DEPARTMENT OF CIVIL ENGINEERING, IIT BOMBAY

CE 221 Solid Mechanics

Tutorial Sheet = 2 Instructor : A. Laskar / Naresh K. Chandiramani

1. Determine the deflection of the free end of the steel rod shown in Fig. 1 under the given load (E = 200 GPa).

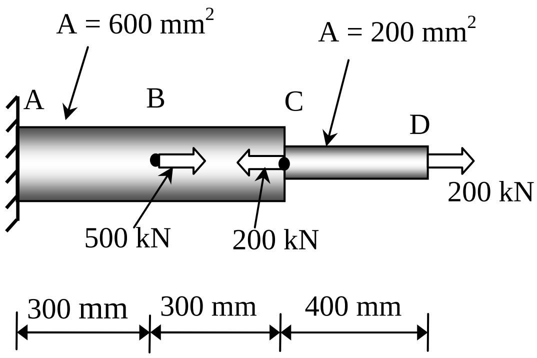


Fig. 1

2. A uniform timber pile which has been driven to depth L in clay carries an applied load of F at top. This load is resisted entirely by friction along the pile, which varies in the parabolic manner $f = ky^2$ (origin at bottom). Show that total shortening of the pile is FL/4AE. AE is the axial rigidity of the pile.

3. Show that the total elongation of a slender elastic bar of constant cross sectional area A, length 2L, unit weight γ is given by following expression when it is rotated in a horizontal plane with an angular velocity of ω radians per second about its middle point.

$$\Delta = \frac{2\gamma\omega^2 L^3}{3Eg}$$

E = Modules of elasticity and g = acceleration due to gravity.

4. The rigid bar BDE (Fig. 2) is supported by two links AB and CD. Link AB is made of aluminum (E=70 GPa) and has a cross-sectional area of 500 mm^2; link CD is made of steel (E=200 GPa) and has a cross-sectional area of 600 mm^2. For the 30 kN force shown, determine the deflection of point B, D and E.

5. A composite bar as shown in Fig 3 is firmly attached to unyielding supports at the ends and is subjected to the axial load F. If the aluminum is stressed to 70 MPa, what is the stress in the steel?.

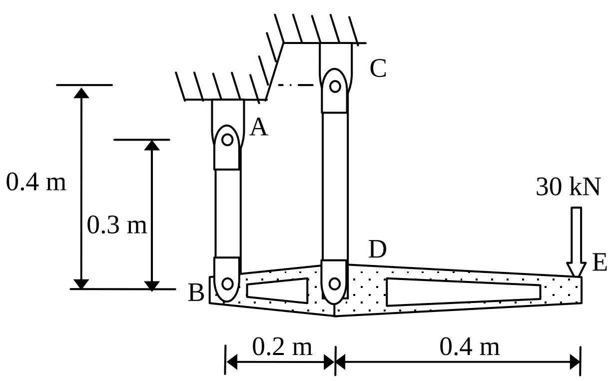


Fig. 2

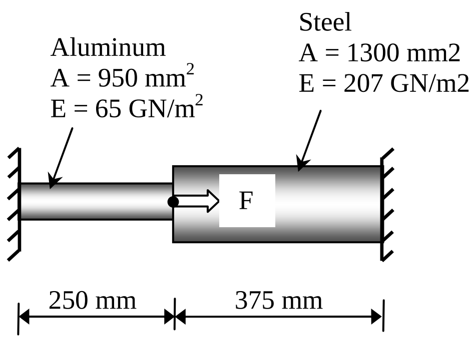


Fig. 3

6. Determine the stresses in each wires supporting the rigid bar shown in Fig. 4 if F = 20 kN.

7. The rigid bar ABCD is suspended from three identical wires as shown in Fig. 5. Knowing that $a = 2b$, determine the tension in each wire caused by the load P applied at C.

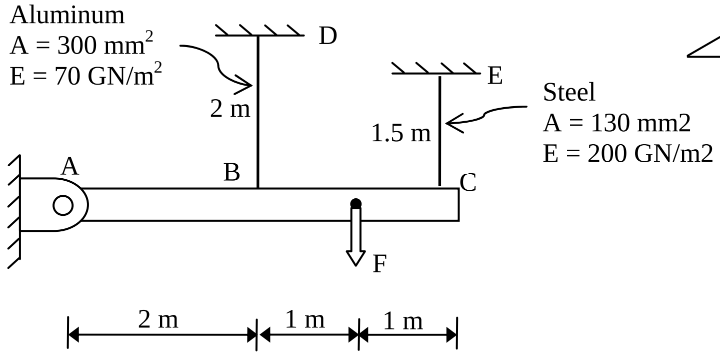


Fig. 4

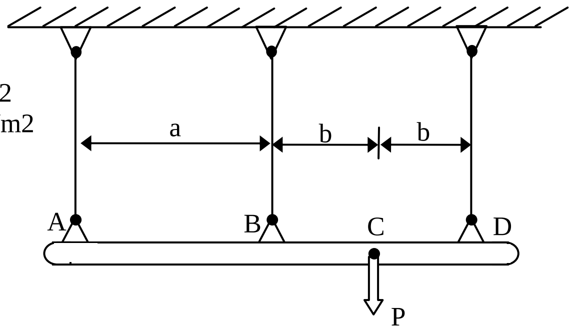


Fig. 5

8. A rod consisting of two cylindrical portion AB and BC (Fig. 6) is restrained at both ends. Portion AB is made of steel (E = 200 GPa, α = 11.7 x 10^{-6} /°C) and portion BC of brass (E=105 GPa, α = 20.9 x 10^{-6} /°C). Knowing that the rod is initially unstressed, determine (a) the normal stresses induced in portions AB and BC by a temperature rise of 50°C, (b) the corresponding deflection of point B.

9. A rigid floor slab with mass of 3,200 kg rests on three columns as shown in Fig. 7. What is the compressive stress in each of the members (a) at installation and (b) after a temperature decrease of 20 °C?

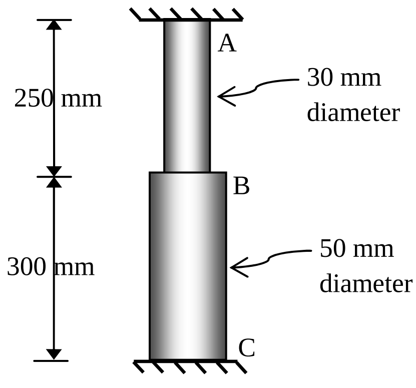


Fig. 6

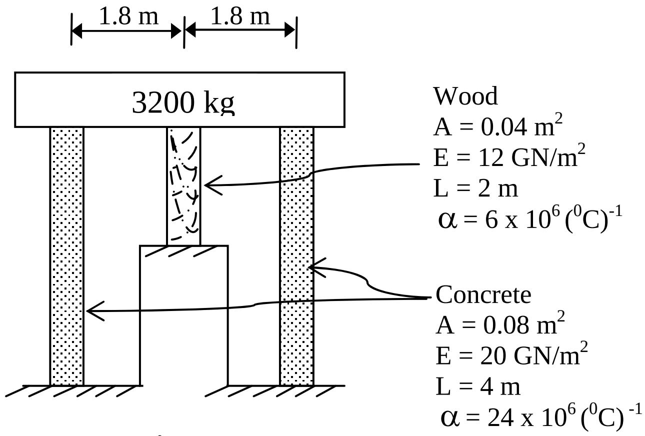


Fig. 7

10. The bar shown in Fig. 8 is cut from a 10 mm thick piece of steel. At the change in cross-section at A and B the approximate stress concentration factors are 2.25 and 2, respectively. What is the maximum force F the bar can be subjected? Take allowable stress for axial tension in the bar as 150 MPa.

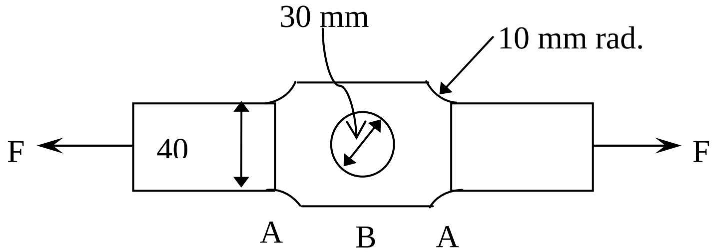
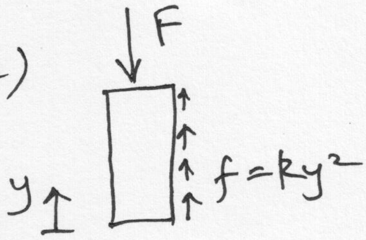


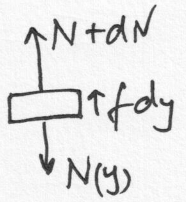
Fig. 8

CE221 TUTORIAL #2

1) $\Delta = \sum \frac{P_i L_i}{(AE)_i} = \frac{(200E3)(400)}{(200)(200E3)} + \frac{(500E3)(300)}{(600)(200E3)} = 3.25\,mm$

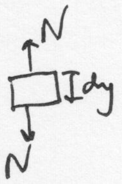
2)





$dN + f\,dy = 0$

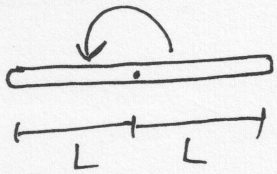
$\int_0^{N(y)} dN = \int_0^y -ky^2\,dy \Rightarrow N(y) = \frac{-ky^3}{3}$

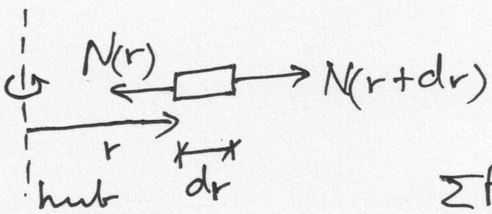


$\delta = \int_0^L \frac{N\,dy}{AE} = \int_0^L \frac{ky^3}{3}\frac{dy}{AE} = \frac{kL^4}{12AE} = \frac{FL}{4AE}$

$\Sigma F_y = 0$ for whole pile $\Rightarrow F = \int_0^L ky^2\,dy = \frac{kL^3}{3}$

3)





$dm = A\frac{\gamma}{g}dr$

$\Sigma F_r: (dm)r\omega^2 = -dN$

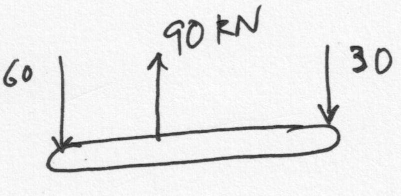
$A\frac{\gamma}{g}\omega^2 \int_L^r r\,dr = -\int_0^{N(r)} dN$

$\Rightarrow A\frac{\gamma}{g}\frac{\omega^2}{2}(r^2 - L^2) = -N(r)$

$\delta = 2\int_0^L \frac{N\,dr}{AE} = 2A\frac{\gamma}{g}\frac{\omega^2}{2}\int_0^L \frac{(L^2 - r^2)}{AE}dr = \frac{\gamma\omega^2}{gE}\left(L^3 - \frac{L^3}{3}\right)$

$= \frac{2}{3}\frac{\gamma\omega^2 L^3}{gE}$

4) Equil:



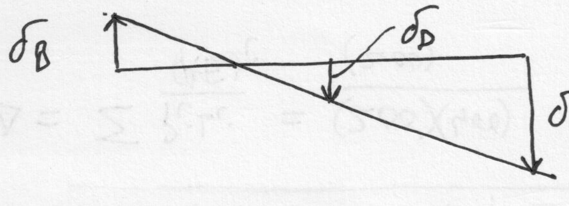
$F_{AB} = \frac{(30)(0.4)}{0.2} = 60\,(C)$

$F_{CD} = (60+30) = 90\,(T)$.

Displacements & compatibility:

$\delta_B = \frac{(60)(0.3)(E6)}{70E3 * 500}(\uparrow)$, $\delta_D = \frac{(90)(0.4)(E6)}{(200E3)(600)}(\downarrow)$

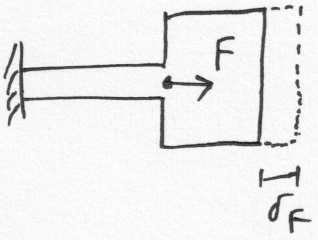
$= 0.5143\,mm$ $\qquad = 0.3\,mm$

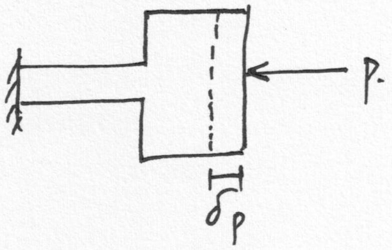


$\delta_E = \frac{\delta_D + \delta_B}{0.2} * 0.4 - \delta_B$

$= 1.1143\,mm = \delta_r$

P5).





$$\delta_F = \frac{F(250)}{950 * 65E3} \quad ; \quad \delta_p = P\left(\frac{250}{(950)(65E3)} + \frac{375}{(1300)(207E3)}\right)$$

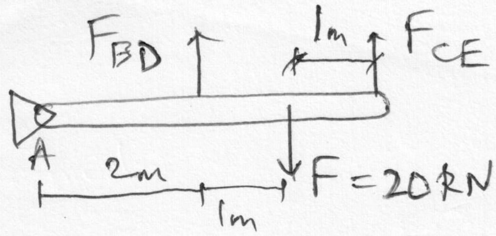
Compatibility: $\delta_F = \delta_p \Rightarrow P = 0.7439F$

$$\sigma_{AL} = 70 \Rightarrow N_{al} = 70 * 950 = F - P = 0.2561F$$

$$\Rightarrow F = 259700$$

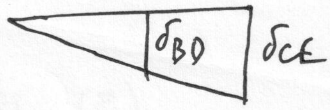
$$\sigma_{St} = \frac{P}{1300} = \frac{0.7439 * 259700}{1300} = 148.61 \text{ (C)}$$

P6



$3\cancel{F} = 2F_{BD} + 4F_{CE} \rightarrow$ 1
$\quad\downarrow$
$\quad 20$

$\dfrac{\delta_{BD}}{2} = \dfrac{\delta_{CE}}{4} \rightarrow$ 2



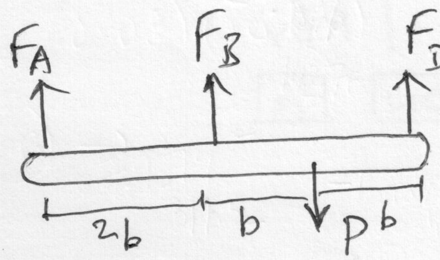
$\dfrac{1}{2}\dfrac{F_{BD}(2)\cancel{(1000)}}{(300)(70\cancel{E3})} = \dfrac{1}{4}\dfrac{F_{CE}(1.5)\cancel{(1000)}}{(130)(200\cancel{E3})} \rightarrow$ 3

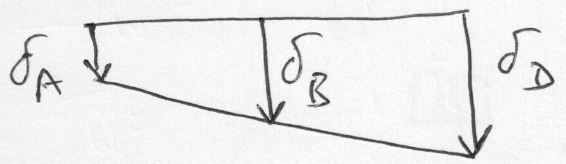
1, 3 $\Rightarrow F_{BD} = 3.94$ kN
$F_{CE} = 13.002$ kN

$\sigma_{BD} = \dfrac{F_{BD}}{300} = 13.13$ MPa

$\sigma_{CE} = \dfrac{F_{CE}}{150} = 86.68$ MPa

P7





$P = F_A + F_B + F_D \longrightarrow$ 1

$F_A(2\cancel{b}) + P(\cancel{b}) = F_D(2\cancel{b}) \rightarrow$ 2

$\dfrac{\delta_B - \delta_A}{\cancel{\alpha}} = \dfrac{\delta_D - \delta_B}{\cancel{\alpha}} \rightarrow$ 3a

$\Rightarrow F_B - F_A = F_D - F_B \rightarrow$ 3 $\left(\because \dfrac{L}{AE}\right.$ same for all wires).

1, 2, 3 $\rightarrow F_A = \dfrac{P}{12}$, $F_B = \dfrac{P}{3}$, $F_D = \dfrac{7P}{12}$

P8 Free thermal expansion $= \delta_T = \left[(11.7\times10^{-6})(250) + (20.9\times10^{-6})(300)\right](50)$ → 1
(say, release at C) (displ of pt. C)
$= 0.45975$

Apply redundant (P)(↑) at C, $\delta_P = P\left[\dfrac{250}{\frac{\pi}{4}(30^2)(200E3)} + \dfrac{300}{\frac{\pi}{4}(50^2)(105E3)}\right]$ 2

(used equil $P_{AB} = P_{BC} = P$ here)

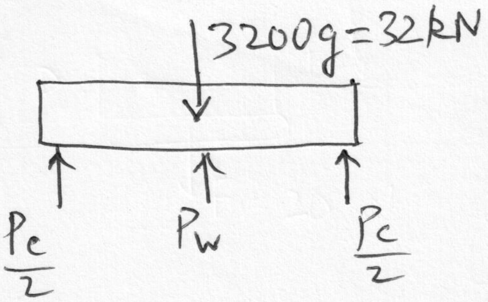
Compatibility → $\delta_T = \delta_P \rightarrow$ 3

1, 2, 3 → $P = 142.995$ kN, $\sigma_{AB} = \dfrac{P}{\frac{\pi}{4}(30)^2} = 202.29$ MPa

$\sigma_{BC} = \dfrac{P}{\frac{\pi}{4}(50)^2} = 72.826$ MPa, $\delta_B = \delta_{AB} = (11.7\times10^{-6})(250)(50) - \dfrac{P(250)}{\pi/4\,(30)^2(200E3)} = 0.14599$ ↓

P9 (a)

assumed all compressive +ve.



$$P_W + P_C = 32 \rightarrow (1)$$

$$\sigma_C = \sigma_W \rightarrow (2)$$

$$\frac{P_C(4)}{2(0.08)(20E6)} = \frac{P_W(2)}{(0.04)(12E6)} \rightarrow (3)$$

(1), (3) $\rightarrow P_W = 7.3841\,kN$, $\frac{P_C}{2} = 12.308\,kN$, $\Rightarrow P_C = 24.6158$, $\sigma_C = \frac{P_C}{2(0.08)} = 0.1538\,MPa$, $\sigma_W = \frac{P_W}{0.04} = 0.1846\,MPa$

0.1538

(b) Only equation (3) changes.

$$\frac{P_C(4)}{2(0.08)(20E6)} + (24E{-6})(4)(20) = \frac{P_W(2)}{(0.04)(12E6)} + (6E{-6})(2)(20) \rightarrow (3a)$$

(1), (3a) $\rightarrow P_W = 317.52\,kN$, $\frac{P_C}{2} = -142.76\,kN \Rightarrow P_C = -285.519$

$\sigma_W = 7.9379\,MPa$, $\sigma_C = -1.7845\,MPa$ (so tensile)

so compressive.

P10 Assume fillets are $\frac{1}{4}$ circles.

At A, $\sigma_{ave} = \frac{\sigma_{max}}{K} = \frac{150}{2.25} = \frac{F}{A} = \frac{F}{(40)(10)} \Rightarrow F = 26.67\,kN$

At B, $\sigma_{ave} = \frac{\sigma_{max}}{K} = \frac{150}{2} = \frac{F}{A} = \frac{F}{(40+10+10-30)(10)} \Rightarrow F = 22.5\,kN$

F_{max} = lower of the two F's = 22.5 kN.