DEPARTMENT OF CIVIL ENGINEERING, IIT BOMBAY

CE 221 Solid Mechanics: QUIZ 1

Note: **Assume suitable data if not given.**

22/08/2015 | Total Marks:10 | Duration: 60 mins | Instructors A. Laskar / N.K.Chandiramani

Problem 1 (5 marks):

A composite bar of square cross-section is constructed of two different materials having moduli of elasticity $\boldsymbol{E_1}$ and $\boldsymbol{E_2}$ as shown in **Fig. 1**. Both parts of the bar have the same cross-sectional dimensions (i.e., $\boldsymbol{2b}$ x $\boldsymbol{b}$). The bar is loaded with a compressive force $\boldsymbol{P}$ acting at an eccentricity $\boldsymbol{e}$ measured from the interface of the two materials, as shown. Assuming that the end plates are rigid:

(a) Derive a formula for the eccentricity $\boldsymbol{e}$ of the load $\boldsymbol{P}$ so that each part of the bar is stressed uniformly in compression.

(b) Under these conditions what part of the load $\boldsymbol{P}$ does each material carry?

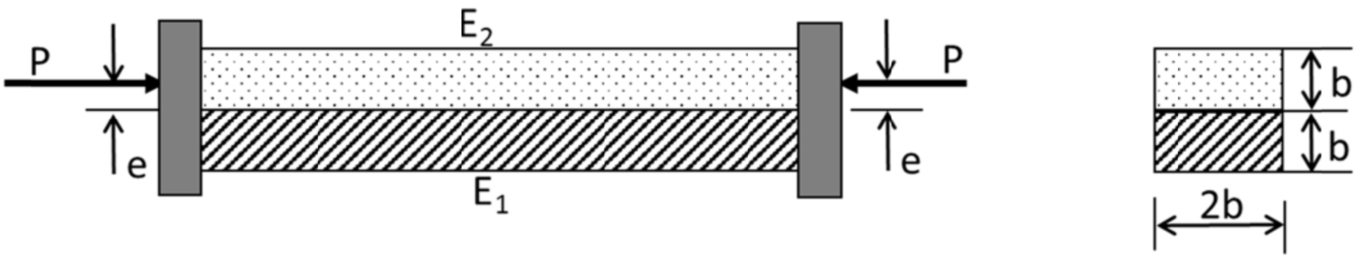

Fig. 1

Problem 2 (5 marks):

A piece of **50 mm** x **250 mm** x **10 mm** steel plate is subjected to uniformly distributed stresses along its edges as shown in **Fig. 2**. If $\boldsymbol{P_y = 2P_x}$ and the maximum shear stress acting on the plate under the given loading is **60 MPa**, determine:

(a) the applied load $\boldsymbol{P_x}$,

(b) the change in thickness of the plate under the applied loads,

Assume **E = 200 GPa** and **ν = 0.25**.

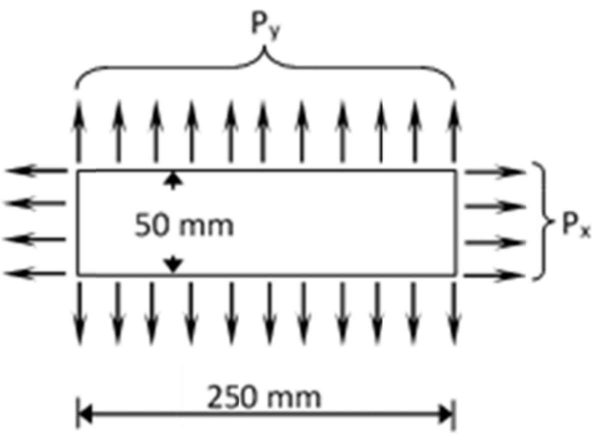

Fig. 2

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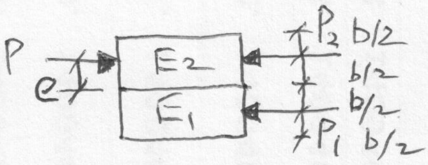
P.1



Equilibrium:

$$P = P_1 + P_2 \longrightarrow ①$$

$$P(e+b) = P_1 \frac{b}{2} + P_2 \frac{3b}{2} \rightarrow ②$$

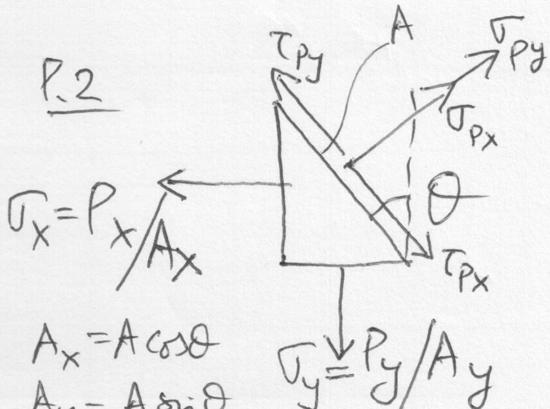
Compatibility: axial strain ε same in E_1, E_2.
+
Hookes law: $P_1 = \sigma_1 A = E_1 \varepsilon A$; $P_2 = E_2 \varepsilon A \rightarrow ③$.

$$\Rightarrow P_1 = P_2 \frac{E_1}{E_2}$$

$$①, ③ \longrightarrow \boxed{P_1 = P\left(\frac{E_1}{E_1+E_2}\right), \; P_2 = P\left(\frac{E_2}{E_1+E_2}\right)} \longrightarrow ④ \rightarrow \text{Ans (b)}$$

$$②, ④ \longrightarrow \boxed{e} = \frac{b}{2} \frac{(E_1 + 3E_2 - 2E_1 - 2E_2)}{(E_1+E_2)} = \boxed{\frac{b}{2}\left(\frac{E_2 - E_1}{E_1+E_2}\right)} \rightarrow \text{Ans (a)}$$

P.2



$A_x = A\cos\theta$
$A_y = A\sin\theta$

$$\tau_{px} A = \sigma_x A_x \sin\theta = \sigma_x A \cos\theta \sin\theta$$

$$\tau_{py} A = \sigma_y A_y \cos\theta = \sigma_y A \sin\theta \cos\theta$$

$$\tau = \tau_{px} - \tau_{py} = \left(\frac{\sigma_x - \sigma_y}{2}\right) \sin 2\theta$$

$$\tau_{max} \text{ at } 2\theta = \frac{\pi}{2}, \quad \tau_{max} = \left(\frac{\sigma_x - \sigma_y}{2}\right).$$

$$60 = \left(\frac{P_x}{500} - \frac{P_y}{2500}\right)\frac{1}{2} = \left(\frac{P_x}{500} - \frac{2P_x}{2500}\right)\frac{1}{2} \Rightarrow P_x = 100000\,N$$

$P_x = 100\,kN$ ◀
$P_y = 200\,kN$.

$$\Delta \text{thickness} = \varepsilon_z (10) = -\frac{\nu}{E}(\sigma_x + \sigma_y)(10)$$

(Plane stress, $\sigma_z = 0$)

$$= \frac{-0.25}{200 * 10^9 * 10^{-6}} \left(\frac{10^5}{500} + \frac{2*10^5}{2500}\right)(10)$$

$$= 3.5 * 10^{-3} \text{ mm}$$