DEPARTMENT OF CIVIL ENGINEERING, IIT BOMBAY

CE 201 Solid Mechanics

Tutorial Sheet = 10 Instructors :A. Laskar /N.K. Chandiramani

1. Determine the maximum allowable load F that may be applied to the aluminium frame shown in Fig. 1. Use Euler's formula and take factor of safety as 2.5. E = 70 GPa.
2. The capacity of the jib crane is given as 20 kN as shown in Fig. 2. What size of steel pipe AB should be used if thickness/diameter ratio of the pipe is 1/10 and E = 200 GPa. Use Euler's formula with a factor of safety 2.5 and neglect the self weight.
3. Using Euler's formula, find the ratio of the critical load of a solid circular column to that of hollow circular cross-section (inner diameter is 0.8 times external diameter) for identical end conditions and length. Both columns have equal area of cross-section.
4. A column of length L and rectangular cross-section (**a** x **b**) has a fixed end B and support a centric load at A as shown in Fig. 3. Two smooth and rounded fixed plates restrain end A from moving in one of the plane of symmetry of the column and allow it to move in the other plane. Determine the ratio **a/b** for most efficient design against buckling.
5. The effective length of a composite column shown in Fig. 4 is 5m. Obtain the dimension b so that the column has equal chances of buckling along the principal directions. Determine the safe load carrying capacity of the column. Take E = 200 GPa and factor of safety as 2.
6. The rigid bar segments of equal length L/2 are connected at the joint and at the bottom by frictionless hinges as shown in Fig. 5. The bars are held in vertical position by two springs. Determine the critical load for the system.

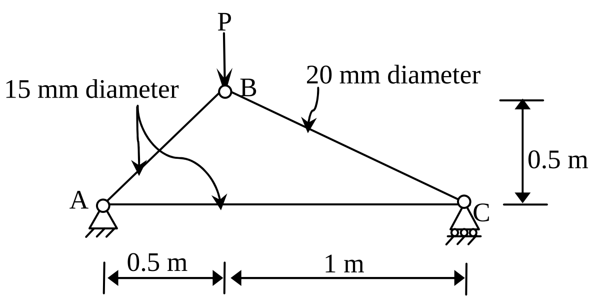


Fig. 1

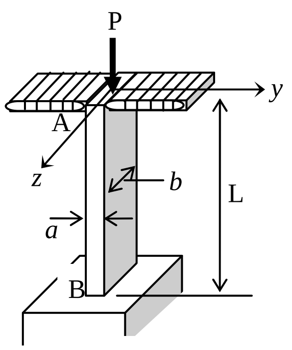


Fig. 3

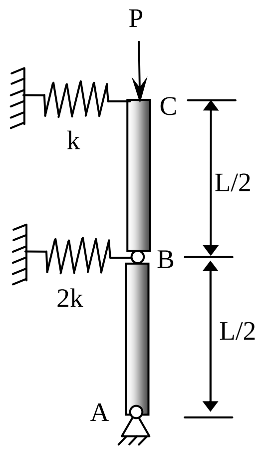


Fig. 5

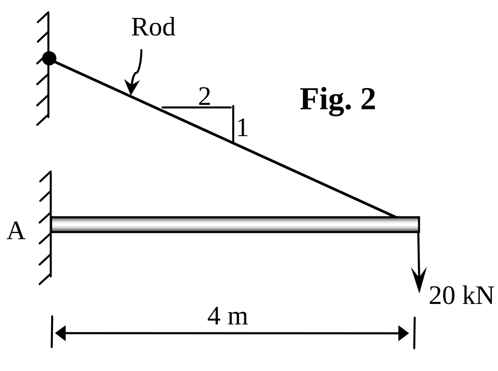
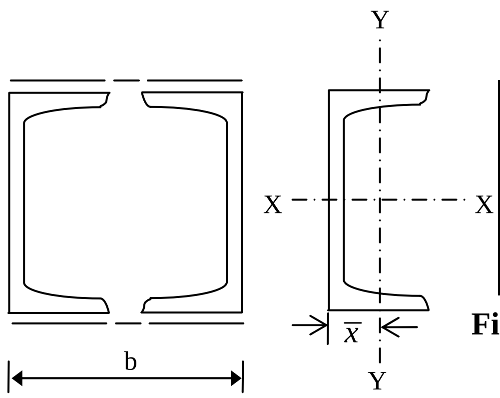


Fig. 2



A = 9480 mm^2
I_{xx} = 168.2 x 10^6 mm^4
I_{yy} = 4.58 x 10^6 mm^4
$\overline{x}$ = 20.3 mm

Fig.4

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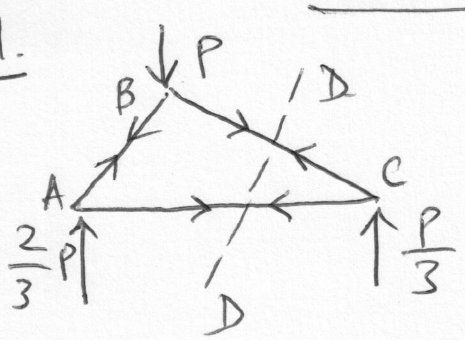
P1.



$\Sigma M_B = 0 = F_{AC}(0.5) - \frac{2P}{3}(0.5)$

$F_{AC} = 2P/3$

Joint A: $F_{AB} = -\frac{2}{3}P\sqrt{2}$

Joint C: $F_{BC} = -\frac{P}{3}\sqrt{5}$

$$P_{cr} = \frac{\pi^2 E I}{L_e^2} = \pi^2 E \frac{\pi}{4}\frac{r^4}{L_e^2}$$

In-plane buckling: Simply supported

AB buckles $\rightarrow \frac{2}{3}P\sqrt{2} = \pi^3 \frac{E}{4}\frac{(7.5/1000)^4}{(0.5\sqrt{2})^2} \Rightarrow P = 3641$

BC buckles $\rightarrow \frac{P}{3}\sqrt{5} = \pi^3 \frac{E}{4}\frac{(10/1000)^4}{(0.5\sqrt{5})^2} \Rightarrow P = 5824$

$\Rightarrow P_{max} = \frac{3641}{FS} = \frac{3641}{2.5} = 1456.4\,N.$

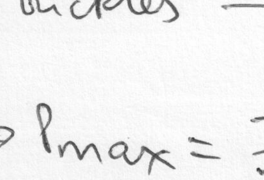
Out-of-plane buckling: Fixed–Fixed (∵ pin-joints ~~not~~ ball & socket)

Recall: $L_e = 4L.$

$\Rightarrow P_{max} = 4 * 1456.4 = 5825.6\,N.$

So In-plane buckling is critical.

P2



$F_{AB} = -F_{BC}\frac{2}{\sqrt{5}} = -20\sqrt{5}\cdot\frac{2}{\sqrt{5}} = -40\,kN.$

$P_{cr} = 40 * FS = 40 * 2.5 = \frac{\pi^2 E\pi(r_o^4 - r_i^4)/4}{L_e^2}$

For thin pipe, $(r_o^4 - r_i^2) = (r_o - r_i)(r_o + r_i)(r_o^2 + r_i^2)$

$\simeq t\,4\bar{r}^3$ ($\bar{r}$ = mean radius)

$\Rightarrow 100\,kN = \frac{\pi^3 E\,4t\bar{r}^3/4}{L_e^2} = \frac{\pi^3 E\,\frac{\bar{r}}{5}\bar{r}^3}{L_e^2}$, use $L_e = L$ (ie SS BC's).

$\Rightarrow \bar{r} = 33.7\,mm \Rightarrow d = 67.4\,mm \rightarrow$ So use 70 mm dia 7 mm thk.

P3 $\dfrac{I_s}{I_H} = \dfrac{R^4}{R_o^4 - (0.8R_o)^4} = \dfrac{P_{cr,s}}{P_{cr,H}}$

$A_s = A_H \Rightarrow R^2 = R_o^2 - (0.8R_o)^2$

$\Rightarrow \dfrac{P_{cr,s}}{P_{cr,H}} = \dfrac{\left[R_o^2 - (0.8R_o)^2\right]^2}{R_o^4 - (0.8R_o)^4} = \dfrac{(1-0.8^2)^2}{1-0.8^4} = 0.2195$

P4 Done in class

P5 $I_{xx} = I_{yy}$

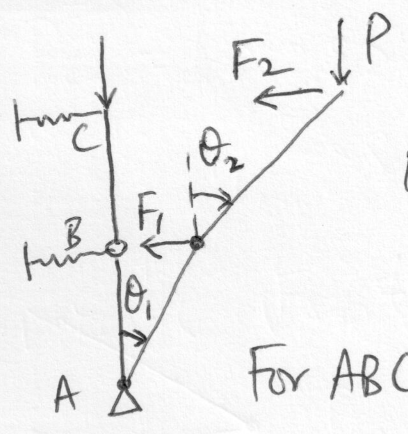
$2(168.2E6) = 2\left(4.58E6 + \overset{A}{9840}\left[\frac{b}{2} - \overset{\bar{X}}{20.3}\right]^2\right)$

$b = 298.5\,mm.$

$P_{cr} = \dfrac{\pi^2 EI}{L_e^2}$, $E = 200\,GPa$, $I = I_{xx} = I_{yy}$, $L_e = 5$
$I_{xx} = 2(168.2E6)$

$P_{max} = \dfrac{P_{cr}}{2} = 13280.5\,kN.$

P6



$F_1 = k\frac{L}{2}\theta_1$, $F_2 = k\frac{L}{2}(\theta_1 + \theta_2)$

For BC: $\Sigma M_B = 0 = P\frac{L}{2}\theta_2 - F_2\frac{L}{2}$

$\Rightarrow 0 = -\frac{kL^2}{4}\theta_1 + \left(\frac{PL}{2} - \frac{kL^2}{4}\right)\theta_2$

For ABC: $\Sigma M_A = 0 = P\frac{L}{2}(\theta_1+\theta_2) - F_1\frac{L}{2} - F_2 L$

$\Rightarrow 0 = \left(\frac{PL}{2} - \frac{kL^2}{2} - \frac{kL^2}{2}\right)\theta_1 + \left(\frac{PL}{2} - \frac{kL^2}{2}\right)\theta_2$

Let $\frac{PL}{2} = \lambda$, $\frac{kL^2}{4} = a$

$\Rightarrow \begin{bmatrix} -a & \lambda - a \\ \lambda - 4a & \lambda - 2a \end{bmatrix} \begin{Bmatrix} \theta_1 \\ \theta_2 \end{Bmatrix} = 0 \Rightarrow \det[\;\;] = 0$ for non trivial $\begin{Bmatrix} \theta_1 \\ \theta_2 \end{Bmatrix}$

$$\det[\quad] = 0 \Rightarrow \lambda^2 - 4a\lambda + 2a^2 = 0$$

$$\Rightarrow \lambda = 2a \pm \sqrt{2}\,a$$

$$P_{cr} = \frac{2}{L}\lambda = \frac{2}{L}(2 \pm \sqrt{2})a = \frac{2}{L}(2\pm\sqrt{2})\frac{kL^2}{4}$$

$$= kL\left(1 \pm \frac{1}{\sqrt{2}}\right)$$

So $P_{buckling} = \text{lower } P_{cr} = kL(0.2928)$

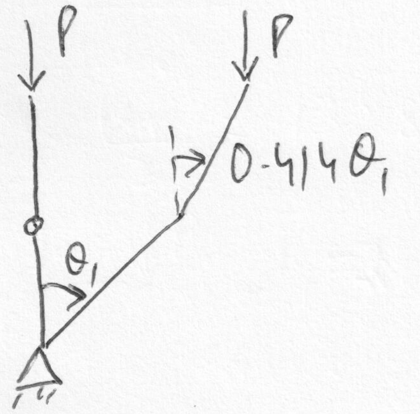
<u>Extra:</u>

Buckling modes:

For $\lambda = (2+\sqrt{2})a \rightarrow -a\theta_1 + (\lambda - a)\theta_2 = 0$

$$\theta_2 = \left(\frac{a}{\lambda - a}\right)\theta_1 = \left(\frac{1}{1+\sqrt{2}}\right)\theta_1 = 0.4142\theta_1$$

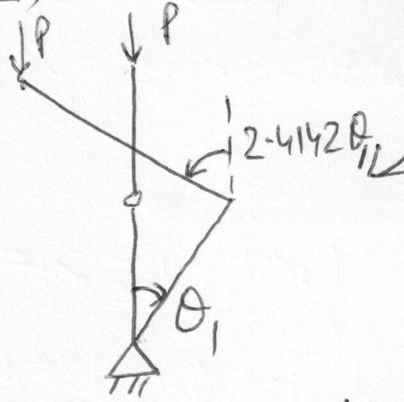
For $\lambda = (2-\sqrt{2})a \longrightarrow \theta_2 = \left(\frac{1}{1-\sqrt{2}}\right)\theta_1 = -2.4142\theta_1$



1st Buckling mode

$P_{cr} = 0.2928kL$

This is critical one.



2nd Buckling mode

$P_{cr} = 1.707kL$