DEPARTMENT OF CIVIL ENGINEERING

CE-221 SOLID MECHANICS

Mid-Sem Exam **11/09/17** **PAPER CODE: A**

Note: Write your name & roll no. on answerbook and on summary-answer-sheet provided with the question paper.
You must submit the summary-answer-sheet along with the answerbook.
Closed book, closed notes exam. No formula sheet allowed. No mobile phones allowed in the exam hall.
Assume suitable data if required and state the same clearly.

Problem 1 (a)

Calculate the increase in the volume ΔV of a bar with uniform cross-section and length **1 m** hanging vertically under its own weight of **20 kN**. The modulus of elasticity of the bar is **200 GPa** and and Poisson's ratio is **0.2**. (**5 Marks**)

Problem 1 (b)

A steel rod ***AB*** of diameter **14 mm** is stretched tightly between two supports so that the tensile stress in the rod is **80 MPa**. Then an axial force ***P*** is applied gradually to the rod at an intermediate location ***C*** as shown in **Fig. 1**. Calculate the value of this load ***P*** when the entire rod yields, if the material is elastic-plastic with yield stress σ_y **= 300 MPa**.

(**5 Marks**)

Fig. 1

Problem 2

Two rigid bars ***AB*** and ***CD*** are connected by linear elastic springs and are supported at ***A*** and ***D*** by hinge supports (**Fig. 2**). When no loads are acting, the bars are horizontal and the springs are unstressed. Determine the vertical deflection δ at point ***C*** when a load is applied at ***C*** as shown.

(**10 Marks**)

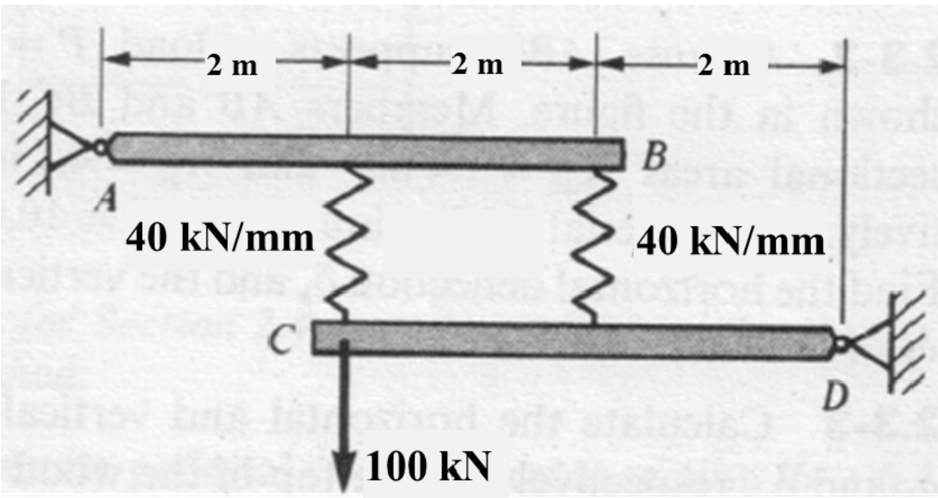

Fig. 2

Problem 3

A steel shaft (**G_s = 80 GPa**) of total length ***L* = 4.0 m** is encased over half its length by a brass tube (**G_b = 40 GPa**) that is securely bonded to the steel (**Fig. 3)**. The diameters of the shaft and tube are **d_1 = 70 mm** and **d_2 = 90 mm**, respectively. Determine the allowable torque ***T*** on the given assembly if the shear stress in the brass and steel are limited to **τ_b = 100 MPa** and **τ_s = 80 MPa**, respectively, and the angle of twist ϕ between the ends ***A*** and ***C*** is limited to **ϕ = 12 degrees**. (**10 Marks**)

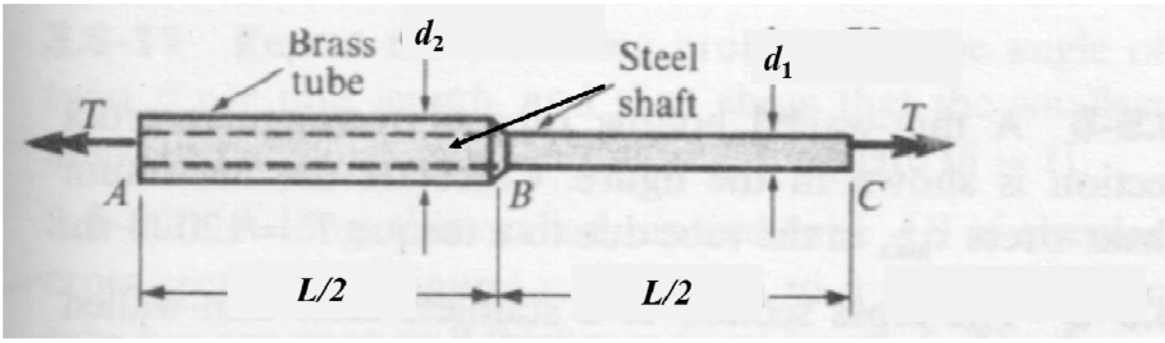

Fig. 3

Problem 4

Draw the shear force and bending moment diagrams of the lower beam ***ABCD*** loaded as shown in **Fig. 4**. The upper beam has a pin support at its left end and a roller at ***D.*** The lower beam has an internal hinge at ***B***. (**10 Marks**)

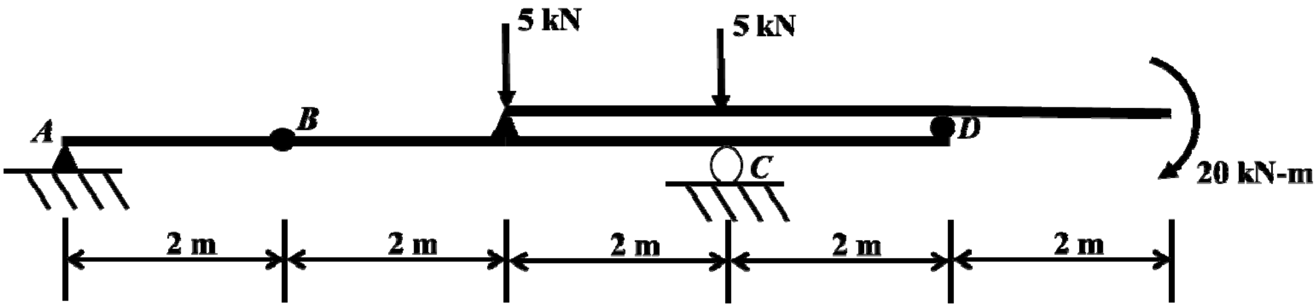

Fig. 4

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Mid-Sem Exam **11/09/17** **PAPER CODE: B**

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Problem 1 (a)

Calculate the increase in the volume ΔV of a bar with uniform cross-section and length **2 m** hanging vertically under its own weight of **15 kN**. The modulus of elasticity of the bar is **300 GPa** and and Poisson's ratio is **0.3**. (**5 Marks**)

Problem 1 (b)

A steel rod ***AB*** of diameter **12 mm** is stretched tightly between two supports so that the tensile stress in the rod is **80 MPa**. Then an axial force ***P*** is applied gradually to the rod at an intermediate location ***C*** as shown in **Fig. 1**. Calculate the value of this load ***P*** when the entire rod yields, if the material is elastic-plastic with yield stress σ_y **= 200 MPa**.

(**5 Marks**)

Fig. 1

Problem 2

Two rigid bars ***AB*** and ***CD*** are connected by linear elastic springs and are supported at ***A*** and ***D*** by hinge supports (**Fig. 2**). When no loads are acting, the bars are horizontal and the springs are unstressed. Determine the vertical deflection δ at point ***C*** when a load is applied at ***C*** as shown.

(**10 Marks**)

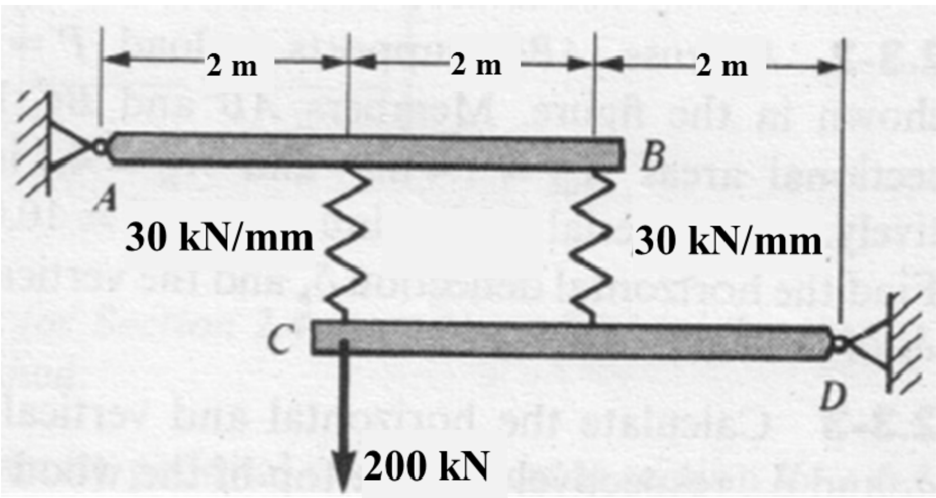

Fig. 2

Problem 3

A steel shaft (**G_s = 60 GPa**) of total length ***L*** **= 6.0 m** is encased over half its length by a brass tube (**G_b = 30 GPa**) that is securely bonded to the steel (**Fig. 3)**. The diameters of the shaft and tube are **d_1 = 90 mm** and **d_2 = 110 mm**, respectively. Determine the allowable torque ***T*** on the given assembly if the shear stress in the brass and steel are limited to **τ_b = 120 MPa** and **τ_s = 100 MPa**, respectively, and the angle of twist ϕ between the ends ***A*** and ***C*** is limited to **ϕ = 6 degrees**. (**10 Marks**)

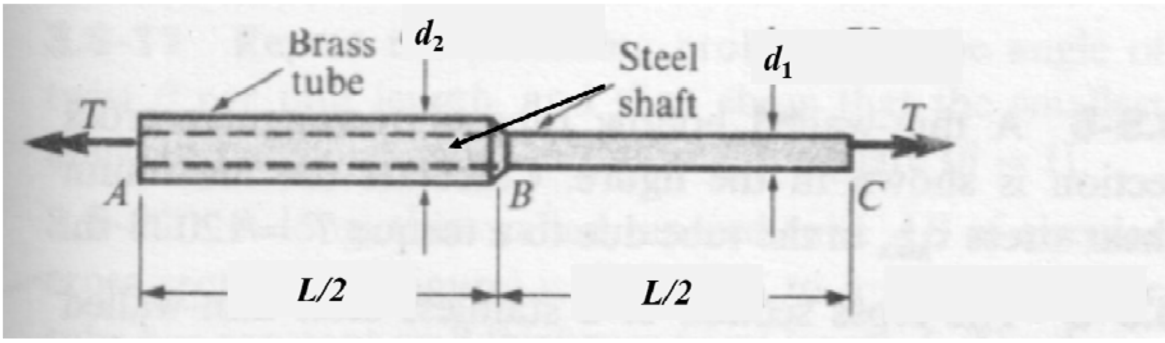

Fig. 3

Problem 4

Draw the shear force and bending moment diagrams of the lower beam ***ABCD*** loaded as shown in **Fig. 4**. The upper beam has a pin support at its left end and a roller at ***D.*** The lower beam has an internal hinge at ***B***. (**10 Marks**)

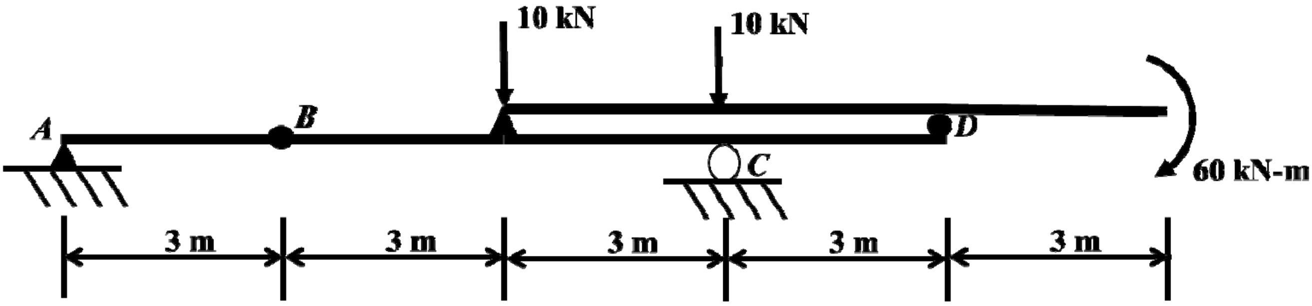

Fig. 4

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CE-221 SOLID MECHANICS

Mid-Sem Exam **11/09/17** **PAPER CODE: C**

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Closed book, closed notes exam. No formula sheet allowed. No mobile phones allowed in the exam hall.
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Problem 1 (a)

Calculate the increase in the volume ΔV of a bar with uniform cross-section and length **3 m** hanging vertically under its own weight of **25 kN**. The modulus of elasticity of the bar is **250 GPa** and and Poisson's ratio is **0.35**. (**5 Marks**)

Problem 1 (b)

A steel rod ***AB*** of diameter **8 mm** is stretched tightly between two supports so that the tensile stress in the rod is **80 MPa**. Then an axial force ***P*** is applied gradually to the rod at an intermediate location ***C*** as shown in **Fig. 1**. Calculate the value of this load ***P*** when the entire rod yields, if the material is elastic-plastic with yield stress σ_y **= 100 MPa**.

(**5 Marks**)

Fig. 1

Problem 2

Two rigid bars ***AB*** and ***CD*** are connected by linear elastic springs and are supported at ***A*** and ***D*** by hinge supports (**Fig. 2**). When no loads are acting, the bars are horizontal and the springs are unstressed. Determine the vertical deflection δ at point ***C*** when a load is applied at ***C*** as shown.

(**10 Marks**)

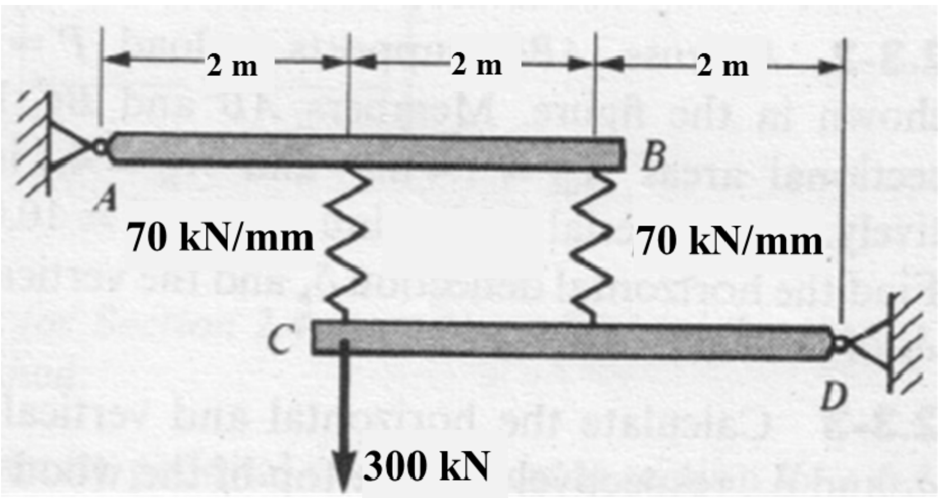

Fig. 2

Problem 3

A steel shaft (**G_s = 100 GPa**) of total length **L = 8.0 m** is encased over half its length by a brass tube (**G_b = 50 GPa**) that is securely bonded to the steel (**Fig. 3)**. The diameters of the shaft and tube are **d_1 = 50 mm** and **d_2 = 80 mm**, respectively. Determine the allowable torque ***T*** on the given assembly if the shear stress in the brass and steel are limited to τ_b = **140 MPa** and τ_s = **120 MPa**, respectively, and the angle of twist ϕ between the ends ***A*** and ***C*** is limited to **ϕ = 8 degrees**. (**10 Marks**)

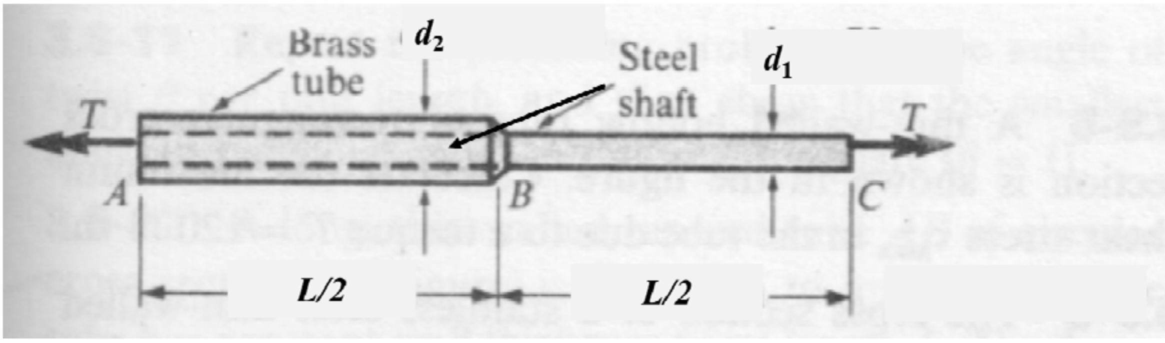

Fig. 3

Problem 4

Draw the shear force and bending moment diagrams of the lower beam ***ABCD*** loaded as shown in **Fig. 4**. The upper beam has a pin support at its left end and a roller at ***D.*** The lower beam has an internal hinge at ***B***. (**10 Marks**)

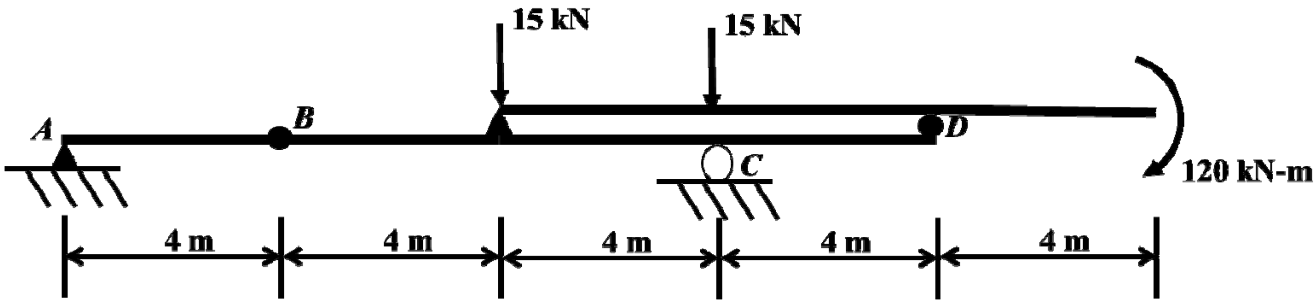

Fig. 4

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CE-221 SOLID MECHANICS

Mid-Sem Exam **11/09/17** **PAPER CODE: D**

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Closed book, closed notes exam. No formula sheet allowed. No mobile phones allowed in the exam hall.
Assume suitable data if required and state the same clearly.

Problem 1 (a)

Calculate the increase in the volume ΔV of a bar with uniform cross-section and length **4 m** hanging vertically under its own weight of **30 kN**. The modulus of elasticity of the bar is **150 GPa** and and Poisson's ratio is **0.4**. (**5 Marks**)

Problem 1 (b)

A steel rod ***AB*** of diameter **6 mm** is stretched tightly between two supports so that the tensile stress in the rod is **80 MPa**. Then an axial force ***P*** is applied gradually to the rod at an intermediate location ***C*** as shown in **Fig. 1**. Calculate the value of this load ***P*** when the entire rod yields, if the material is elastic-plastic with yield stress σ_y **= 400 MPa**.

(**5 Marks**)

Fig. 1

Problem 2

Two rigid bars ***AB*** and ***CD*** are connected by linear elastic springs and are supported at ***A*** and ***D*** by hinge supports (**Fig. 2**). When no loads are acting, the bars are horizontal and the springs are unstressed. Determine the vertical deflection δ at point ***C*** when a load is applied at ***C*** as shown.

(**10 Marks**)

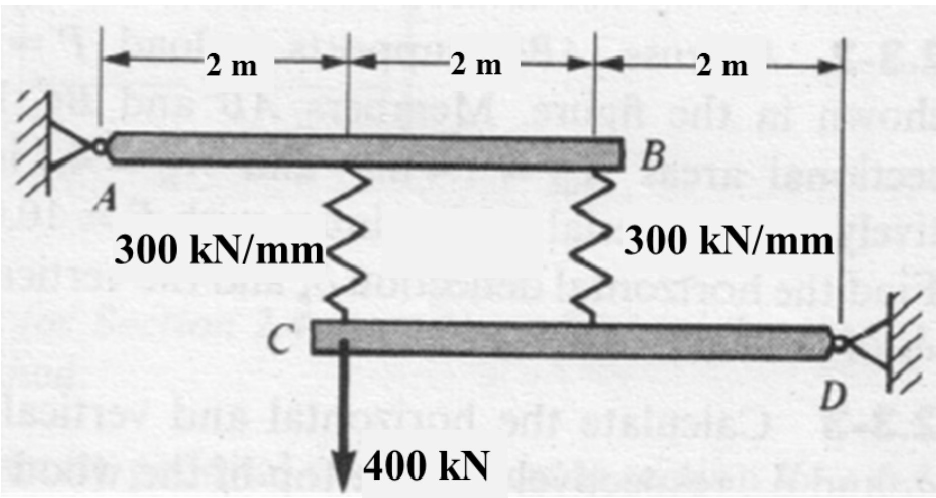

Fig. 2

Problem 3

A steel shaft (**G_s = 120 GPa**) of total length ***L* = 10.0 m** is encased over half its length by a brass tube (**G_b = 70 GPa**) that is securely bonded to the steel (**Fig. 3)**. The diameters of the shaft and tube are **d_1 = 100 mm** and **d_2 = 150 mm**, respectively. Determine the allowable torque ***T*** on the given assembly if the shear stress in the brass and steel are limited to τ_b = **160 MPa** and τ_s = **130 MPa**, respectively, and the angle of twist ϕ between the ends ***A*** and ***C*** is limited to **ϕ = 10 degrees**. (**10 Marks**)

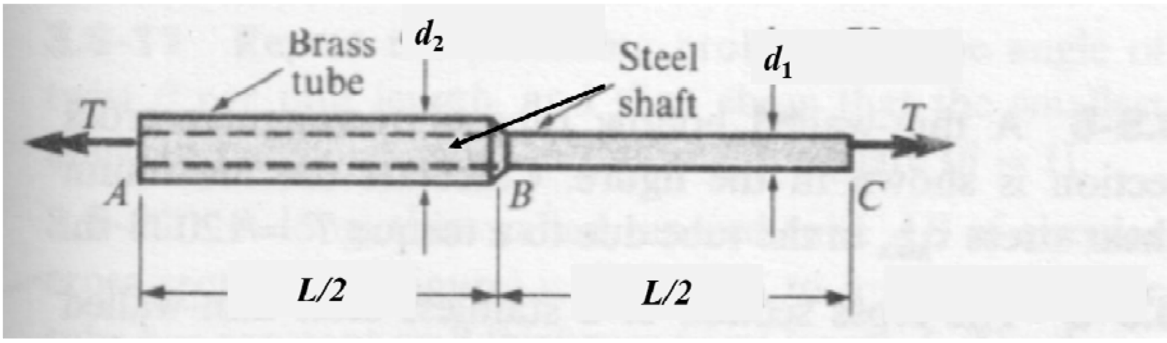

Fig. 3

Problem 4

Draw the shear force and bending moment diagrams of the lower beam ***ABCD*** loaded as shown in **Fig. 4**. The upper beam has a pin support at its left end and a roller at ***D.*** The lower beam has an internal hinge at ***B***. (**10 Marks**)

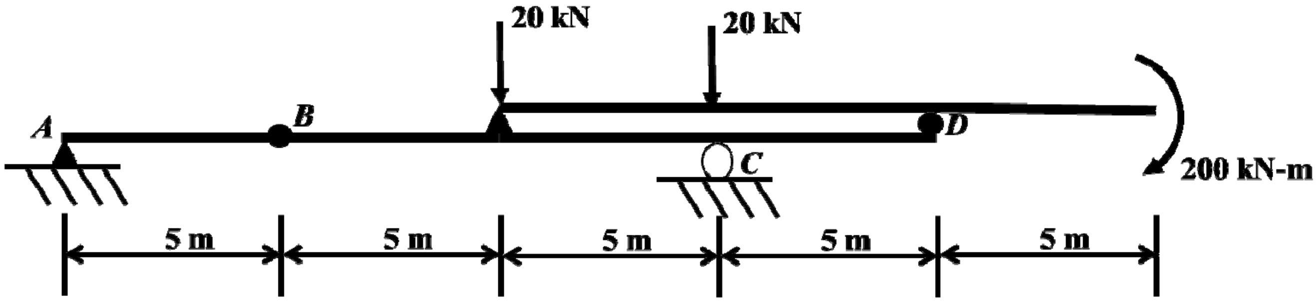

Fig. 4

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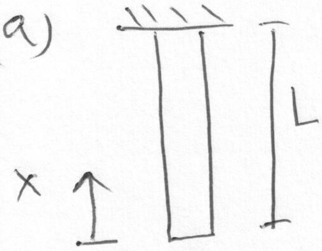
P1(a)



$$\sigma_x = \frac{W}{L} x \cdot \frac{1}{A}$$

Consider slice of area A, thickness dx.

Cubical dilatation $\rightarrow \frac{\text{change in vol}}{\text{original vol}} = \frac{d(\Delta V)}{dV} = \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz}$

$$= \frac{\sigma_x}{E}(1-2\nu)$$

$\because$ strains are constant throughout ~~throughout~~ the slice of thickness dx,

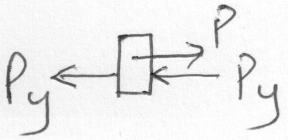
we can put $dV = A\,dx$

$$\Rightarrow \frac{d(\Delta V)}{dV} = \frac{d(\Delta V)}{A\,dx} = \frac{\sigma_x}{E}(1-2\nu) = \frac{W}{LA} \cdot x \frac{(1-2\nu)}{E}$$

integrate, $\Delta V = \frac{W}{E}\frac{(1-2\nu)}{L}\frac{L^2}{2} = \frac{WL}{2}\left(\frac{1-2\nu}{E}\right)$ ◄

Code: A $\rightarrow 30\,mm^3 = \Delta V$; B $\rightarrow 20\,mm^3$; C $\rightarrow 45\,mm^3$; D $\rightarrow 80\,mm^3$

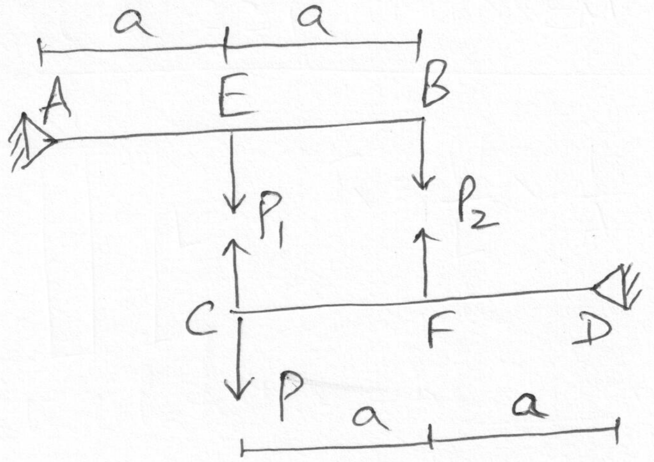
P1(b) Entire rod yields $\Rightarrow \sigma = \sigma_y$ throughout.



$$\Rightarrow P = 2P_y = 2\sigma_y \frac{\pi}{4}(d^2)$$

Code: A $\rightarrow$ P = 92.36 kN ; B $\rightarrow$ 45.24 kN ; C $\rightarrow$ 10.05 kN ; D $\rightarrow$ 22.62 kN

P.2



Equilibrium: $P_1 = -2P_2$ $\left(\sum M_{A\ \text{upper FBD}} = 0\right)$

$2(-2P_2 - P) = -P_2$ $\left(\sum M_{D\ \text{lower FBD}} = 0\right)$.

$\Rightarrow P_2 = -\frac{2}{3}P$; $P_1 = \frac{4}{3}P$

Compatibility: $\because$ bars are rigid,

$\delta_B = 2\delta_E$

$\delta_C = 2\delta_F$

Equil + Compat:

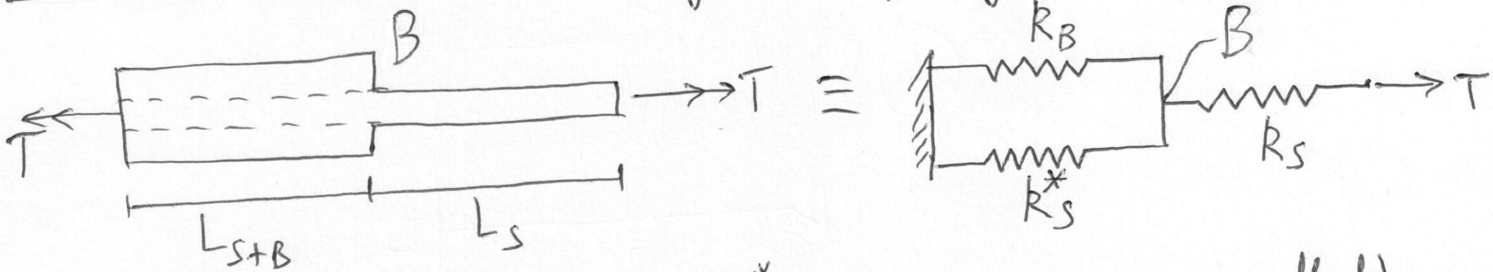
$\Rightarrow k(\delta_C - \delta_E) = \frac{4}{3}P$; $k(\delta_F - \delta_B) = -\frac{2}{3}P$

$\Rightarrow \delta_C = \delta_E + \frac{4}{3}\frac{P}{k}$; $\delta_F = \frac{\delta_C}{2} = -\frac{2}{3}\frac{P}{k} + 2\delta_E$

$\Rightarrow \delta_C\left(2 - \frac{1}{2}\right) = \frac{8}{3}P + \frac{2}{3}\frac{P}{k} \Rightarrow \delta_C = \frac{20}{9}\frac{P}{k}$ ◄

Code: A $\rightarrow$ $\delta_C = 5.56$mm ; B $\rightarrow$ 14.81mm ; C $\rightarrow$ 9.52mm ; D $\rightarrow$ 2.96mm

P.3 <u>Short solution</u>: using concept of equivalent springs.



$$k_S = \frac{G_S J_S}{L_S} \; ; \quad k_{S+B} = k_B + k_S^* \quad (\because \text{ springs in parallel})$$

$$\hookrightarrow = \frac{G_B J_B}{L_{S+B}} + \frac{G_S J_S}{L_{S+B}}$$

$$k_{eff} = \frac{k_S \, k_{S+B}}{k_S + k_{S+B}} \quad (\because k_S \;\&\; k_{S+B} \text{ are in series}).$$

$$(T_{all})_{\theta_{all}} = k_{eff}\, \theta_{all} \longrightarrow (1)$$

$$(T_{all})_{\tau_S} = (\tau_{all})_S \frac{J_S}{r_S} \longrightarrow (2)$$

$$\frac{(T_B)_{S+B}}{(T_S)_{S+B}} = \frac{G_B J_B / L_{S+B}}{G_S J_S / L_{S+B}} = \lambda^{-1}$$ (from displ compatibility at any section between A & B).

$$(T_{all})_{\tau_B} = \frac{(\tau_{all})_B \, J_B}{(r_o)_B} (1+\lambda) \longrightarrow (3)$$ (from equilibrium at any section between A & B, i.e. $T = T_S + T_B$).

Choose lowest of (1), (2), (3) to get T_{max}.

<u>Detailed solution</u>

For composite section, at any point between A & B,

$$\theta_B = \theta_S \Rightarrow \frac{(T_S)_{S+B}\, l}{G_S J_S} = \frac{(T_B)_{S+B}\, l}{G_B J_B} \Rightarrow \frac{(T_S)_{S+B}}{(T_B)_{S+B}} = \lambda, \text{ as above.}$$

$$\theta_{C/A} = \theta = \frac{T L_S}{G_S J_S} + \frac{(T_S)_{B+S} L_{B+S}}{G_S J_S} \longrightarrow$$

$$T = (T_S)_{B+S} + (T_B)_{B+S} \longrightarrow$$

$$\Rightarrow \theta = T \left[\frac{L_S + \frac{\lambda}{1+\lambda} L_{B+S}}{G_S J_S} \right] \longrightarrow (4)$$

($\hookrightarrow k_{eff}^{-1}$)

→ It is obvious that $(\tau_s)_s > (\tau_s)_{B+S}$ since Brass shares the load. So for $(\tau_{all})_{\tau_s}$ criteria we get same expression as (2) for $(\tau_{all})_{\tau_s}$.

→ For θ_{all} criteria use (4) with $\theta = \theta_{all}$ to get $(T_{all})_{\theta_{all}}$, which is same as (1) ∵ $[\ ]$ in (4) is k_{eff}^{-1}

→ For $(T_{all})_{\tau_B}$ criteria, $T = (T_B)_{B+S} + (T_S)_{B+S} = (T_B)_{B+S}[1+\lambda]$

$\left(\text{for } (T_{all})_{\tau_B},\ (T_B)_{B+S} = \frac{(\tau_{all})_B J_B}{(r_0)_B}\right) \Rightarrow T = \frac{(\tau_{all})_B J_B}{(r_0)_B}[1+\lambda]$ → same as (3)

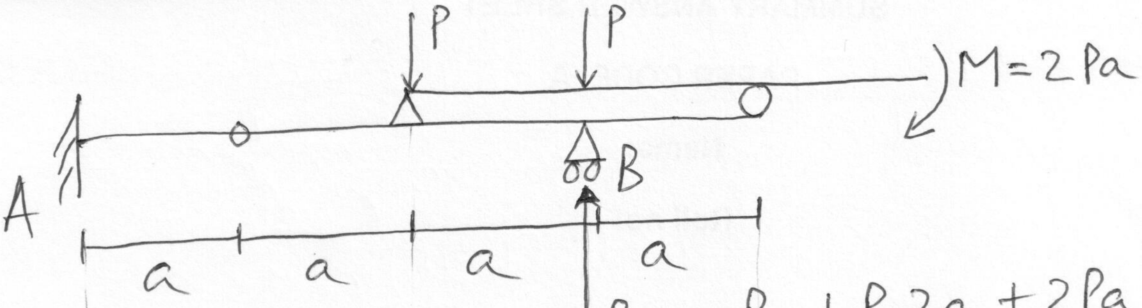
So both ways yield same result.

Code	$(T_{all})_{\theta_{all}}$	$(T_{all})_{\tau_s}$	$(T_{all})_{\tau_B}$
A →	$= 12.86$	$= \boxed{5.388}$	$= 19.55$
B →	$= \boxed{8.333}$	$= 14.31$	$= 45.41$
C →	$= \boxed{1.693}$	$= 2.945$	$= 16.22$
D →	$= 31.71$	$= \boxed{25.52}$	$= 120.99$

all in kN.m

Circled ones are the lowest, i.e, the answer for the respective paper code.

P4



$$R_B = \frac{Pa + P.2a + 2Pa}{2a} = \frac{5}{2}P$$

(from $\sum_{\text{hinge}} M = 0$)