DEPARTMENT OF CIVIL ENGINEERING

CE-221 SOLID MECHANICS

Mid-Sem Exam **06/09/16** **PAPER CODE: A**

Note: Write your name & roll no. on answerbook and on summary-answer-sheet provided with the question paper.
You must submit the summary-answer-sheet along with the answerbook.
Closed book, closed notes exam. No formula sheet allowed. **No mobile phones allowed in the exam hall**.
All questions carry equal marks. Assume suitable data if required and state the same clearly.
Use formulae from provided tables, **if applicable**.

Problem 1

A symmetric bimetallic bar consisting of an inner copper layer securely bonded to two outer steel layers (**Fig. 1**) is heated uniformly to increase its temperature by **100 ^{o}C**. Assuming that the width of the bar is **30 mm**, the length is **500 mm,** and the thickness of each of the three layers is **10 mm**, determine the normal stresses σ_s and σ_c in steel and copper, respectively. Coefficient of thermal expansion of steel is **18 x 10^{-6} /^{o}C** and copper is **25 x 10^{-6} /^{o}C**. Modulus of elasticity of steel is **190 GPa** and copper is **75 GPa**.

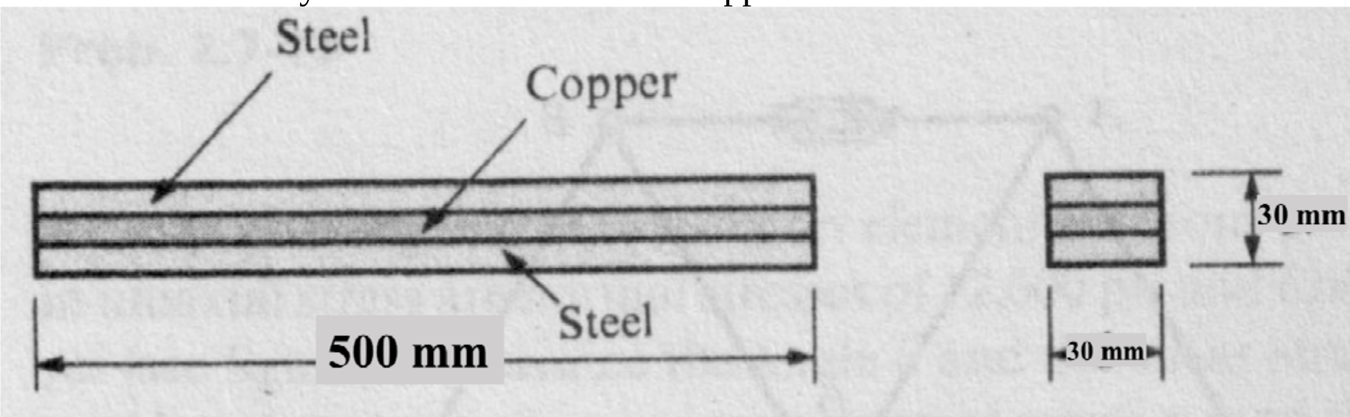

Fig. 1

Problem 2

The movement of a **100 mm x 50 mm** plate is restrained along the x-direction by supports along two edges as shown in **Fig. 2**. There is no restraint to the movement of the plate in the y and z directions at the supports. The coefficient of thermal expansion is **25 x 10^{-6}/^{o}C**, modulus of elasticity is **200 GPa,** and Poisson's ratio is **0.25. Calculate the change in length of the plate along the y direction (δ_y)** due to a temperature change of **100 ^{o}C**.
Note that the normal strains in the plate along any direction is the summation of strains due to mechanical stresses and thermal changes, i.e. $\varepsilon_x = \sigma_x/E - \nu\sigma_y/E - \nu\sigma_z/E + \alpha\Delta T$ and similarly for ε_y and ε_z.

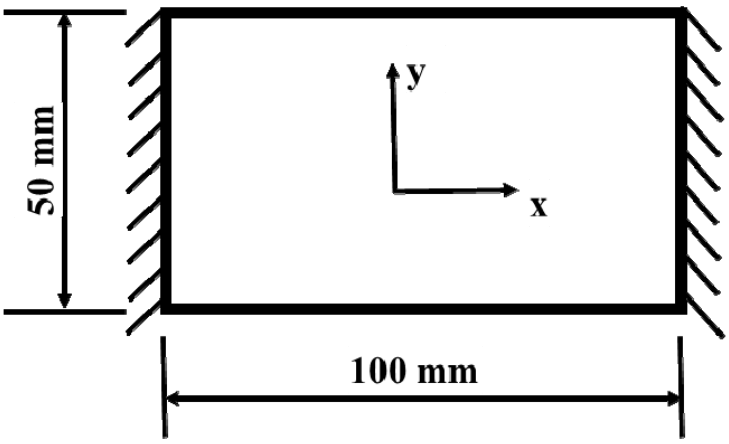

Fig. 2

Problem 3

A system of two concentric torsion members connected at their ends by rigid disks ***G*** is shown in **Fig. 3**. The outside member ***C*** is a tube with outside diameter **300 mm** and thickness **10 mm.** The inside member consists of solid shafts ***A*** of diameter **100 mm** and ***B*** of diameter **200 mm**. Shafts ***A*** and ***B*** and tube ***C*** are made of the same material having shear modulus **100 GPa**. At the right end the rigid disk is fixed in support ***D***. A torque of **200 kN-m** is applied to the outside tube as shown in **Fig. 3**. **Determine the maximum shear stress in tube *C* ($\tau_{C,max}$) and the rotation of the <u>left end</u> rigid disk (ϕ_G).**

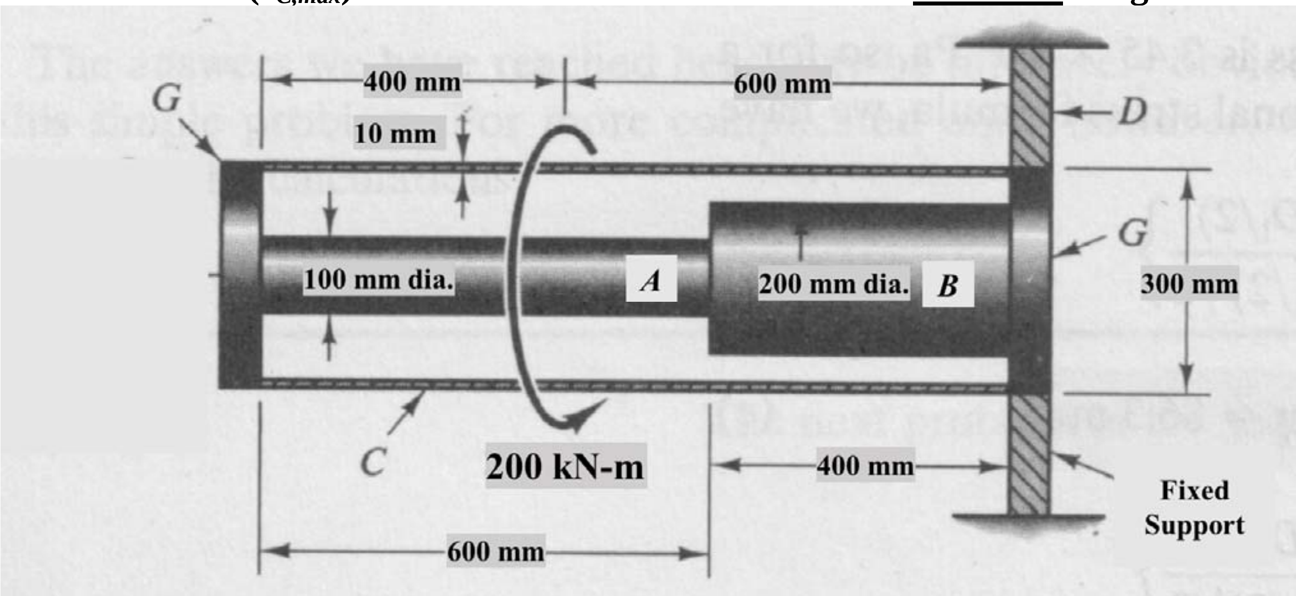

Fig. 3

Problem 4

Draw the shear force and bending moment diagrams of the 8 m long lower continuous beam that has an internal hinge/pin as shown in **Fig. 4**. The BMD and SFD for the upper simply supported beam is not required.

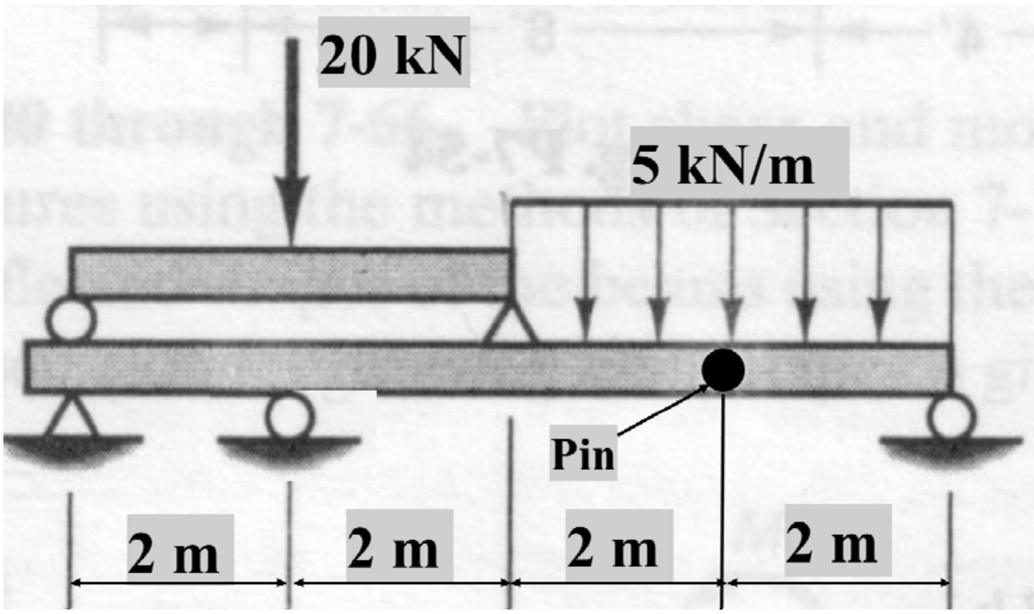

Fig. 4

SUMMARY ANSWER SHEET

PAPER CODE: A

Name:

Roll no:

Problem 1

$\sigma_s =$

$\sigma_c =$

Problem 2

$\delta_y =$

Problem 3

$\tau_{C,max} =$

$\phi_G =$

DEPARTMENT OF CIVIL ENGINEERING

CE-221 SOLID MECHANICS

Mid-Sem Exam **06/09/16** **PAPER CODE: B**

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Closed book, closed notes exam. No formula sheet allowed. No mobile phones allowed in the exam hall.
All questions carry equal marks. Assume suitable data if required and state the same clearly.
Use formulae from provided tables, **if applicable**.

Problem 1

A symmetric bimetallic bar consisting of an inner copper layer securely bonded to two outer steel layers (**Fig. 1**) is heated uniformly to increase its temperature by **150 ^{o}C**. Assuming that the width of the bar is **35 mm**, the length is **600 mm,** and the thickness of each of the three layers is **15 mm**, determine the normal stresses σ_s and σ_c in steel and copper, respectively. Coefficient of thermal expansion of steel is **22 x 10^{-6} /^{o}C** and copper is **28 x 10^{-6} /^{o}C**. Modulus of elasticity of steel is **195 GPa** and copper is **80 GPa**.

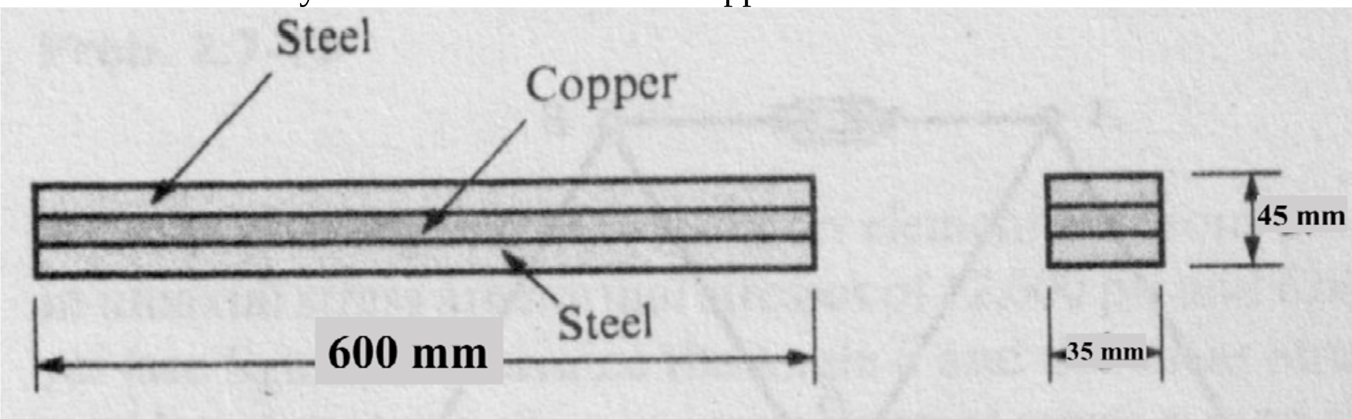

Fig. 1

Problem 2

The movement of a **150 mm x 75 mm** plate is restrained along the x-direction by supports along two edges as shown in **Fig. 2**. There is no restraint to the movement of the plate in the y and z directions at the supports. The coefficient of thermal expansion is **30 x 10^{-6}/^{o}C,** modulus of elasticity is **210 GPa,** and Poisson's ratio is **0.30. Calculate the change in length of the plate along the y direction (δ_y)** due to a temperature change of **125 ^{o}C**.
Note that the normal strains in the plate along any direction is the summation of strains due to mechanical stresses and thermal changes, i.e. $\varepsilon_x = \sigma_x/E - \nu\sigma_y/E - \nu\sigma_z/E + \alpha\Delta T$ and similarly for ε_y and ε_z.

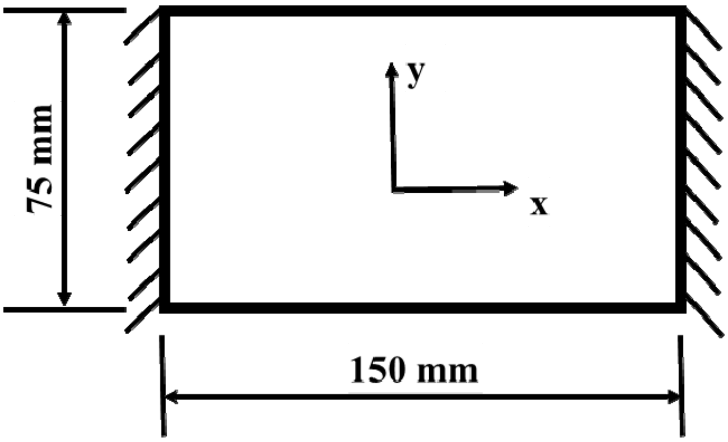

Fig. 2

Problem 3

A system of two concentric torsion members connected at their ends by rigid disks **G** is shown in **Fig. 3**. The outside member **C** is a tube with outside diameter **400 mm** and thickness **15 mm.** The inside member consists of solid shafts **A** of diameter **125 mm** and **B** of diameter **250 mm**. Shafts **A** and **B** and tube **C** are made of the same material having shear modulus **110 GPa**. At the right end the rigid disk is fixed in support **D**. A torque of **250 kN-m** is applied to the outside tube as shown in **Fig. 3**. **Determine the maximum shear stress in tube C ($\tau_{C,max}$) and the rotation of the <u>left end</u> rigid disk (ϕ_G).**

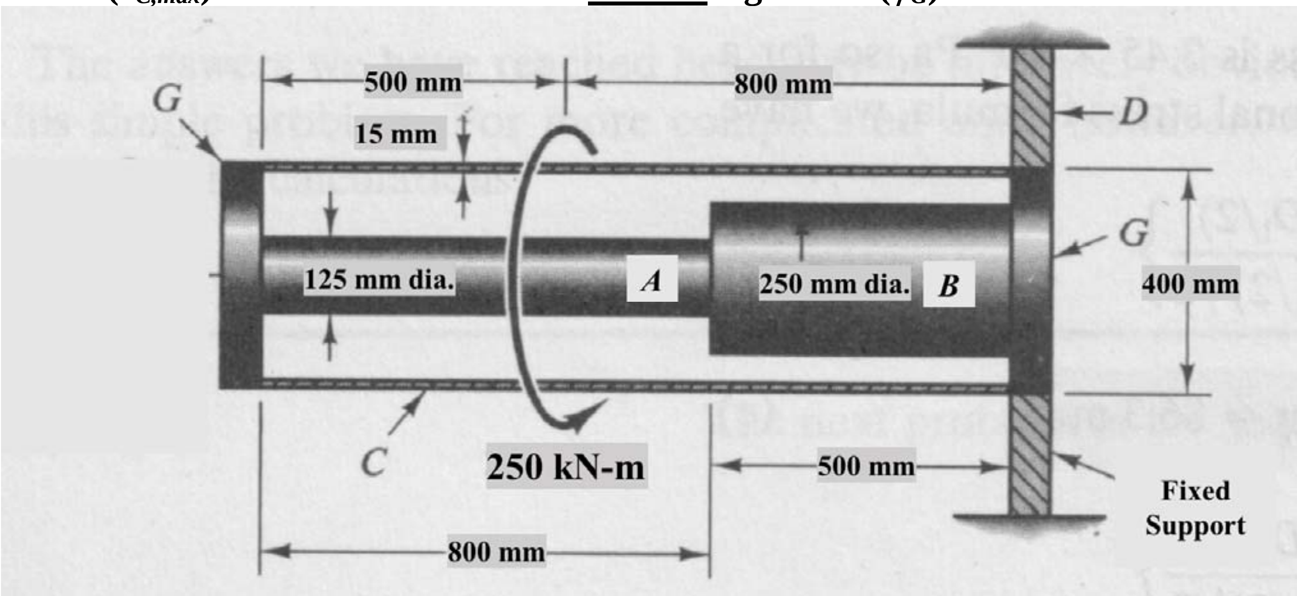

Fig. 3

Problem 4

Draw the shear force and bending moment diagrams of the 12 m long lower continuous beam that has an internal hinge/pin as shown in **Fig. 4**. The BMD and SFD for the upper simply supported beam is not required.

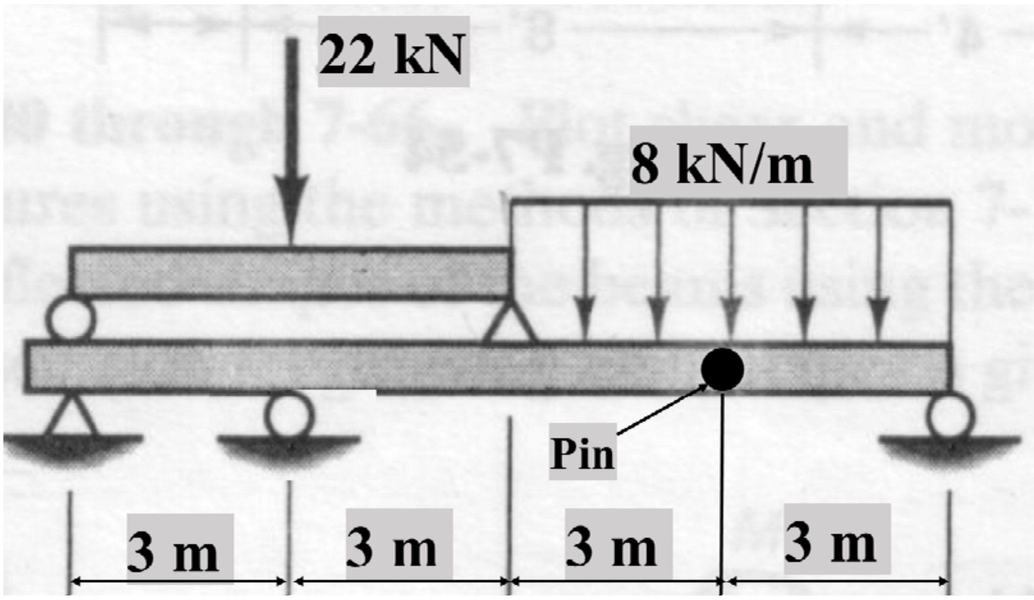

Fig. 4

SUMMARY ANSWER SHEET

PAPER CODE: B

Name:

Roll no:

Problem 1

$\sigma_s =$

$\sigma_c =$

Problem 2

$\delta_y =$

Problem 3

$\tau_{A,max} =$

$\tau_{B,max} =$

$\tau_{C,max} =$

$\phi_G =$

DEPARTMENT OF CIVIL ENGINEERING

CE-221 SOLID MECHANICS

Mid-Sem Exam **06/09/16** **PAPER CODE: C**

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Closed book, closed notes exam. No formula sheet allowed. No mobile phones allowed in the exam hall.
All questions carry equal marks. Assume suitable data if required and state the same clearly.
Use formulae from provided tables, **if applicable**.

Problem 1

A symmetric bimetallic bar consisting of an inner copper layer securely bonded to two outer steel layers (**Fig. 1**) is heated uniformly to increase its temperature by **200 ^{o}C**. Assuming that the width of the bar is **40 mm**, the length is **700 mm,** and the thickness of each of the three layers is **20 mm**, determine the normal stresses σ_s and σ_c in steel and copper, respectively. Coefficient of thermal expansion of steel is **26 x 10^{-6} /^{o}C** and copper is **32 x 10^{-6} /^{o}C**. Modulus of elasticity of steel is **200 GPa** and copper is **85 GPa**.

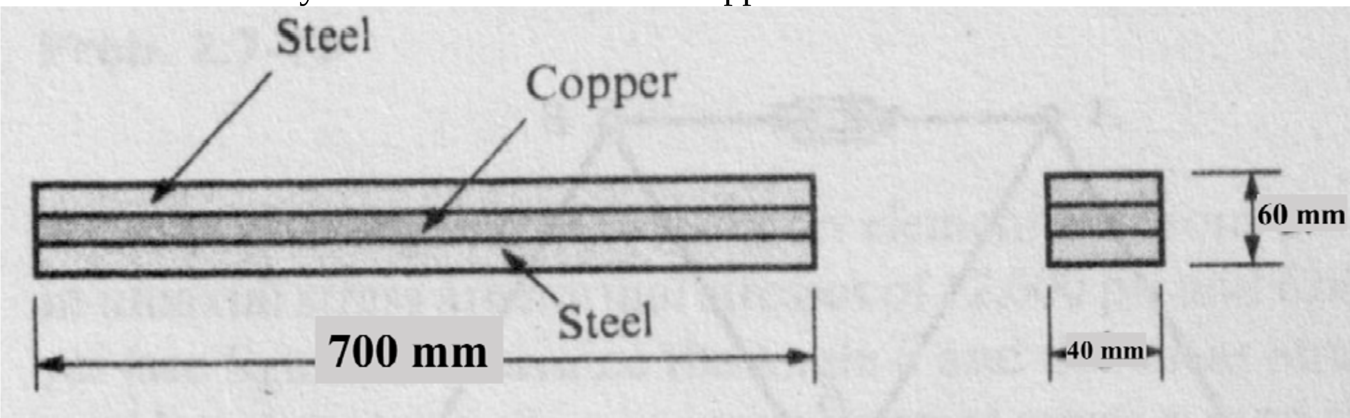

Fig. 1

Problem 2

The movement of a **200 mm x 100 mm** plate is restrained along the x-direction by supports along two edges as shown in **Fig. 2**. There is no restraint to the movement of the plate in the y and z directions at the supports. The coefficient of thermal expansion is **35 x 10^{-6}/^{o}C**, modulus of elasticity is **220 GPa,** and Poisson's ratio is **0.35. Calculate the change in length of the plate along the y direction (δ_y)** due to a temperature change of **150 ^{o}C**.

Note that the normal strains in the plate along any direction is the summation of strains due to mechanical stresses and thermal changes, i.e. $\varepsilon_x = \sigma_x/E - \nu\sigma_y/E - \nu\sigma_z/E + \alpha\Delta T$ and similarly for ε_y and ε_z.

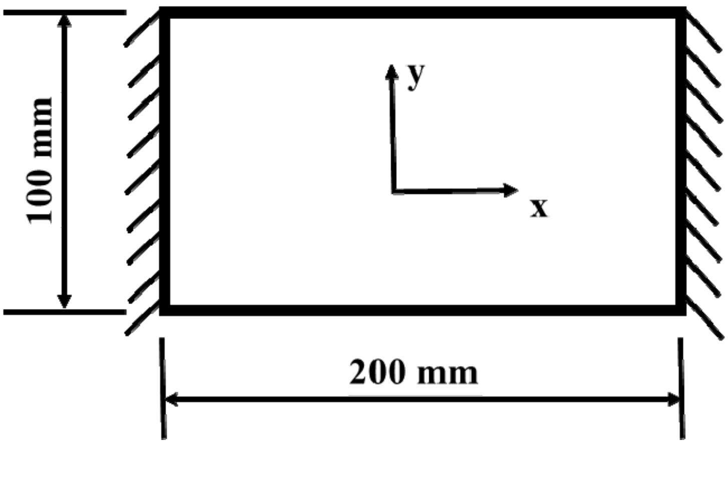

Fig. 2

Problem 3

A system of two concentric torsion members connected at their ends by rigid disks ***G*** is shown in **Fig. 3**. The outside member ***C*** is a tube with outside diameter **500 mm** and thickness **20 mm.** The inside member consists of solid shafts ***A*** of diameter **150 mm** and ***B*** of diameter **300 mm**. Shafts ***A*** and ***B*** and tube ***C*** are made of the same material having shear modulus **120 GPa**. At the right end the rigid disk is fixed in support ***D***. A torque of **300 kN-m** is applied to the outside tube as shown in **Fig. 3**. **Determine the maximum shear stress in tube *C* ($\tau_{C,max}$) and the rotation of the <u>left end</u> rigid disk (ϕ_G).**

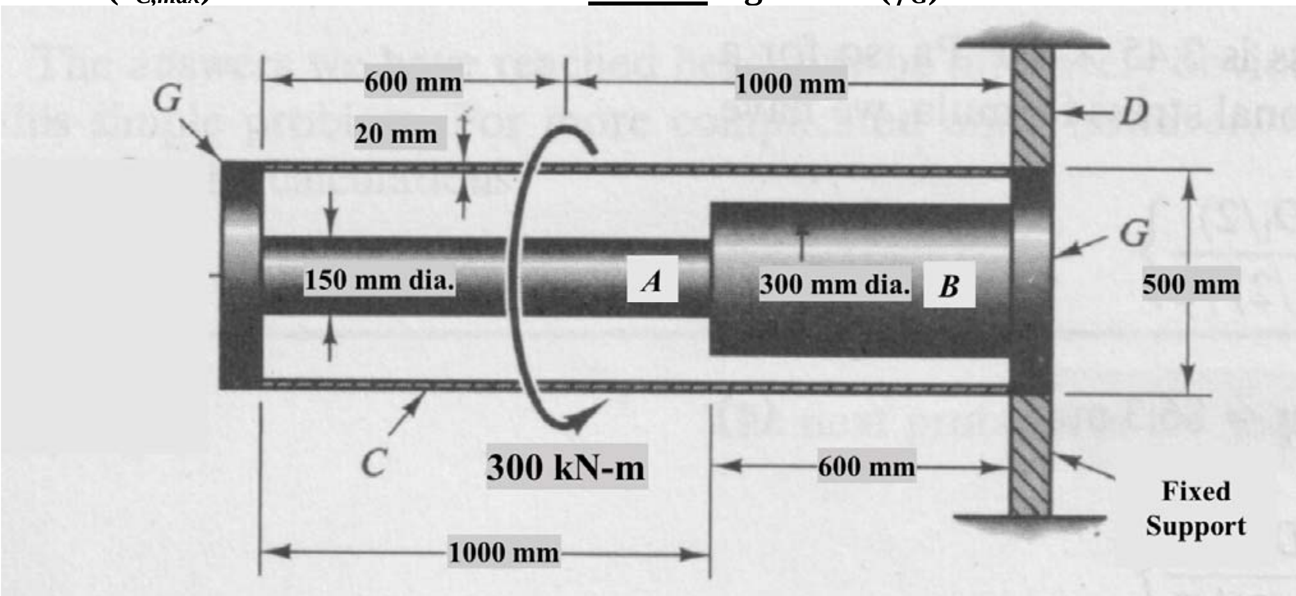

Fig. 3

Problem 4

Draw the shear force and bending moment diagrams of the 16 m long lower continuous beam that has an internal hinge/pin as shown in **Fig. 4**. The BMD and SFD for the upper simply supported beam is not required.

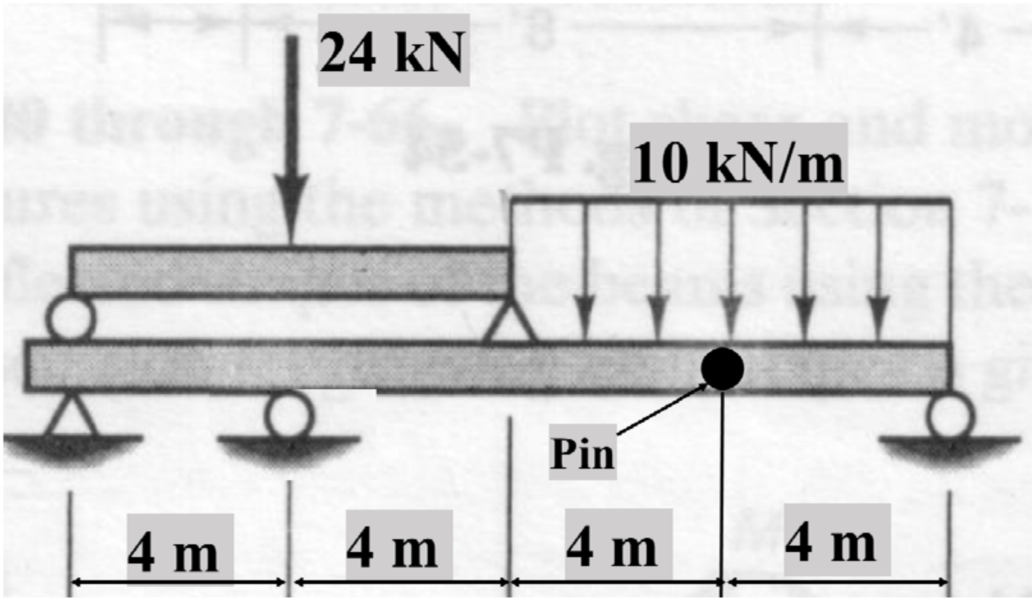

Fig. 4

SUMMARY ANSWER SHEET

PAPER CODE: C

Name:

Roll no:

Problem 1

$\sigma_s =$

$\sigma_c =$

Problem 2

$\delta_y =$

Problem 3

$\tau_{A,max} =$

$\tau_{B,max} =$

$\tau_{C,max} =$

$\phi_G =$

DEPARTMENT OF CIVIL ENGINEERING

CE-221 SOLID MECHANICS

Mid-Sem Exam **06/09/16** **PAPER CODE: D**

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Closed book, closed notes exam. No formula sheet allowed. No mobile phones allowed in the exam hall.
All questions carry equal marks. Assume suitable data if required and state the same clearly.
Use formulae from provided tables, **if applicable**.

Problem 1

A symmetric bimetallic bar consisting of an inner copper layer securely bonded to two outer steel layers (**Fig. 1**) is heated uniformly to increase its temperature by **250 ^{o}C**. Assuming that the width of the bar is **45 mm**, the length is **800 mm,** and the thickness of each of the three layers is **25 mm**, determine the normal stresses σ_s and σ_c in steel and copper, respectively. Coefficient of thermal expansion of steel is **29 x 10^{-6} /^{o}C** and copper is **35 x 10^{-6} /^{o}C**. Modulus of elasticity of steel is **205 GPa** and copper is **90 GPa**.

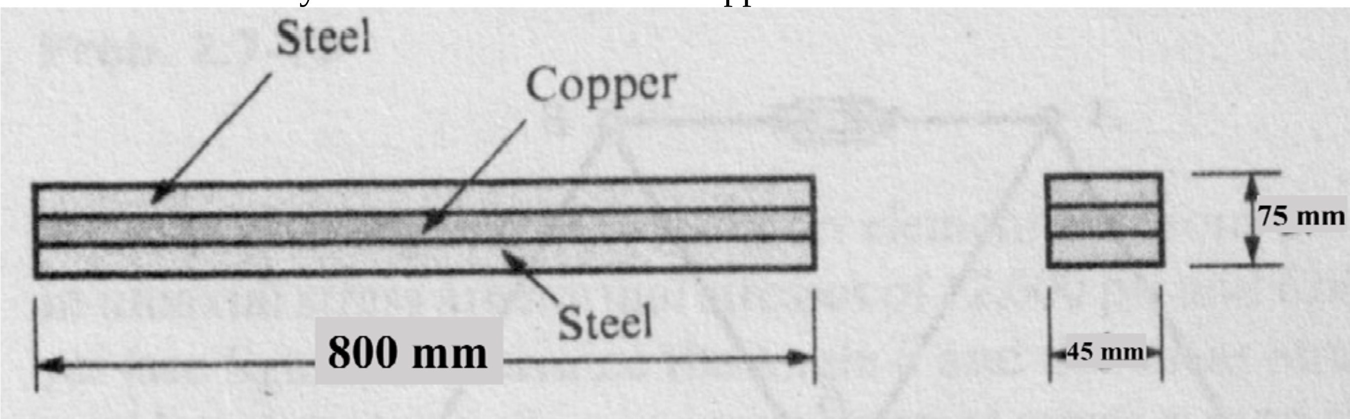


Fig. 1

Problem 2

The movement of a **250 mm x 125 mm** plate is restrained along the x-direction by supports along two edges as shown in **Fig. 2**. There is no restraint to the movement of the plate in the y and z directions at the supports. The coefficient of thermal expansion is **40 x 10^{-6}/^{o}C**, modulus of elasticity is **230 GPa,** and Poisson's ratio is **0.4. Calculate the change in length of the plate along the y direction (δ_y)** due to a temperature change of **175 ^{o}C**.

Note that the normal strains in the plate along any direction is the summation of strains due to mechanical stresses and thermal changes, i.e. $\varepsilon_x = \sigma_x/E - \nu\sigma_y/E - \nu\sigma_z/E + \alpha\Delta T$ and similarly for ε_y and ε_z.

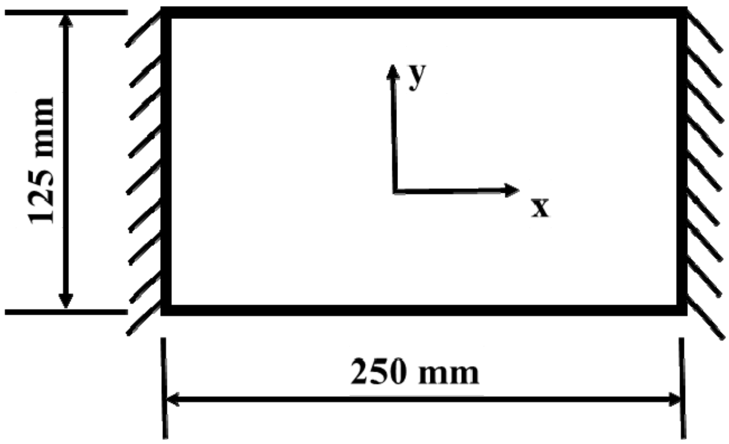


Fig. 2

Problem 3

A system of two concentric torsion members connected at their ends by rigid disks ***G*** is shown in **Fig. 3**. The outside member ***C*** is a tube with outside diameter **600 mm** and thickness **25 mm.** The inside member consists of solid shafts ***A*** of diameter **175 mm** and ***B*** of diameter **350 mm**. Shafts ***A*** and ***B*** and tube ***C*** are made of the same material having shear modulus **130 GPa**. At the right end the rigid disk is fixed in support ***D***. A torque of **350 kN-m** is applied to the outside tube as shown in **Fig. 3**. **Determine the maximum shear stress in tube *C* ($\tau_{C,max}$) and the rotation of the <u>left end</u> rigid disk (ϕ_G).**

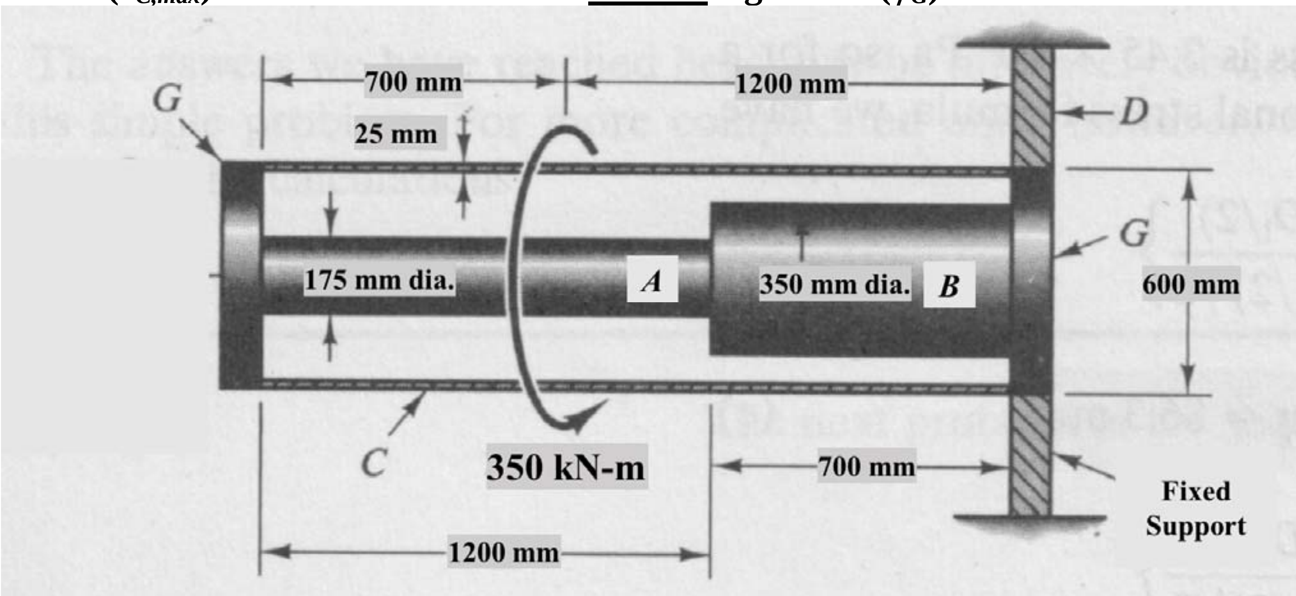


Fig. 3

Problem 4

Draw the shear force and bending moment diagrams of the 20 m long lower continuous beam that has an internal hinge/pin as shown in **Fig. 4**. The BMD and SFD for the upper simply supported beam is not required.

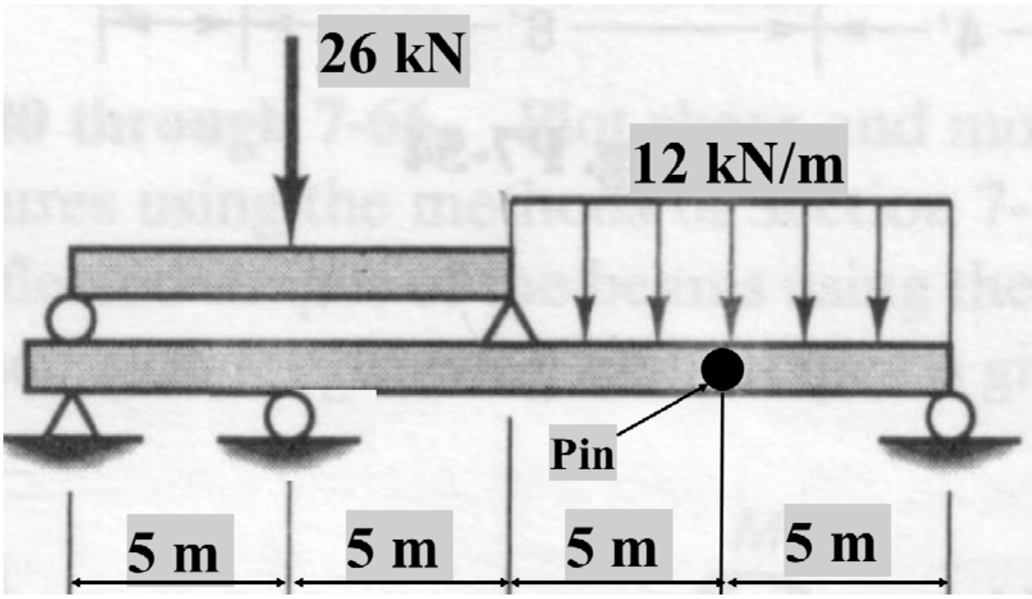


Fig. 4

SUMMARY ANSWER SHEET

PAPER CODE: D

Name:

Roll no:

Problem 1

$\sigma_s =$

$\sigma_c =$

Problem 2

$\delta_y =$

Problem 3

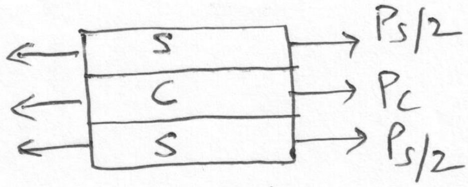
$\tau_{A,max} =$

$\tau_{B,max} =$

$\tau_{C,max} =$

$\phi_G =$

P1



Equil: $P_s + P_c = 0$.

Compat: $\delta_s = \delta_c$

$$\Rightarrow \alpha_s \Delta T L + \frac{P_s L}{2AE_s} = \alpha_c \Delta T L + \frac{\cancel{P_c}^{(-P_s)} L}{AE_c}$$

$A_s = 2A_c = 2A$.

$$\boxed{\sigma_s = \frac{P_s}{2A} = \frac{(\alpha_c - \alpha_s)\Delta T}{\frac{1}{E_s} + \frac{2}{E_c}}} \; ; \; \boxed{\sigma_c = \frac{P_c}{A} = \frac{-P_s}{A} = -2\sigma_s}$$

($\because \alpha_c > \alpha_s$, σ_s (T), σ_c (C), as expected).

Codes: A →	$\sigma_s = 21.92$,	$\sigma_c = -43.85$	MPa
B →	29.87,	−59.74	
C →	42.06,	−84.12	
D →	55.35,	−110.70	

P2 $\varepsilon_x = 0$, $\sigma_y = \sigma_z = 0$ ($\because$ constrained in x-direction only).

$\delta_y = \varepsilon_y \cdot L_y$

$\varepsilon_y = \cancel{\sigma_y}^{0}/E - \nu \sigma_x/E - \nu \cancel{\sigma_z}^{0}/E + \alpha \Delta T$

$\varepsilon_x = \sigma_x/E - \nu \cancel{\sigma_y}^{0}/E - \nu \cancel{\sigma_z}^{0}/E + \alpha \Delta T$

$\Rightarrow \varepsilon_y = -\nu(-E\alpha\Delta T)/E + \alpha \Delta T = \alpha \Delta T(1+\nu)$

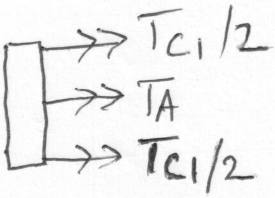
$\delta_y = \alpha \Delta T (1+\nu) L_y$.

Code A:	$\delta_y = 0.156250$	mm.
B:	$= 0.365625$	
C:	$= 0.708750$	
D:	$= 1.225$	

P3. Let L_1, L_2 denote lengths of hollow shaft to the left & right of applied torque, respectively, and T_{C1} & T_{C2} denote corresponding internal torques in C. Int. torque in A & B is $T_A = T_B$.

Equil:

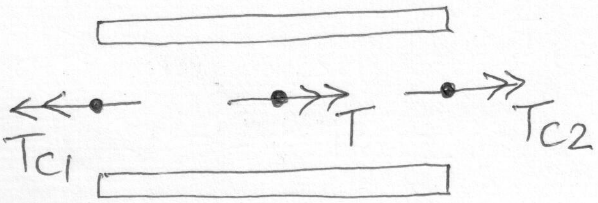
left disk G



($T_{C1}/2$ shown for convenience, actually its T_{C1} in hollow shaft C).

$$T_A + T_{C1} = 0$$

hollow shaft C



$$T_{C1} = T + T_{C2}$$

($T_{C2} = T_{C1} - T$ used)

Compatibility: at left disk G,

$$\phi_A = \phi_C$$

$$T_A\left[\frac{L_A}{(GJ)_A} + \frac{L_B}{(GJ)_B}\right] = \frac{T_{C1}L_1 + (T_{C1}-T)L_2}{(GJ)_C}$$

$$\Rightarrow \boxed{T_{C1} = \frac{TL_2}{(GJ)_C}\left[\frac{L_A}{(GJ)_A} + \frac{L_B}{(GJ)_B} + \frac{L_1+L_2}{(GJ)_C}\right]^{-1}} \rightarrow (*)$$

$$\boxed{\phi_G = \phi_A = \phi_C = -(T_{C1})\left[\frac{L_A}{(GJ)_A} + \frac{L_B}{(GJ)_B}\right].}$$

$$(\tau_C)_{max} = \max\left[\tau_{C1}, \tau_{C2}\right].$$

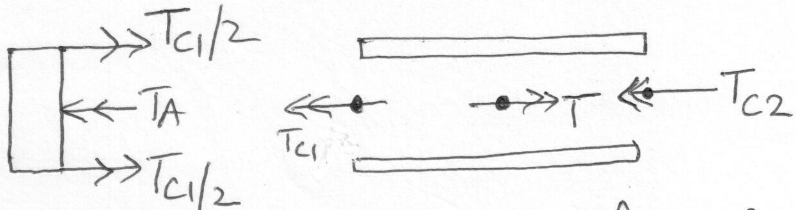
For given dimensions, $|T_{C2}| > |T_{C1}|$, so $(\tau_C)_{max} = \tau_{C2}$

$$\boxed{(\tau_C)_{max} = \tau_{C2} = \frac{T_{C2}\, r_{o,c}}{J_c}, \text{ where } T_{C2} = T_{C1} - T}$$

$$J_A = \frac{\pi}{32}(d_A)^4, \quad J_B = \frac{\pi}{32}(d_B)^4, \quad J_C = \frac{\pi}{32}\left(d_{o,c}^4 - d_{i,c}^4\right).$$

Codes: A →	$T_{C_1} = 9.0847$,	$T_{C2} = -190.9153$,	$(\tau_c)_{max} = 149.32$,	$\phi_G = 5.78$
	$= 8.1131$,	$= -241.8869$,	$= 71.85$,	$= 2.56$
	$= 7.9101$,	$= -292.0899$,	$= 41.96$,	$= 1.38$
	$= 8.0127$,	$= -341.9873$,	$= 27.43$,	$= .833$
only $(\tau_c)_{max}$, ϕ_G	kN.m.	kN.m	MPa	$*10^{-3}$ rad

If you do with physically correct directions,



$T_{C_1} = T_A$

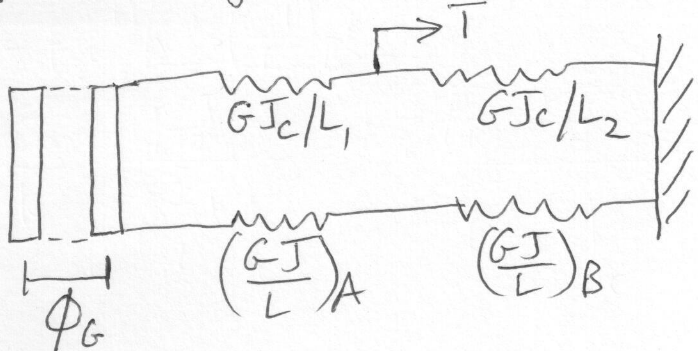
$T_{C_1} + T_{C2} = T.$

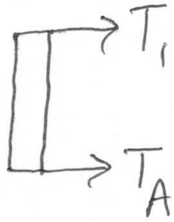
Compatibility : $\phi_A = -\phi_C$ ($\because$ rotations in opp directions as per FBD's)

$$T_A\left[\frac{L_A}{(GJ)_A} + \frac{L_B}{(GJ)_B}\right] = -\left[\frac{T_{C_1}L_1}{(GJ)_C} - \frac{T_{C2}L_2}{(GJ)_C}\right]$$

after substituting these you get same result as (*) on previous pg.

If you do by equivalent springs,





$T_1 = -T_A$ for equil.

T_A = (effective stiffness of lower springs) $* \phi_G$

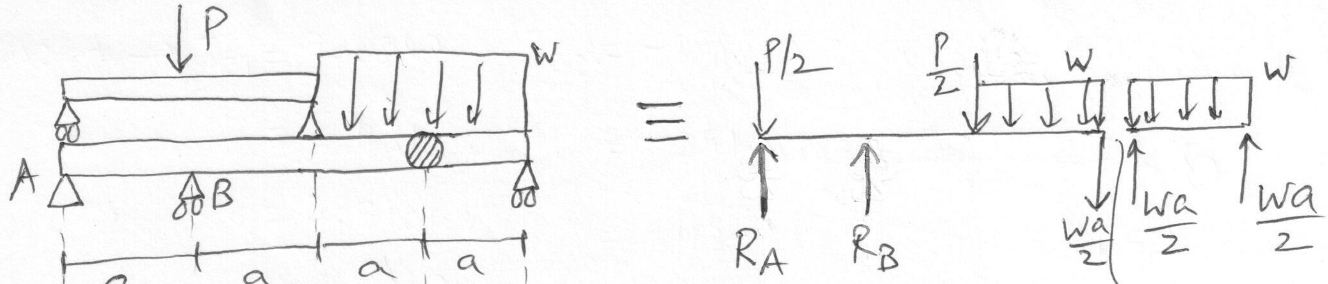
$$T_A = \left[\frac{1}{(GJ/L)_A} + \frac{1}{\left(\frac{GJ}{L}\right)_B}\right]^{-1}\phi_G$$

Also, ϕ_G = extension of left upper spring + ext of right upper spring

$$\phi_G = \frac{T_1}{GJ_c/L_1} + \frac{T_1 - T}{GJ_c/L_2}.$$

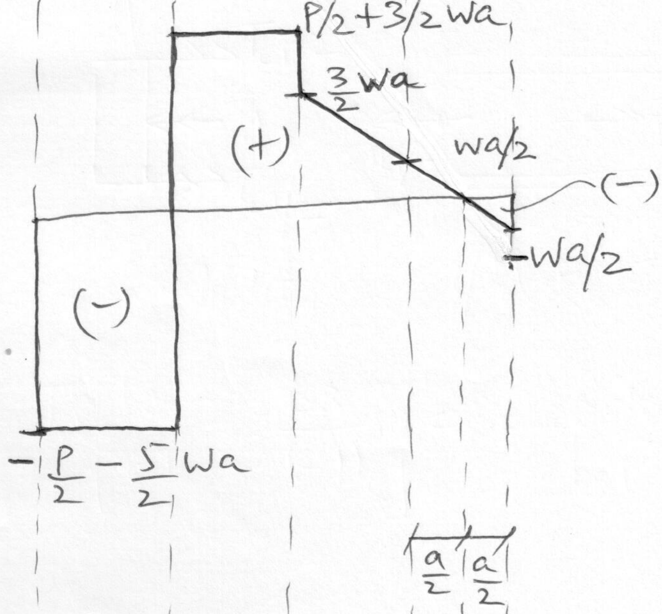
Eliminate ϕ_G and T_A and get T_1 same as T_{C_1} in (*) on previous pg.

P4.



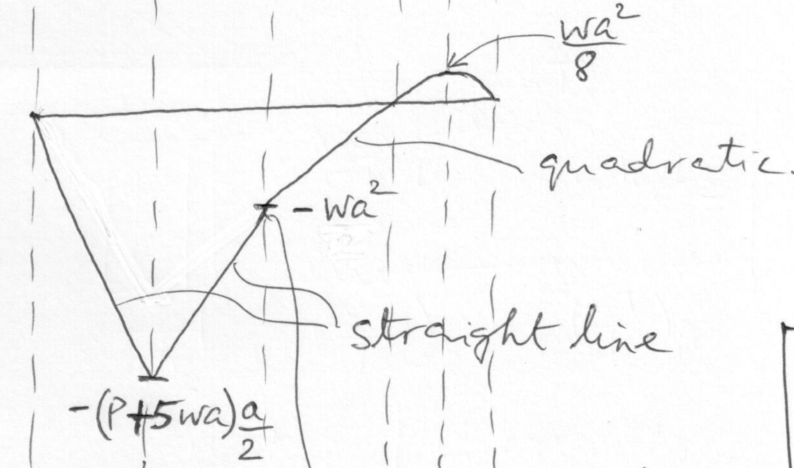
hinge, so only internal shear wa/2, no bending moment.

$$\sum M_A = 0 \Rightarrow R_B = \frac{\frac{wa}{2}(3a) + wa\left(\frac{5}{2}a\right) + \frac{P}{2}(2a)}{a} = 4wa + P$$



← starting from right end proceeding leftward using $V' = -w$.

Values, left to right are:
Code A: −35, 25, 15, 5, −5
B: −71, 47, 36, 12, −12
C: −112, 72, 60, 20, −20
D: −163, 103, 90, 30, −30



kink. Slope before kink is $\frac{P}{2} + \frac{3}{2}wa$, slope after kink is $\frac{3}{2}wa$.

← starting from left end proceeding rightward using $M' = V$, $M_R - M_L = \int V\,dx$.

Values, left to right are:
Code A: −70, −20, 2.5
B: −213, −72, 9
C: −448, −160, 20
D: −815, −300, 37.5