DEPARTMENT OF CIVIL ENGINEERING, IIT BOMBAY

CE 221 Solid Mechanics

Tutorial Sheet = 4 Instructor : A. Laskar / N. K. Chandiramani

1. A hollow steel shaft with external diameter 150 mm is required to transmit 1MW at 300 rpm. Calculate a suitable internal diameter for the shaft if its shear stress is not to exceed 70 MPa. Compare the torque carrying capacity of this shaft with a solid steel shaft having the same weight per unit length and limiting shear stress. Take G = 80 GPa.

2. A steel shaft consists of a hollow shaft 2m long, with an outside diameter of 100 mm and an inside diameter of 70 mm, rigidly attached to a solid shaft of 1.5 m length and 70 mm diameter. Determine the maximum power that can be transmitted by the shaft at a speed of 100 rpm without exceeding a shear stress of 70 MPa and a twist of 2.5^0 in the 3.5 m length. G = 80 GPa.

3. Compare the torque transmitted by a solid shaft of diameter d and the torque transmitted by a shaft of annular cross-section of the same material with mean diameter d, the weight per unit length being same for the two shafts.

4. A solid tapered circular shaft of length **L** is fixed at one end and free on the other. The diameter of shaft at free end is **d** and increases linearly to **2d** at fixed end. The shaft is subjected to a torque **T** at the free end. Determine the angle of twist at the free end.

5. The solid cylindrical shaft of variable size as shown in mm in Fig. 1 is acted upon by the torque indicated. What is the maximum shearing stress developed in the shaft, and between what two pulleys it occurs. Also determine the relative twist between the end pulleys. Take G = 80 GPa.

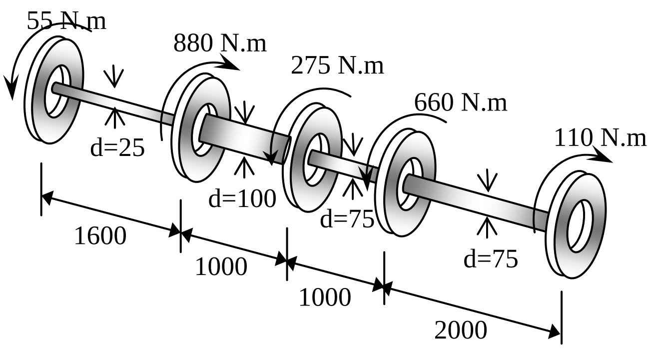

Fig. 1

6. A steel shaft and an aluminium tube are connected to a fixed support and rigid disk as shown in Fig. 2. Determine the maximum torque which may be applied to the disk without exceeding the shearing stresses of 120 MPa and 70 MPa in steel and aluminium tube respectively. Take G = 80 GPa for steel and 27 GPa for aluminium.

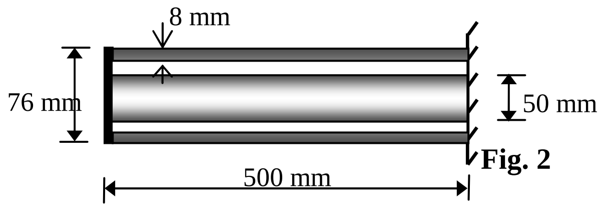

Fig. 2

7. A circular shaft AB consists of a 250 mm long, 20 mm diameter steel cylinder, in which a 125 mm long, 16 mm diameter cavity is drilled from end B (Fig. 3). The shaft is attached to fixed supports at both ends, and a 120 N.m torque is applied at its mid-section Determine the torque exerted by the shaft on both supports.

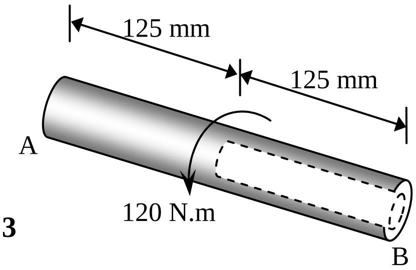

Fig. 3

8. The composite shaft shown in Fig. 4 consists of a 5 mm thick brass jacket (G_b = 39 GPa) bonded to a 40 mm diameter steel core (G_{st} = 77 GPa). The shaft is subjected to a 600 N.m torque, determine (i) the maximum shearing stress in the brass jacket, (ii) the maximum shearing stress in steel core, (iii) the angle of twist of B relative to A.

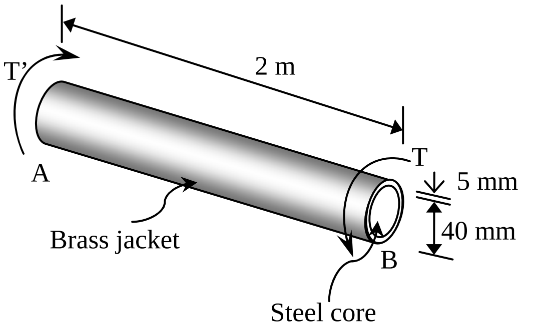

Fig. 4

Torsion Tutorial Sheet # 4

1) $T = \frac{P}{\omega} = \frac{10^6}{300 * \frac{2\pi}{60}} = 31831 \text{ N.m}$

$$\tau_{max} = \frac{T\,(150/2000)}{\frac{\pi}{32}\left(\left(\frac{150}{1000}\right)^4 - d_i^4\right)} = 70*10^6 \Rightarrow d_i = 0.1123 \text{ m}$$

solid shaft, $d = \sqrt{0.15^2 - d_i^2} = 0.0994 \text{ m}$, $\frac{(\tau_{max})_{solid}}{(\tau_{max})_{hollow}} = \frac{d^4\,(0.075)}{(0.15^4 - d_i^4)(d/2)} = 0.4243$

✓ 2) $P_{max} = T_{max}\left(\frac{100 * 2\pi}{60}\right)$

$70 \text{ Mpa} = \tau_{max} = \frac{Tr}{J}$, so shaft with higher $\frac{r}{J}$ is critical for τ_{max} criteria.

$$\frac{2.5\pi}{180} = \theta_{max} = \sum \frac{TL}{GJ} = \frac{T}{\underset{80E9}{G}}\left(\frac{2}{(0.1^4 - 0.07^4)} + \frac{1.5}{0.07^4}\right)\cdot\frac{32}{\pi} \Rightarrow T = 3859.5 \text{ N.m}$$

Tmax from θ criteria

$$\therefore \left(\frac{70/2}{\frac{\pi}{32}70^4}\right)_{\frac{r}{J}\text{ solid}} > \left(\frac{100/2}{\frac{\pi}{32}(100^4 - 70^4)}\right)_{\frac{r}{J}\text{ hollow}}$$, so solid shaft is critical for τ_{max} criteria.

$$T = \frac{(70)\left(\frac{\pi}{32}\cdot 70^4\right)}{(70/2)} = 4714350 \text{ N.mm} = 4714.35 \text{ N.m}$$

Tmax from τ_{max} criteria

Choose lower T, ie $T_{max} = 3859.5 \text{ Nm} \Rightarrow P_{max} = 40416 \text{W}$

✓ 3) unit wt same $\Rightarrow (d_o^2 - d_i^2) = (d^2)_{solid}$

Also given $d = \frac{d_o + d_i}{2} \Rightarrow d_o - d_i = \frac{d^2}{2d} = \frac{d}{2}$

$\Rightarrow d_o = \frac{5}{4}d.$

$d_i = \frac{3}{4}d$

$$\frac{T_{solid}}{T_{hollow}} = \frac{J_{solid}}{d/2}\cdot\frac{d_o/2}{J_{hollow}}$$

$$= \frac{d^4}{d}\cdot\frac{5/4\,d}{\left(\left(\frac{5}{4}\right)^4 - \left(\frac{3}{4}\right)^4\right)d^4} = \frac{10}{17}$$

✓ 4) Same as Ex 3.11 done in class, answer = $\theta = \dfrac{7TL * 16}{12\pi G d^4} = \dfrac{28}{3}\dfrac{TL}{\pi G d^4}$

✓ 5) $\tau_1 = \dfrac{55(1000)(12.5)}{\frac{\pi}{32}(25)^4} = 17.927\,MPa$, $\tau_2 = \dfrac{825(1000)(50)}{\frac{\pi}{32}(100)^4} = 4.202\,MPa$

$\tau_3 = \dfrac{550(1000)(37.5)}{\frac{\pi}{32}(75)^4} = 6.640\ MPa$, $\tau_4 = \dfrac{110(1000)(37.5)}{\frac{\pi}{32}(75)^4} = 1.328\ MPa$

$\tau_{max} = 17.927$ MPa between first two pulleys on left.

$$\theta = \frac{55(1000)(1600)}{80E3\ \frac{\pi}{32}(25)^4} - \frac{825(1000)(1000)}{80E3\ \frac{\pi}{32}(100)^4} - \frac{550(1000)(1000)}{80E3\ \frac{\pi}{32}(37.5)^4}$$

$$+ \frac{110(1000)(2000)}{80E3\ \frac{\pi}{32}(37.5)^4} = 6.386 \times 10^{-3}\,rad$$

(positive CW when looking from 75mm shaft towards 25mm shaft)

✓ 6) Same as class Ex 3.6, only $G = 80\,GPa$ for steel instead of 77 GPa.
(so replace 77 → 80 in class notes)

Answer $T = T_S + T_A = 2945.2 + 3244.78 = 6189.98$ N.m.

✓ 7) same as class notes, slide 22.

$$T_A = 120\left(1 + \frac{L_1 J_2}{L_2 J_1}\right)^{-1} = 120\left(1 + \frac{(125)(20^4 - 16^4)}{(125)(20)^4}\right)^{-1} = 75.45\ Nm$$

$T_B = 120 - T_A = 44.55$ Nm

✓ 8) Same as Ex (3.8) of slides, only data changed.

$600 = T_{Br} + T_S$; $\theta_{Br} = \theta_S \Rightarrow \dfrac{T_S(2)(1000)}{(77)\frac{\pi}{32}(40)^4} = \dfrac{T_{Br}(2)(1000)}{(39)\frac{\pi}{32}(\ \ ^4 - 40^4)}$

$T_{Br} = 253.19$ N.m, $T_S = 346.81$ N.m

$(\tau_{Br})_{max} = \dfrac{T_{Br}(1000)(2\ \ )}{\frac{\pi}{32}(\ \ ^4 - 40^4)} = 17.47$ MPa

$(\tau_S)_{max} = \dfrac{T_S(1000)(20)}{\pi/32\,(40)^4} = 27.598$ MPa, $\theta_{B/A} = \theta_{Br} = \theta_S = 0.0358$ rad