DEPARTMENT OF CIVIL ENGINEERING, IIT BOMBAY

CE 201 Solid Mechanics

Tutorial Sheet = 3 Instructor : A. Laskar/ Naresh K. Chandiramani

1. A 500 mm long, 16 mm diameter rod is observed to increase in the length by 300 μm, and to decrease in the diameter by 2.4 μm when subjected to an axial load of 25 kN. Determine the modulus of elasticity, Poisson's ratio and shear modulus.

2. A piece of 50 x 250 x 10 mm steel plate is subjected to uniformly distributed stresses along its edges as shown in Fig. 1. (a) If P_x = 100 kN and P_y = 200 kN then determine the change in thickness of plate. (b) Determine the P_x alone which will cause the same change in thickness as in (a). Let E=200 GPa and ν=0.25.

3. A structural steel plate with E = 210 GPa and ν=0.3 has the dimension as shown in Fig. 2 before loading. The plate is then subjected to a state of plane stress in xy plane with σ_x=150 MPa. For what value of stress σ_y will the dimension Y of the plate remain unchanged? What are the final dimension of the plate in the other two directions?

4. The steel block shown in Fig. 3 is subjected to a uniform pressure on all its faces. Knowing that the change in length of edge AB is -24 μm determine (a) the change in length of the other two edges, (b) the pressure applied to the faces of the block. Let E=200 GPa and ν=0.29.

5. A circle of diameter 200 mm is scribed on an unstressed 18 mm thick aluminium plate as shown in Fig. 4. Forces acting in the plane of the plate later cause normal stresses σ_x = 85 MPa and σ_z = 150 MPa. Determine the change in (i) the length of diameter AB and CD, (ii) thickness of plate and (iii) volume of plate. Let E=70 GPa and ν=1/3.

6. For the state of plane stress shown in Fig. 5, determine the corresponding ε_x, ε_y and γ_{xy}. E=80 GPa and ν=0.35.

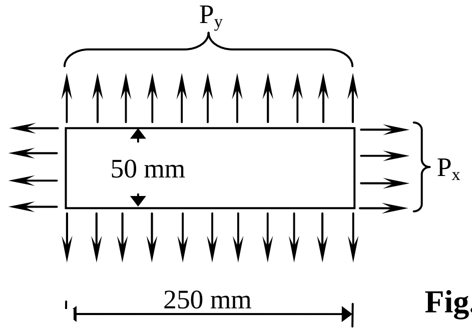

Fig. 1

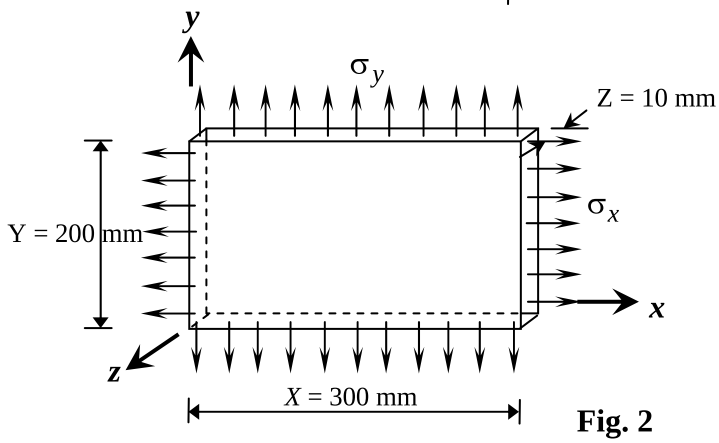

Fig. 2

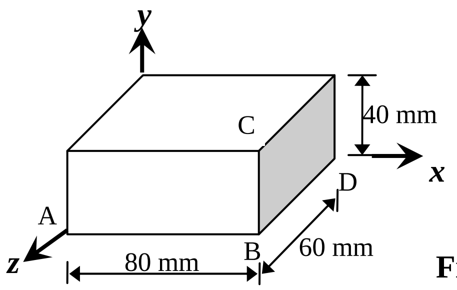

Fig. 3

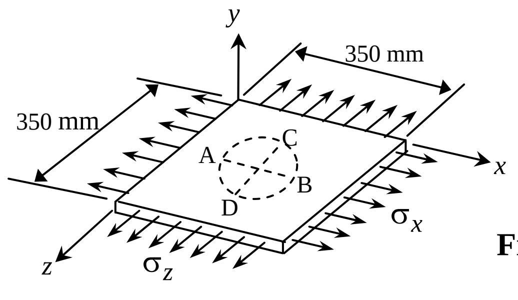

Fig. 4

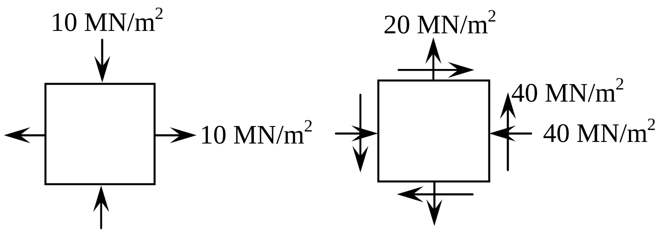

Fig. 5

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P1 $\sigma_x = \dfrac{25*1000}{\frac{\pi}{4}(16^2)} = 124.339\ \text{MPa}$; $\varepsilon_x = \dfrac{300*10^{-3}}{500} = 6*10^{-4}$

$$\varepsilon_y = \varepsilon_z = \frac{-2.4*10^{-3}}{16} = -1.5*10^{-4}$$

$$\varepsilon_x = \frac{\sigma_x}{E} - \frac{\nu}{E}\cancelto{0}{\sigma_y} - \frac{\nu}{E}\cancelto{0}{\sigma_z} \rightarrow ①$$

$$\varepsilon_y = \cancelto{0}{\frac{\sigma_y}{E}} - \frac{\nu}{E}\sigma_x - \frac{\nu}{E}\cancelto{0}{\sigma_z} \rightarrow ② \quad ; \quad \varepsilon_z = \cancelto{0}{\frac{\sigma_z}{E}} - \frac{\nu}{E}\sigma_x - \frac{\nu}{E}\cancelto{0}{\sigma_y}$$

give same eqn.

①, ②, $\rightarrow$ $E = 2.07*10^5\ \text{MPa}$, $\nu = 0.2497$

P2 (a) $\varepsilon_z = \cancelto{0}{\dfrac{\sigma_z}{E}} - \dfrac{\nu}{E}(\sigma_x + \sigma_y)$; $\sigma_x = \dfrac{100}{(50)(10)} = 0.2\ \dfrac{kN}{mm^2}$

Use $E = 200\ kN/mm^2$, $\nu = 0.25$; $\sigma_y = \dfrac{200}{(250)(10)} = 0.08\ \dfrac{kN}{mm^2}$

get $\varepsilon_z = -3.5*10^{-4}$

change in thk $= \varepsilon_z(10)\ mm = -3.5*10^{-3}\ mm$

(b) So ε_z same as above, so new σ_x same as $(\sigma_x + \sigma_y)$ above

So $P_x = 100 + \underbrace{40}_{P_{add}} = 140\ kN$. (ie $\dfrac{P_{add}}{(50)(10)} = \dfrac{200}{(250)(10)}$).

P3 $\varepsilon_y = 0 = \dfrac{\sigma_y}{E} - \dfrac{\nu}{E}\left(\cancelto{150}{\sigma_x} + \cancelto{0}{\sigma_z}\right) \Rightarrow \sigma_y = (0.3)(150) = 45\ \text{MPa}$

$$\varepsilon_x = \cancelto{150}{\frac{\sigma_x}{E}} - \frac{\nu}{E}\left(\cancelto{45}{\sigma_y} + \cancelto{0}{\sigma_z}\right) = \frac{150 - 0.3*45}{210E3} = 6.5*10^{-4}$$

$$\varepsilon_z = \cancelto{0}{\frac{\sigma_z}{E}} - \frac{\nu}{E}\left(\cancelto{150}{\sigma_x} + \cancelto{45}{\sigma_y}\right) = -\frac{195*0.3}{210E3} = -2.786*10^{-4}$$

$X_{new} = (1+\varepsilon_x)\cancelto{300}{X} = 300.195\ mm$, $Z_{new} = (1+\varepsilon_z)\cancelto{10}{Z} = 9.997\ mm$

$Y_{new} = Y_{old} = 200\ mm$

P4 $\varepsilon_x = \dfrac{-24 * 10^{-3}}{80} = \left(\dfrac{1-2\nu}{E}\right)(-P) \Rightarrow P = 0.143$ GPa

(used $\sigma_x = \sigma_y = \sigma_z = -P$)

$\varepsilon_z = \varepsilon_y = \varepsilon_x$ when uniform pressure applied.

$\Delta_y = \varepsilon_y(40) = -0.012$ mm, $\Delta_z = \varepsilon_z(60) = -0.018$ mm.

P5 $\varepsilon_x = \dfrac{85}{70E3} - \dfrac{1}{3}\cdot\left(\dfrac{150}{70E3}\right) = 5*10^{-4}$, $\varepsilon_z = \dfrac{150}{70E3} - \dfrac{1}{3}\left(\dfrac{85}{70E3}\right) = 1.74*10^{-3}$

$(AB)_{new} = (1+\varepsilon_x)(200) = 200.1$ mm, $(CD)_{new} = (1+\varepsilon_z)(200) = 200.348$ mm

$\Delta_{AB} = \varepsilon_x(200) = 0.1$ mm $\Delta_{CD} = \varepsilon_z(200) = 0.348$ mm

$\varepsilon_y = -\dfrac{1}{3}\cdot\left(\dfrac{1}{70E3}\right)(150+85) = -1.119\times 10^{-3}$

$(Thk)_{new} = (1+\varepsilon_y)(18) = 17.979$ mm $\Delta_{thk} = \varepsilon_y(18) = -0.0201$

$(Vol)_{new} = (1+\underbrace{\varepsilon}_{\text{cubical dilatation}})(Vol)_{old} = (1+\varepsilon_x+\varepsilon_y+\varepsilon_z)(350^2)(18) = 2.207*10^6 \text{ mm}^3$

$\Delta_{vol} = \varepsilon(350^2)(18) = 2471.8 \text{ mm}^3$

also, $(Vol)_{new} = \cancel{(AB)_{new}} \underbrace{((1+\varepsilon_x)(350))((1+\varepsilon_y)(18))((1+\varepsilon_z)(350))}_{\text{new length of sides}} = 2.207*10^6$

$\Delta_{vol} = (Vol)_{new} - (Vol)_{old} =$

The two results will be close since higher order terms in $\varepsilon_x, \varepsilon_y, \varepsilon_z$ are small.

P6 (a) $\varepsilon_x = \dfrac{10*10^6}{80*10^9} - \dfrac{0.35}{80*10^9}(-10*10^6) = 1.6875*10^{-4}$

$\varepsilon_y = \dfrac{-10*10^6}{80*10^9} - \dfrac{0.35}{80*10^9}(10*10^6) = -1.6875*10^{-4}$, $\gamma_{xy} = 0$.

(b) $\gamma_{xy} = \tau_{xy}/G = \tau_{xy}*2(1+\nu)/E = 40*10^6 * \dfrac{2(1+0.35)}{80*10^9} = 1.35*10^{-3}$

$\varepsilon_x = -\dfrac{40*10^6}{80*10^9} - \dfrac{0.35}{80*10^9}(20*10^6) = -5.875*10^{-4}$

$\varepsilon_y = \dfrac{20*10^6}{80*10^9} - \dfrac{0.35}{80*10^9}(-40*10^6) = 4.25*10^{-4}$