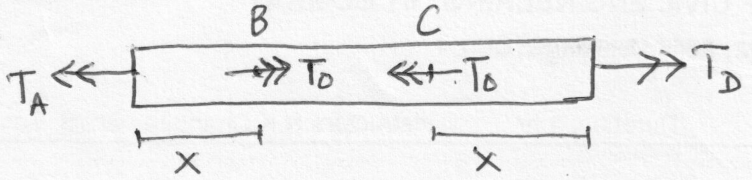
Put $K_1 = 0.15E9$, $K_2 = 0.2E9$, $L_1 = 4m$, $L_2 = 3m$, $E = 200\,GPa = 200E9$,

$A = \frac{\pi}{4}(100E-3)^2$, $\alpha = 12E-6$, $\Delta T_1 = -50°C$, $\Delta T_2 = 50°C$

get $F_2 = -58213\,N$, $F_1 = 344783\,N$

$$\theta = \frac{\frac{F_2}{K_2} + \frac{F_2 L_2}{AE} + \alpha \Delta T_2 L_2}{9} = 1.5531E-4 \text{ radians}$$

P3



Equilibrium : $T_A = T_D$.

Compatibility : $\theta_D = \dfrac{T_A x}{GJ} + \dfrac{(T_A - T_0)(L-2x)}{GJ} + \dfrac{T_A x}{GJ} = 0$

$$\Rightarrow T_A = \frac{T_0(L-2x)}{L}$$

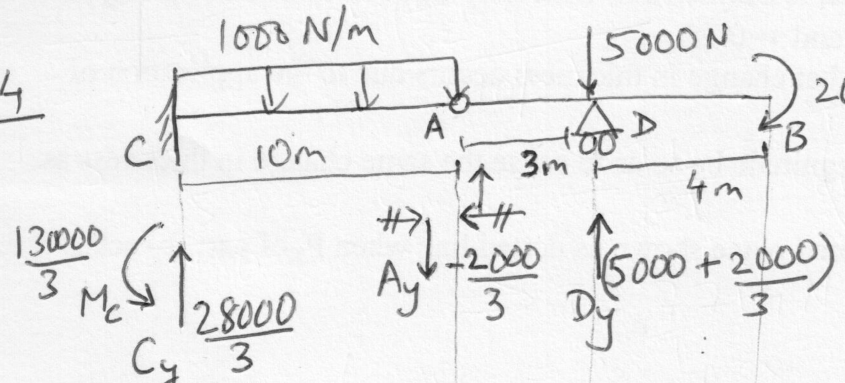
$$\theta_B = \frac{T_A x}{GJ} = \frac{T_0}{GJ}\,\frac{x(L-2x)}{L} = \theta_C$$

$$\frac{d\theta_B}{dx} = 0 \Rightarrow L - 4x = 0 \Rightarrow x = \frac{L}{4}$$

$$\frac{d^2\theta_B}{dx} = -4 < 0 \Rightarrow \text{maxima.}$$

$$\theta_B = \theta_C = \frac{T_0}{GJ}\,\frac{L/4\,(L - 2L/4)}{L} = \frac{1}{8}\frac{T_0 L}{GJ}$$

P4



$A_x = 0$

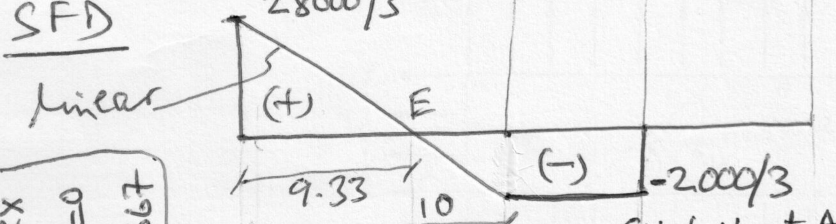
$A_y = -\dfrac{2000}{3}$

$M_C = \dfrac{2000}{3}\cdot 10 - 1000\cdot\dfrac{10^2}{2}$

$= -130000/3$

$V_C = V_A + 1000\cdot 10 = \dfrac{28000}{3}$ ($V_A = -2000/3$)

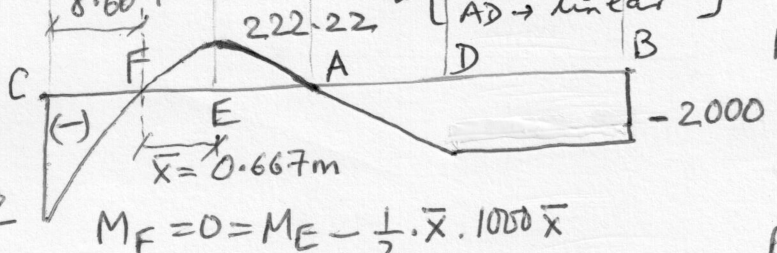
SFD



$x_E = \dfrac{28000/3}{28000/3 + \dfrac{2000}{3}} * 10 = \dfrac{14}{15} * 10$

$= 9.333$

{No kink at A
CA → quadratic
AD → linear}



$-\dfrac{130000}{3} + \dfrac{28000}{3}x - 1000\dfrac{x^2}{2} = 0$ gives, $x = 8.667$

$M_A = M_D + \dfrac{2000}{3}\cdot 3 = 0.$ ($M_D = 2000$)

$M_E = M_A + \dfrac{1}{2}\cdot\dfrac{2000}{3}(10 - 9.333)$

$= 222.2$

$M_F = 0 = M_E - \dfrac{1}{2}\cdot\bar{x}\cdot 1000\bar{x}$

$\Rightarrow \bar{x} = 0.6667m.$

$M_C = M_E - \dfrac{1}{2}\cdot\dfrac{28000}{3}\cdot 9.333$

$= 130000/3$