DEPARTMENT OF CIVIL ENGINEERING, IIT BOMBAY

CE 221 Solid Mechanics: Midsem

Note: **Assume suitable data if not given.**

07/09/2015 Total Marks:30 Duration: 2 hrs Instructors N.K. Chandiramani / A. Laskar

Problem 1 (7.5 Marks):

A block **A** weighs **90,000 N** and is to be supported by three elastic steel members **DC**, **CE** and **EF** as shown in **Fig 1**. Take the block **A** as **rigid** and the modulus of elasticity of the three elastic steel members as **1.93 x 10^{11} Pa**. Neglect the self weight of the three elastic steel members. Assume small displacements and rotations.

Calculate the vertical and horizontal displacements of pins C and F due to the weight of the block.

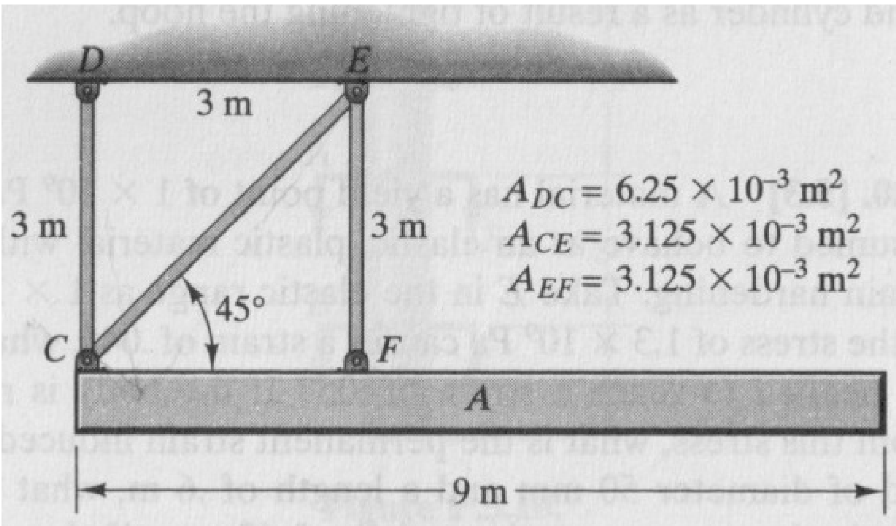

Fig. 1

Problem 2 (7.5 Marks):

Two solid shafts **1** and **2** are rigidly connected to a hollow shaft **3** as shown in **Fig. 2**. Shaft **2** is fixed at the right end while shaft **3** is fixed at the left end. A torque ***T* = 500 N-mm** is applied to the free end of shaft **1**. The shear modulus of all the three shafts is ***G* = 90 GPa**.

- **(i)** **Calculate the torques at the supports of shafts 2 and 3,**
- **(ii)** **Calculate the rotation at the point of application of torque *T*.**

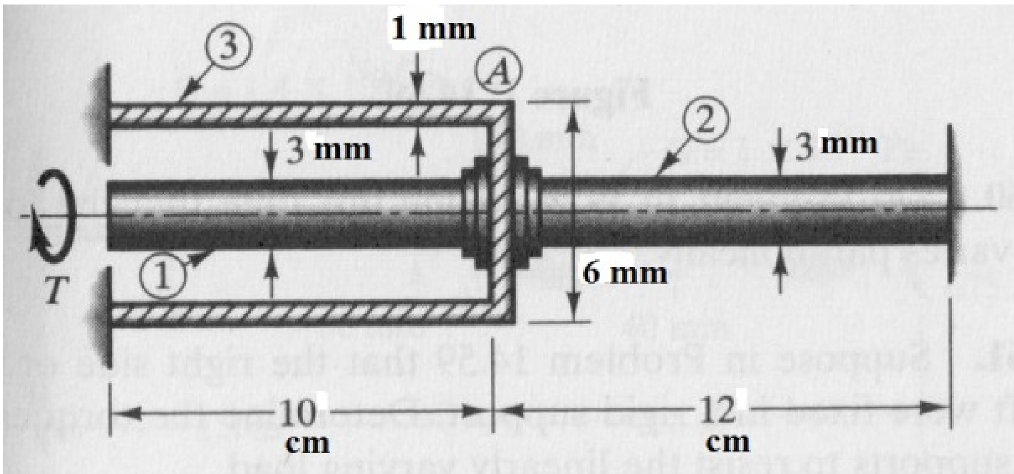

Fig. 2

Problem 3 (7.5 Marks):

Three shafts and four gears are used to form a gear train that will transmit power from the motor at ***A*** to a machine tool at ***F*** as shown in **Fig. 3.** The **radii** of the four gears are shown in **Fig.3**. Bearings supporting the shafts are not shown in **Fig. 3** for clarity. The diameter of each shaft is as follows: d_{AB} **= 16 mm**, d_{CD} **= 20 mm** and d_{EF} **= 28 mm**. The motor operates at **1440 rpm** and the allowable stress for each shaft is **75 MPa**.

Determine the maximum power that can be transmitted at 1440 rpm.

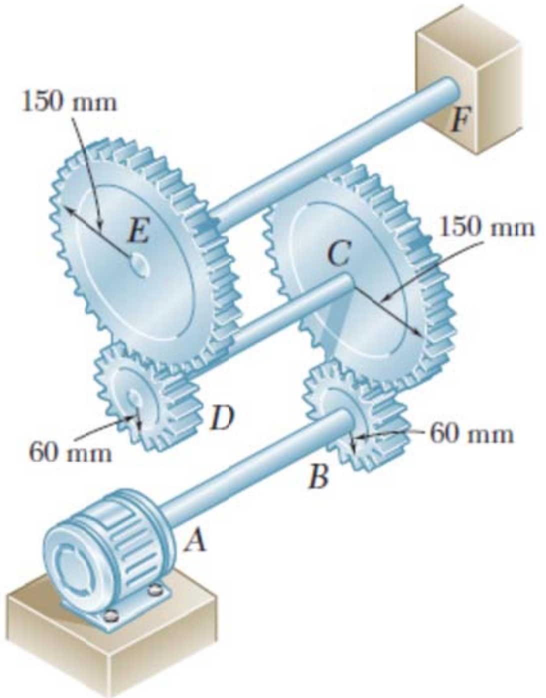

Fig. 3

Problem 4 (7.5 Marks):

The bending moment diagram for a beam of length ***5a*** which is supported by a pin support at **A** and a roller support at **B** is shown in **Fig. 4**. The beam has an overhanging portion to the left of support **A** and to the right of support **B.**

(a) **Draw the shear force diagram.**

(b) **Draw the loading diagram, showing the supports also.**

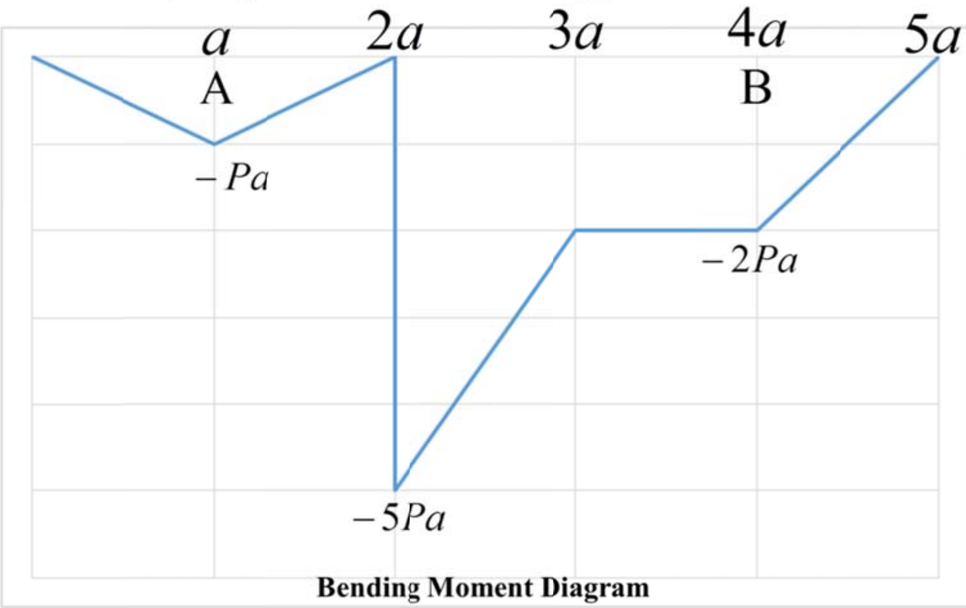

Fig. 4

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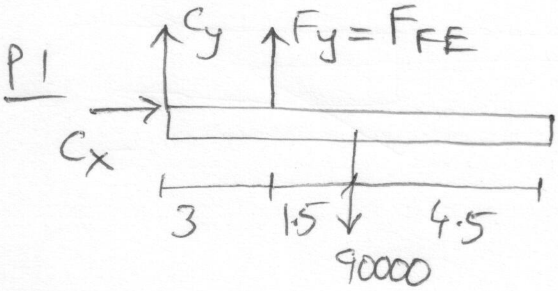
P1

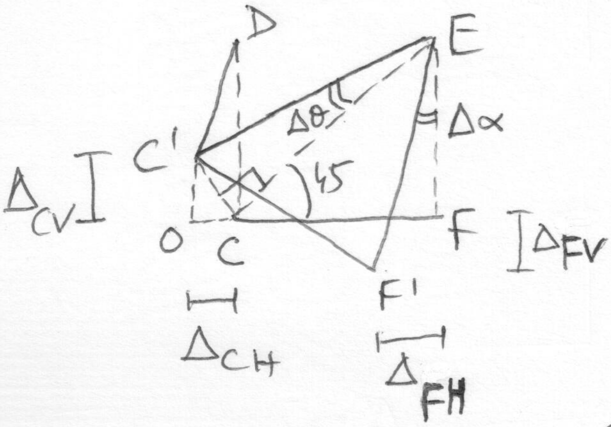


Equil: $\Rightarrow C_x = 0 = F_{CE}$

$C_y = F_{CD}$

$F_{FE} = (90000)\left(\frac{4.5}{3}\right) = 135000, \; F_{CD} = -45000$

Kinematics: $\because F_{CE} = 0$, it doesn't deform, it only "rigid body" rotates.



$\boxed{\Delta_{CV}} \cong$ change in length of CD

$= \dfrac{F_{CD} L_{CD}}{(AE)_{CD}} = \boxed{0.1119\,\text{mm} \uparrow}$

$\boxed{\Delta_{CH} = \Delta_{CV} = 0.1119\,\text{mm}} \leftarrow$

($\because$ OCC' is isoceles).

see proof below.

Assume small displ & rotations.

$\Delta_{FV} \cong$ change in length of EF

$= \dfrac{F_{EF} L_{EF}}{(AE)_{EF}} = \downarrow 0.6715\,\text{mm}.$

$\Delta_{FH} = \Delta_{CH} = 0.1119\,\text{mm} \leftarrow$

Proof:

$$DC' = \sqrt{(DC - \Delta_{CV})^2 + \Delta_{CH}^2} \cong \sqrt{DC^2 - 2\Delta_{CV} DC} = \cancel{DC} - \Delta_{CV}$$

$$\cancel{DC} - \frac{F_{CD} L_{CD}}{(AE)_{CD}} \Rightarrow \Delta_{CV} = \frac{F_{CD} L_{CD}}{(AE)_{CD}}$$

Similarly, $EF' = \cancel{EF} + \dfrac{F_{EF} L_{EF}}{(AE)_{EF}} = \cancel{EF} + \Delta_{FV}$

Similarly, $CF = C'F'$ ($\because$ block 'A' is rigid)

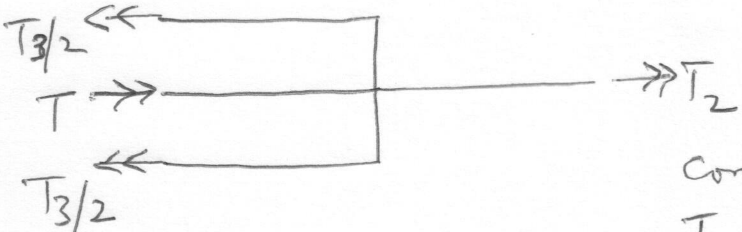
$$= \sqrt{(x_{C'} - x_{F'})^2 + (y_{C'} - y_{F'})^2} = \sqrt{(x_C + \Delta_{CH} - x_F - \Delta_{FH})^2 + (\Delta_{CV} + \Delta_{FV})^2}$$

$$\cong \sqrt{CF^2 + 2CF(\Delta_{CH} - \Delta_{FH})} = CF\left[1 + \frac{1}{2}\left\{\frac{\Delta_{CH} - \Delta_{FH}}{CF}\right\}\right]$$

$$\Rightarrow \Delta_{CH} = \Delta_{FH}.$$

END RESULT: ONLY DISPLACEMENT ALONG MEMBER DIRECTION CONTRIBUTES TO CHANGE IN LENGTH.

P2



Equil: $T + T_2 = T_3 \rightarrow$ ①

compatibility:

$$\frac{T_2 \cdot 12}{G \frac{\pi}{32} (3)^4} = \frac{-T_3 \cdot 10}{G \frac{\pi}{32} \left[6^4 - 4^4\right]} \rightarrow ②$$

Rotation of free end, where T applied:

$$\theta_{free} = \left\{ \frac{T \cdot 10}{G \frac{\pi}{32} (3)^4} + \frac{T_3 \cdot 10}{G \frac{\pi}{32} \left[6^4 - 4^4\right]} \right\} \cdot \frac{100}{10^3} \rightarrow ③$$

Use $T = 500$, $G = 90$

Result:

①, ② $\rightarrow T_3 = 469.53$ N.mm

$T_2 = -30.47$ N.mm

③ $\rightarrow \theta_{free} = 0.07497$ rad

P3

$$\tau_{AB} = \frac{T_{AB}(8)}{\frac{\pi}{32} \cdot 16^4} \; ; \; \tau_{CD} = \frac{\frac{150}{60} T_{AB}(10)}{\frac{\pi}{32} \cdot 20^4} \; ; \; \tau_{EF} = \frac{\left(\frac{150}{60}\right)^2 T_{AB}(14)}{\frac{\pi}{32} \cdot 28^4}$$

$$= 1.22E{-4} \frac{T_{AB}}{(\pi/32)} \; ; \; = 1.5625E{-4} \frac{T_{AB}}{(\pi/32)} \; ; \; = 1.4236E{-4} * \frac{T_{AB}}{\pi/32}$$

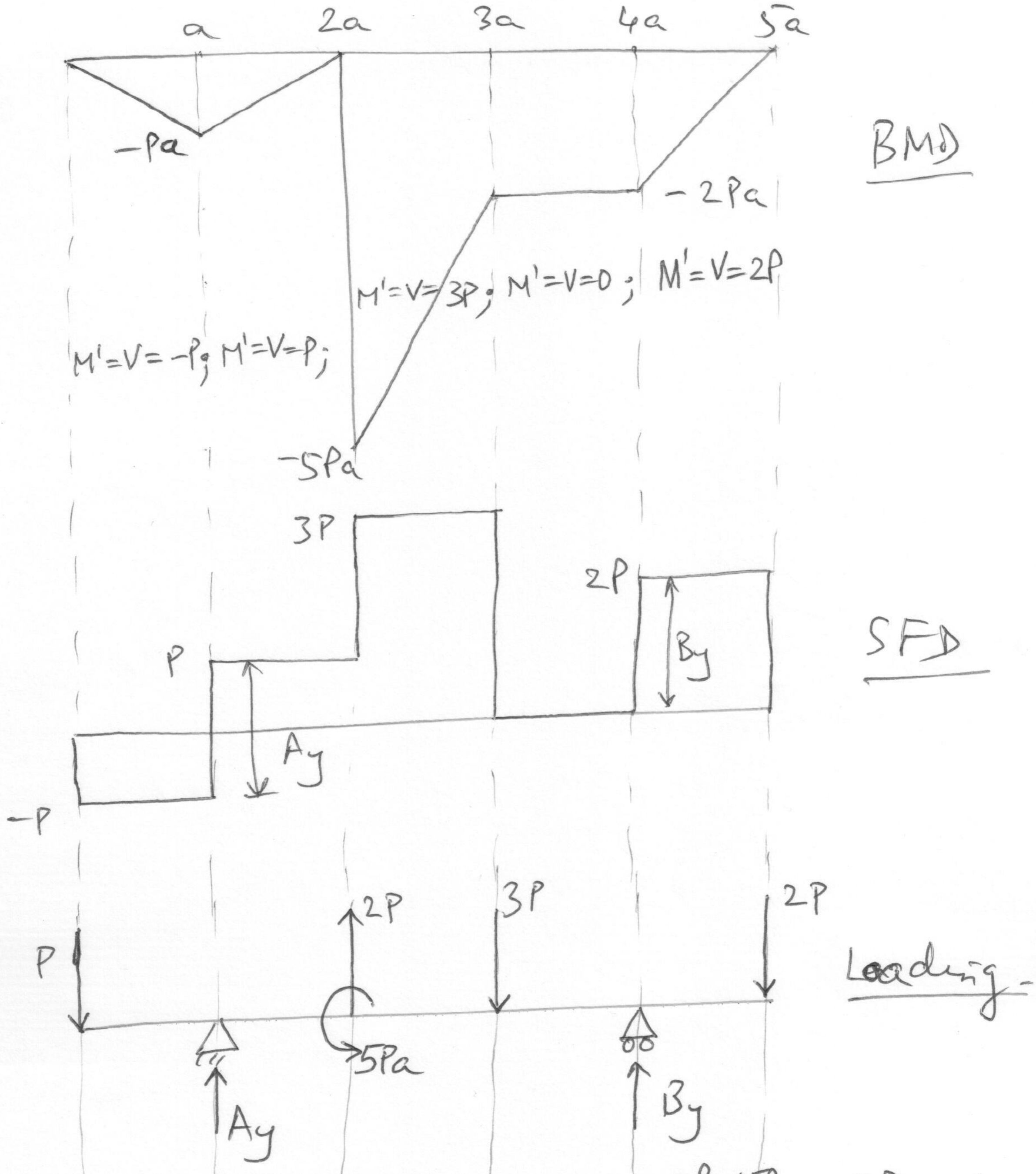
largest, so CD fails first.

$$\Rightarrow 75 = 1.5625E{-4} \frac{(T_{AB})_{max}}{(\pi/32)} \Rightarrow (T_{AB})_{max} = 47123.9 \text{ N.mm}$$

$$P_{max} = \frac{47123.9}{1000} * 1440 \left(\frac{2\pi}{60}\right) = 7.11 * 10^3 \text{ W}$$

$= 7.11$ kW.

P4



Check: $A_y = \dfrac{4Pa - 2P \cdot 2a + 3Pa - 2Pa + 5Pa}{3a} = 2P$ ✓

$B_y = P + 3P + 2P - 2P - A_y = 2P.$ ✓