DEPARTMENT OF CIVIL ENGINEERING, IIT BOMBAY

CE 201 Solid Mechanics

Tutorial Sheet = 5 Instructor: A. Laskar/N. K. Chandiramani

1. For the various beams shown in Fig. 1, draw the shear force and bending moment diagrams indicating salient features such as nature, max values and points of contraflexure etc. Also, find the ratio a/L for minimum value of the maximum bending moment (if applicable).

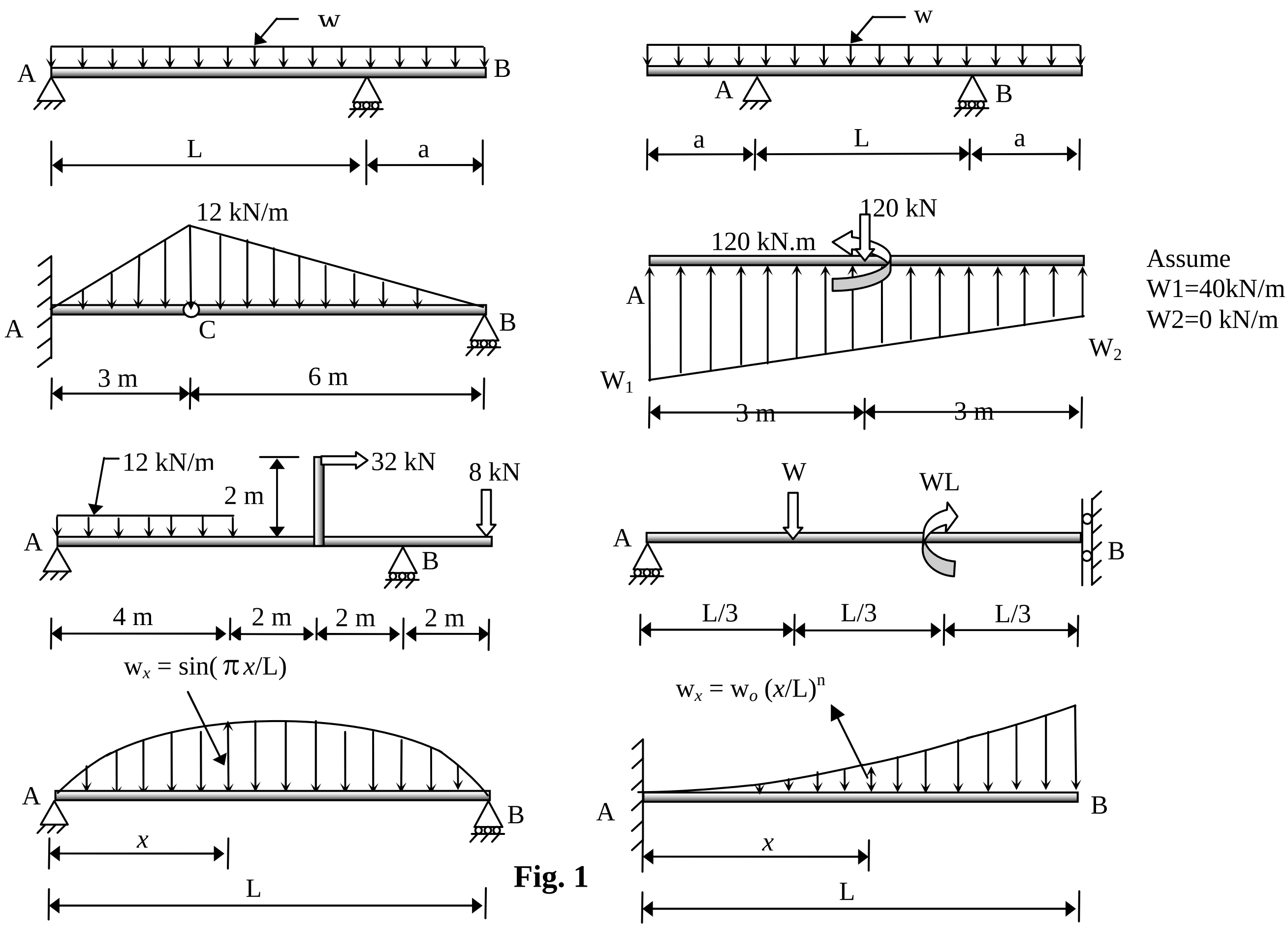


Fig. 1

2. Shear force diagram for simply supported beams at point A and B is shown in Fig. 2. Draw the diagram of beam along with its loading and corresponding bending moment diagram.

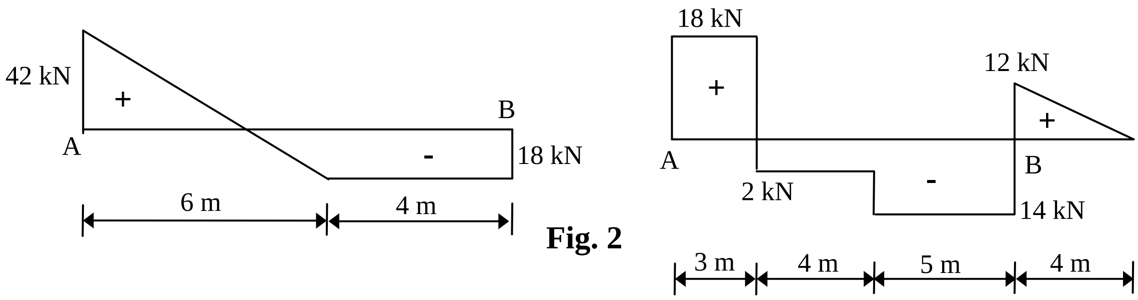
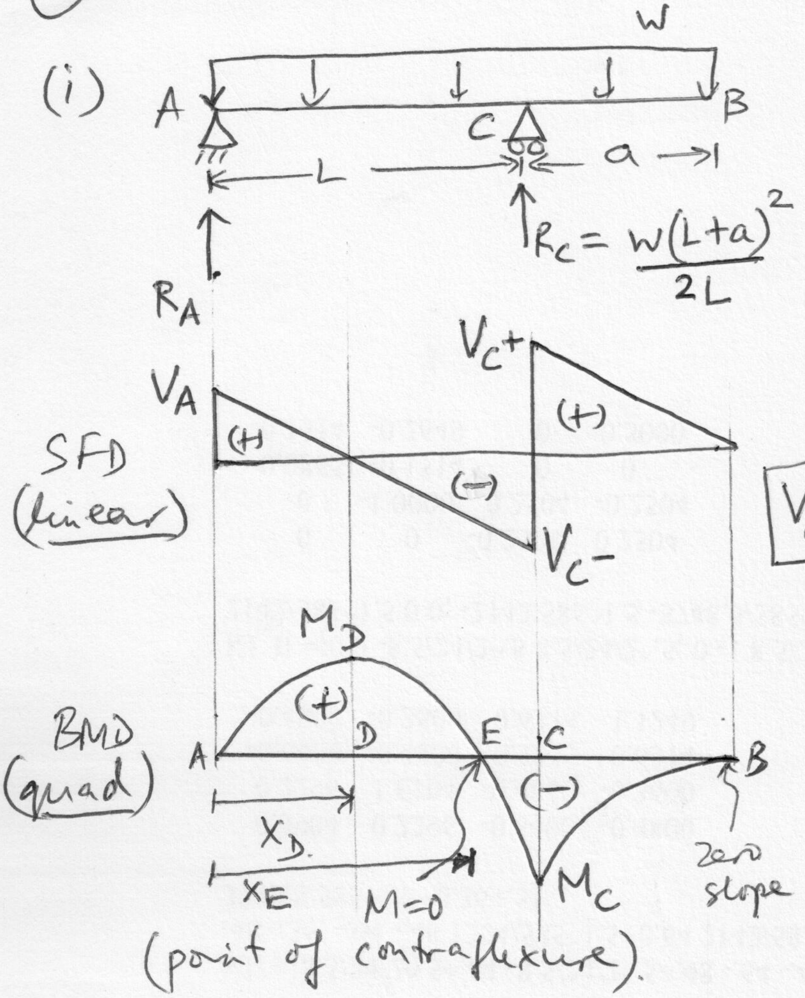


Fig. 2

TUTORIAL # 5 CE221

(i)



$V' = -w$

$\boxed{V_{C^+} = wa}$

$\boxed{V_{C^-}} = wa - w\left(\frac{L}{2} + \frac{a^2}{2L} + a\right) = \boxed{-w\left(\frac{L}{2} + \frac{a^2}{2L}\right)}$

$V_A = V_{C^-} + wL$ (from $V_{C^-} - V_A = \int_0^L -w\,dx$)

$\boxed{V_A = \frac{wL}{2} - \frac{wa^2}{2L}} > 0$ iff $a < L$

$M' = V$

$\boxed{M_C = -\frac{wa^2}{2}}$

At D, $V = 0 \Rightarrow M =$ local extremum

$M_D = \underbrace{M_A}_{0} + \underbrace{\int_0^{x_D} V\,dx}_{\text{area under SFD from A to D.}}$

To find x_D, $\boxed{x_D} = \left(\frac{V_A}{V_A - V_{C^-}}\right)L = \left(\frac{\frac{wL}{2} - \frac{wa^2}{2L}}{wL}\right)L = \boxed{\frac{1}{2}\left(1 - \frac{a^2}{L^2}\right)L}$

So $\boxed{M_D =} \frac{1}{2} V_A x_D$ (area under SFD) $= \boxed{\frac{1}{2}\frac{wL}{2}\left(1 - \frac{a^2}{L^2}\right)^2 \frac{L}{2}}$

To find x_E (pt of contraflexure), when area under SFD equals zero, ie +ve & −ve areas in triangles cancel out, we have $M_E = \underbrace{M_A}_{0} + \underbrace{\text{area under SFD}}_{0} = 0$

$\Rightarrow \boxed{x_E = 2x_D}$.

When $M_D = |M_C|$, max BM is minimum in value.

$\Rightarrow \frac{wL^2}{8}\left(1 - \frac{a^2}{L^2}\right)^2 = \frac{wa^2}{2} \rightarrow \frac{a}{L} = \sqrt{2} - 1.$

Can also write expression for M for $0 \le x \le L$ to get M_D & x_E.

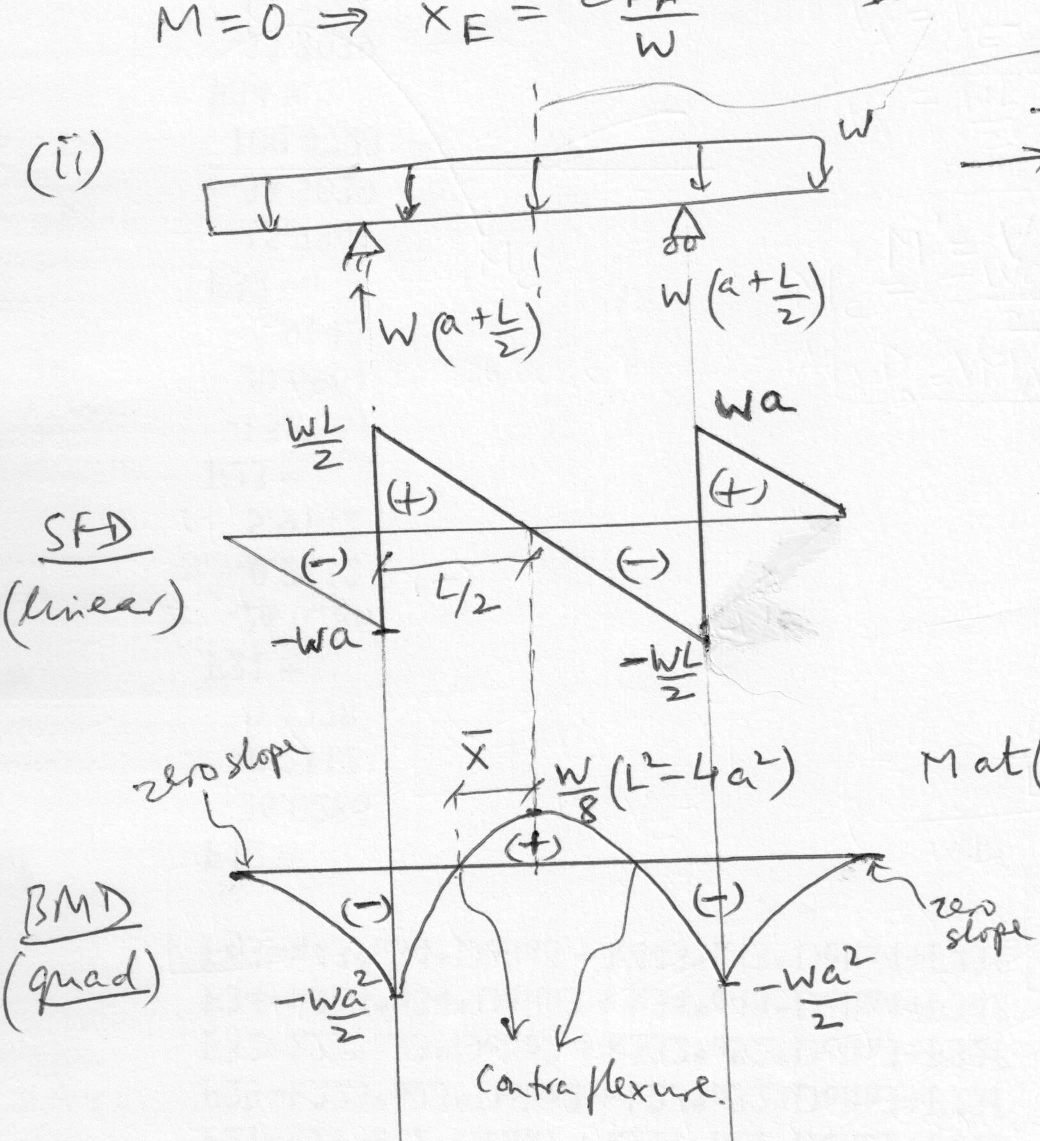
$$M = R_A x - \frac{w x^2}{2} \Rightarrow \frac{dM}{dx} = 0 = R_A - w x \Rightarrow x_D = \frac{R_A}{w}$$

$$R_A = \frac{1}{L}\,\frac{w(L^2-a^2)}{2} \quad \text{from equilibrium} = V_A \quad \text{(check)}$$

$$x_D = \frac{L^2-a^2}{2L} \quad \text{(check) as before}$$

$$M = 0 \Rightarrow x_E = \frac{2R_A}{w} = 2x_D.$$

(ii)



→ Extra (not required)

Whenever structure symmetric about a line, & loading symmetric about same line, SFD antisymmetric & BMD symmetric about that line.

M at $\left(a+\frac{L}{2}\right)$ = area under left two Δ,

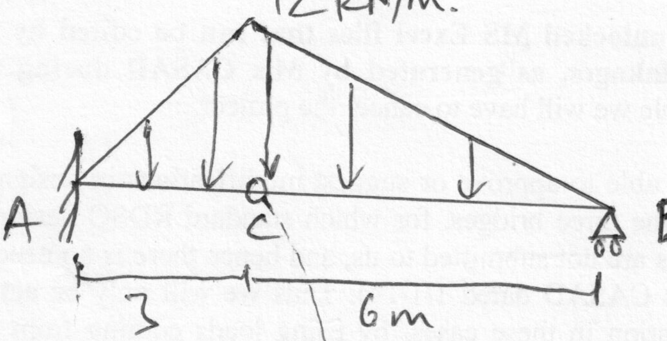
$$= -\frac{wa^2}{2} + \frac{wL^2}{8}$$

For min value of max BMD, $\frac{w}{8}(L^2-4a^2) = \frac{wa^2}{2} \Rightarrow \frac{a}{L} = \frac{1}{\sqrt{8}}$

For Contraflexure point, $\frac{wa^2}{2} = \frac{1}{2}\left(\frac{wL}{2} + \frac{w\bar{x}}{2}\right)\left(\frac{L}{2} - \bar{x}\right)$

and symmetric on right half. Or we can also do M equation but that is more tedious.

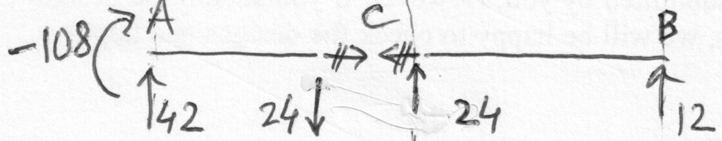
(iii)

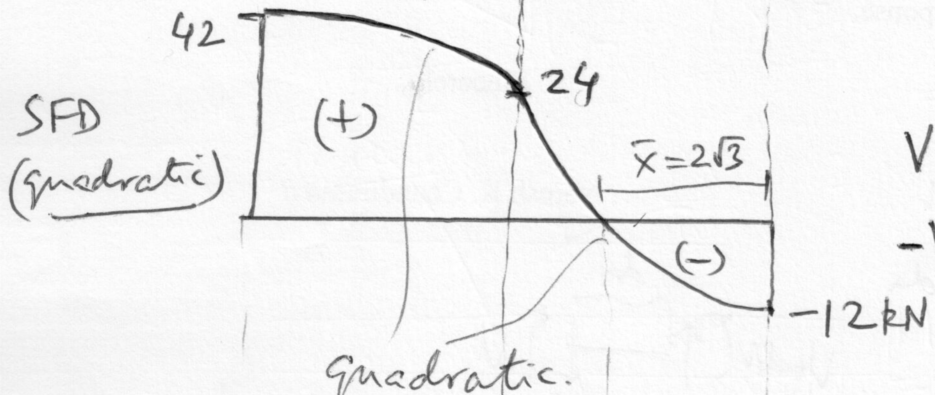


Take moment about hinge C to get R_B.

$$R_B = \frac{1}{6}\left(\frac{1}{2}\cdot 6\cdot 12\cdot \frac{6}{3}\right) = 12$$

$$M_A = -24\cdot 3 - \frac{1}{2}\cdot 12\cdot 3\cdot \frac{2}{3}\cdot 3 = -108$$





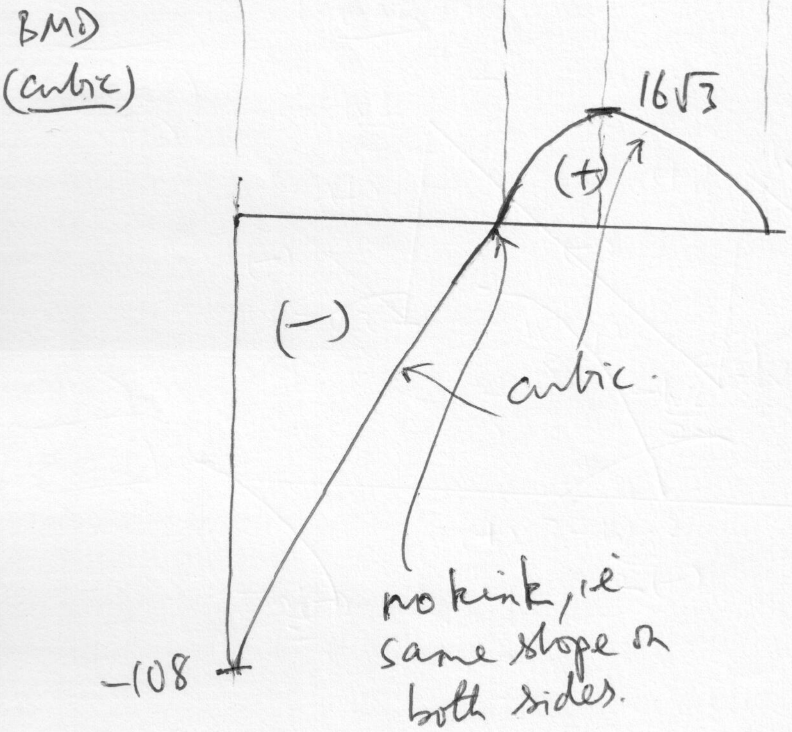
$V' = -w$

$$-V(x) + V_B = \int_x^{x_B} -w\,dx = \underbrace{-\frac{1}{2}\bar{x}\,\frac{12}{6}\bar{x}}_{\text{area of }\Delta}$$

$\bar{x}$ measured from right end B.

So $V(x) = 0 = V_B + \bar{x}^2 = -12 + \bar{x}^2$

$\Rightarrow \bar{x} = 2\sqrt{3}.$



$\cancel{M_B}_{0} - M(\bar{x})$ = area between $\bar{x}$ and x_B in SFD

Note area is −ve until $\bar{x} = 2\sqrt{3}$. $\Rightarrow M(\bar{x})$ +ve

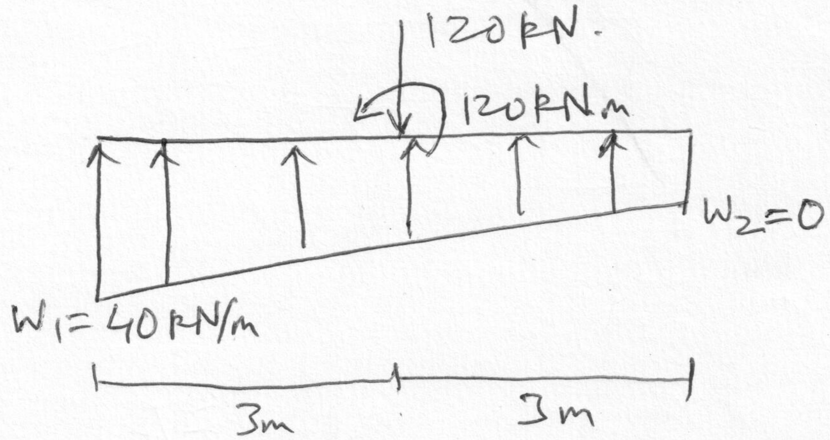
$M' = V$, so $M_{extremum}$ at $V = 0$

M_{max} at $\bar{x} = 2\sqrt{3}$,

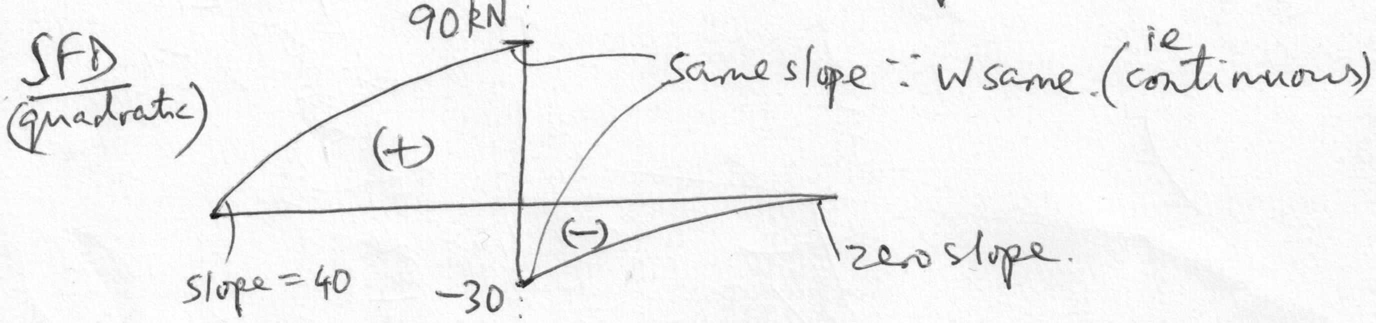
$$M_{max} = R_B\cdot 2\sqrt{3} - \frac{1}{2}\cdot 2\sqrt{3}\cdot 2\cdot 2\sqrt{3} * \frac{2\sqrt{3}}{3}$$

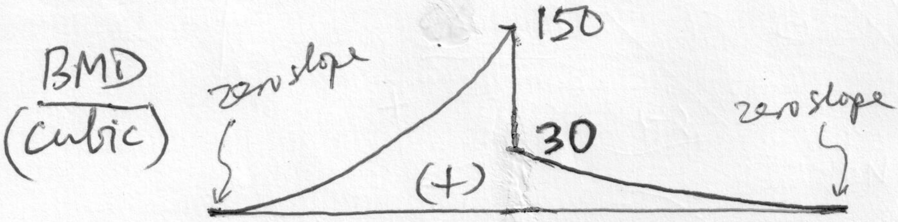
$$= 16\sqrt{3}$$

(iv)



$V' = -W.$

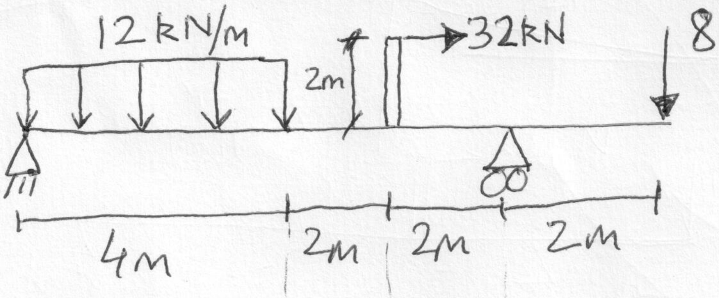




$$M_{x=3m} = \frac{1}{2}.20.3^2 + \frac{1}{2}.3.20.\frac{2}{3}.3$$

$$= 150$$

(v)



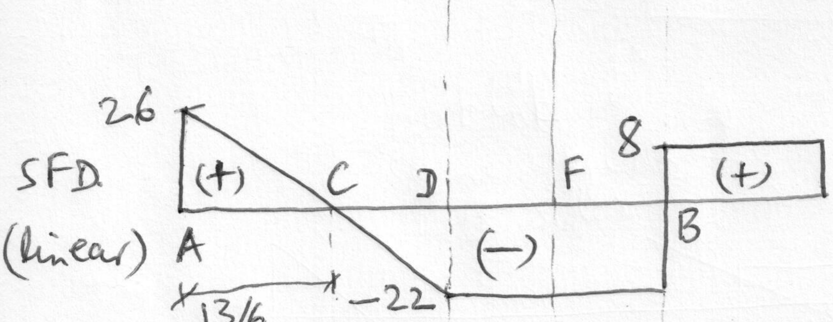
$$R_A = \frac{1}{8}\left(-8.2 + 12.4.6 - 32.2\right)$$

$$= 26$$

$$R_B = 12.4 + 8 - 26 = 30$$

$$V_C = 26 - 12x_C = 0$$

$$x_C = 13/6$$



$$M_C = \underbrace{M_A}_{0} + \frac{1}{2}.26.\frac{13}{6} = 28.167$$

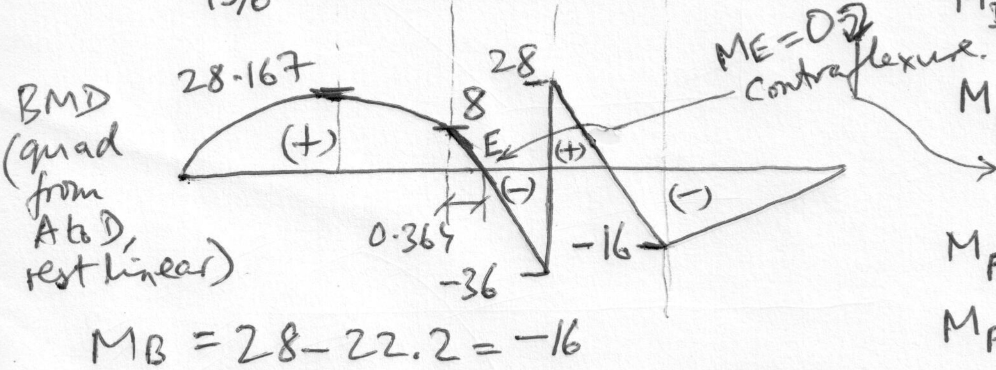
$$M_D = M_C - \frac{1}{2}.22.\left(4 - \frac{13}{6}\right) = 8$$

$$M_E = M_D - 22\,(x_E - 4) = 0$$

$$x_E = 4.364$$

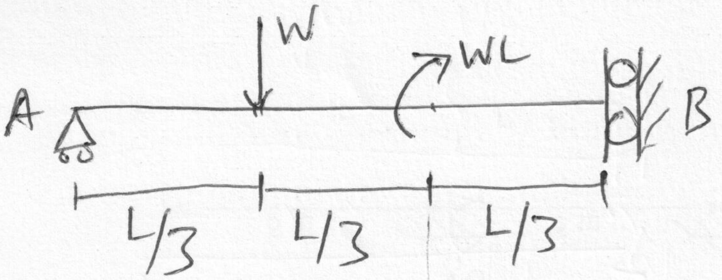
$$M_F = M_D - 22.2 = -36$$

$$M_F^+ = -36 + 32.2 = 28$$

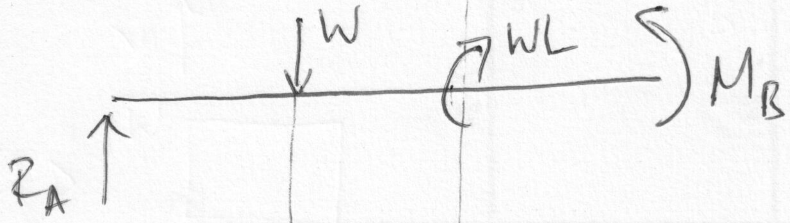


$$M_B = 28 - 22.2 = -16$$

(vi)

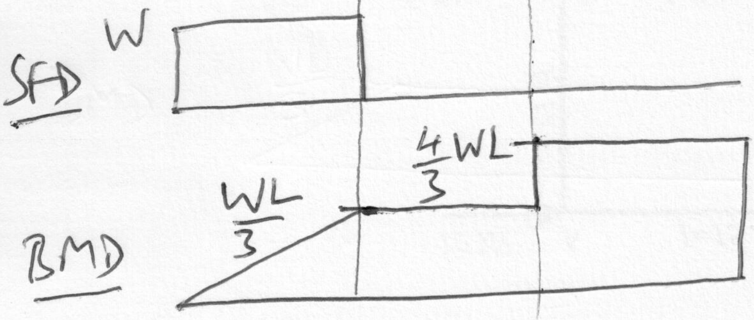


Support at B is fixed in rotation but free in vertical translation.

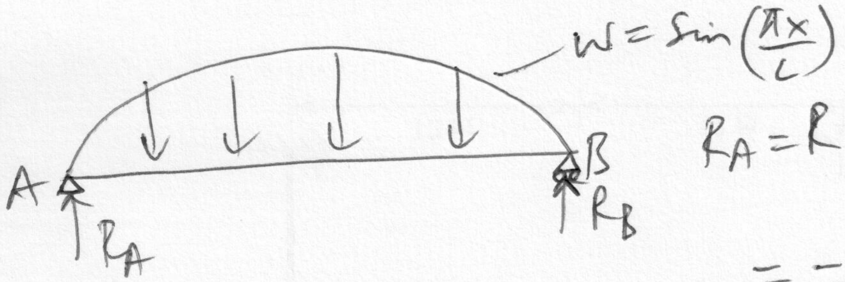


$$M_B = W\frac{L}{3} + WL = \frac{4}{3}WL$$

$$R_A = W$$

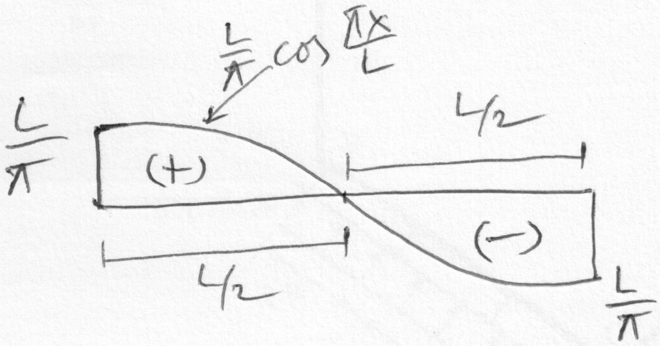


(vii)



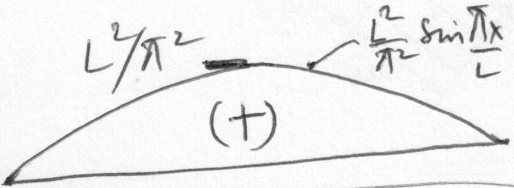
$$R_A = R_B = \frac{1}{2}\int_0^L \sin\frac{\pi x}{L}\,dx = \left[-\frac{1}{2}\frac{L}{\pi}\cos\frac{\pi x}{L}\right]_0^L$$

$$= -\frac{1}{2}\frac{L}{\pi}(-1-1) = \frac{L}{\pi}$$



$$V - V_A = -\int_0^x \sin\frac{\pi x}{L} = +\frac{L}{\pi}\left[\cos\frac{\pi x}{L}\right]_0^x$$

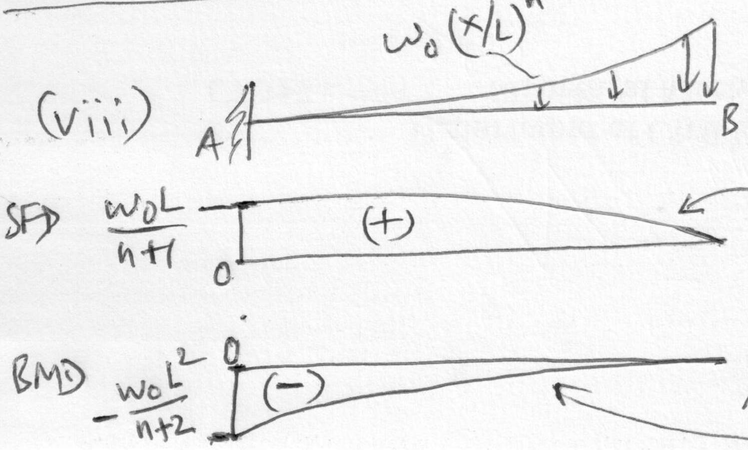
$$V = \frac{L}{\pi} - \left(1 - \cos\frac{\pi x}{L}\right)\frac{L}{\pi} = \frac{L}{\pi}\cos\frac{\pi x}{L}$$



$$M = \cancel{M_A}^{\,0} + \int_0^x V\,dx = \frac{L^2}{\pi^2}\left[\sin\frac{\pi x}{L}\right]_0^x$$

$$= \frac{L^2}{\pi^2}\sin\frac{\pi x}{L}$$

(viii)



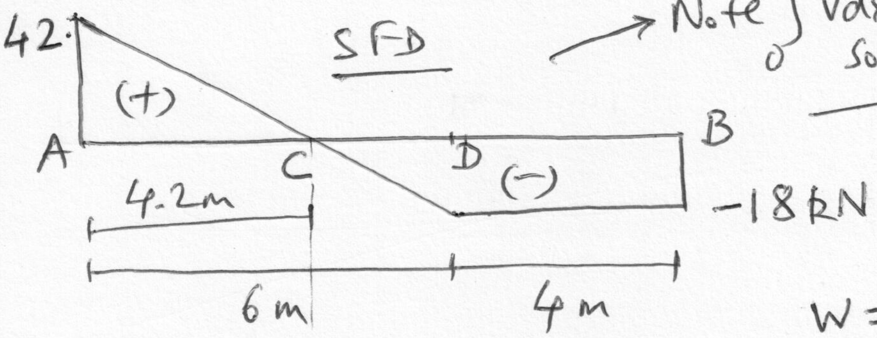
$$\cancel{V_B}^{\,0} - V(x) = -\int_x^L w_0\left(\frac{x}{L}\right)^n dx$$

$$V(x) = \frac{w_0}{n+1}\left(L - \frac{x^{n+1}}{L^n}\right)$$

$$\cancel{M_B}^{\,0} - M(x) = \frac{w_0}{n+1}\left[Lx - \frac{x^{n+2}}{(n+2)L^n}\right]_x^L = \frac{w_0}{n+1}\left[L^2\frac{(n+1)}{(n+2)} - Lx + \frac{x^{n+2}}{(n+2)L^n}\right] = -M(x)$$

(2)

(i)

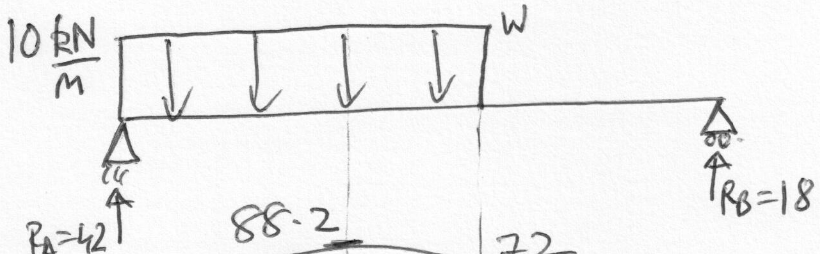


Note $\int_0^L V dx = 0$ in this case
So $M_{x=0} = M_{x=L}$

$X_C = \left(\frac{42}{42+18}\right) \cdot 6 = 4.2$

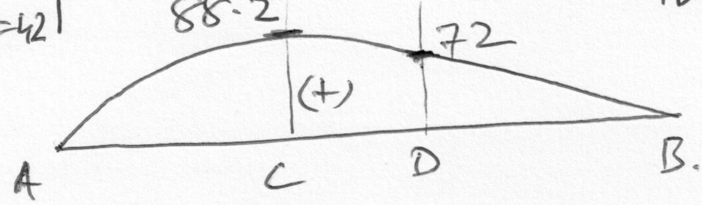
$W = -(\text{slope of SFD})$
$= \frac{(42+18)}{6} = 10 \text{ kN/m}$

Loading



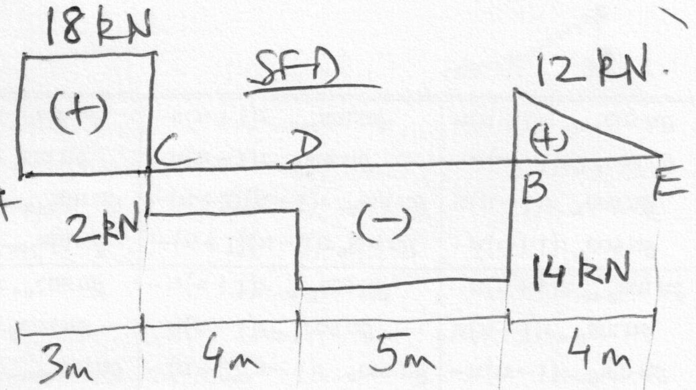
$M_C = \cancel{M_A}^{0} + \frac{1}{2} \cdot 42 \cdot (4.2)$
$= 88.2 \text{ kN.m}$

BMD
(quadratic A to D, linear D to B)



$M_D = \cancel{M_D}^{88.2} - \frac{1}{2} \cdot 18 (6-4.2)$
$= 72 \text{ kN.m.}$
$M_B = 72 - 4 \times 18 = 0.$

(ii)

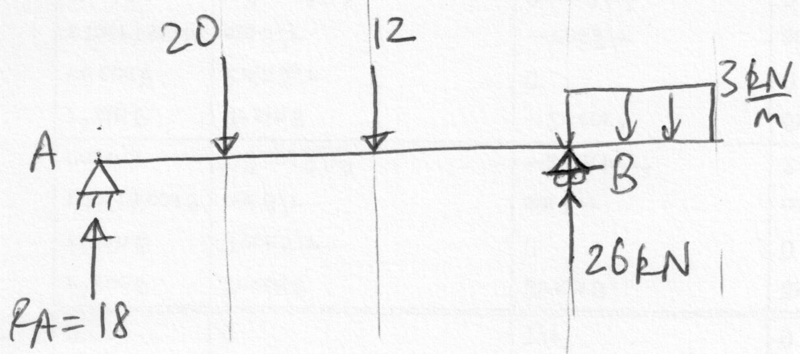


Here also $\int_0^L V dx = 0 \Rightarrow M_{x=0} = M_{x=L}$

Point load/reaction = amount of discontinuity (jump/fall) in SFD

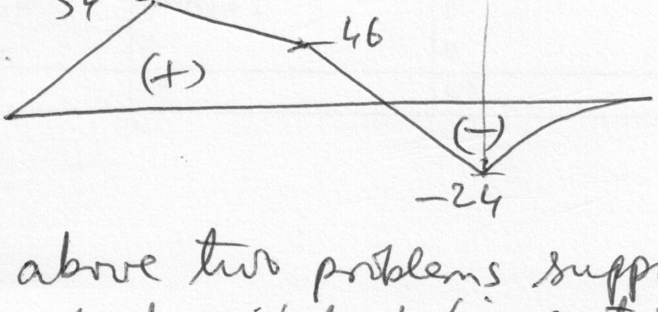
$W = \frac{12}{4} = 3$

Loading



$M_C = 18.3 = 54$
$M_D = 54 - 2.4 = 46$
$M_B = 46 - 5.14 = -24$
$M_E = -24 + \frac{1}{2} \cdot 12 \cdot 4 = 0$

BMD



Note: in above two problems supports can be replaced by equivalent applied point loads (ie reactions). Also in problem 2(ii) supports can be placed wherever any two point loads occur, ie A, B not unique