DEPARTMENT OF CIVIL ENGINEERING, IIT BOMBAY

CE 221 Solid Mechanics

Tutorial Sheet = 6 Instructor :A. Laskar/ N. K. Chandiramani

1. The sections of the beam shown in Fig. 1 are subjected to a positive bending moment of 20 kN.m. Determine (i) the neutral axis and the moment of inertia of the cross-section, (ii) the stresses in extreme fibers, (iii) the resultant compressive force above neutral axis, (iv) the resultant tensile force below neutral axis, and (v) the lever arm of the couple.

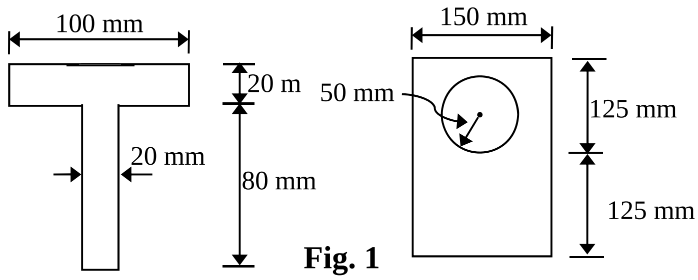


Fig. 1

2. A solid steel beam (Fig. 2) was loaded in laboratory in pure bending about a horizontal neutral axis. Strain measurements showed that the top fibers contracted 0.0003 m/m longitudinally, the bottom fibers elongated 0.0006 m/m longitudinally. Determine the total normal force which acted on the shaded area indicated in the figure. E = 200 GPa.

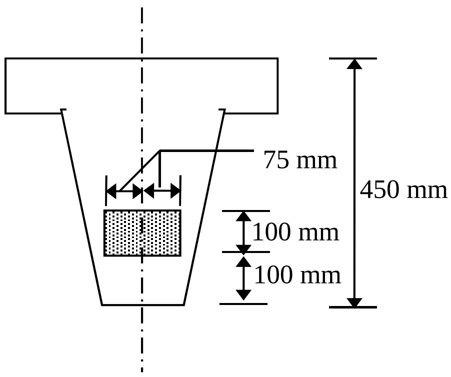
3. A portion of the square bar is removed by milling as shown in Fig. 3. Determine the ratio h/h_0 for which the section has the maximum moment carrying capacity about its horizontal neutral axis.

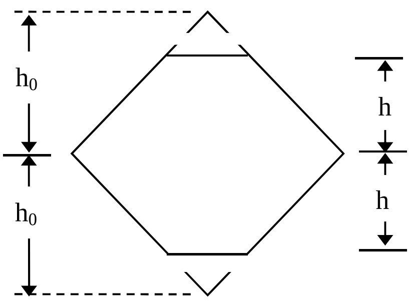
4. A trapezoidal beam section has depth **d,** the top width **b** and bottom width as α**b**. Determine the value of α if the ratio of maximum stress at top to bottom is 1.5.

5. A steel beam in the shape of T has been strengthened by securing bolting to it the two oak timbers shown in Fig. 4. The modules of elasticity is 12.5 GPa for the wood and 200 GPa for the steel. Draw the bending stress diagram of the beam due to positive bending moment 50 kNm.

6. A rectangular beam (b = 60 mm, d = 120 mm) is made of a plastic for which the modules of elasticity in tension is one-half of its value in compression. Determine the maximum tensile and compressive stress due to moment of 5 kN.m.

7. A rectangular beam (b = 100 mm, d = 180 mm) is made of an alloy for which the stress-strain relationship, in both tension and compression may be represented by the relation $\varepsilon = k\sigma^3$. Determine the maximum stress due to moment of 60 kN.m.





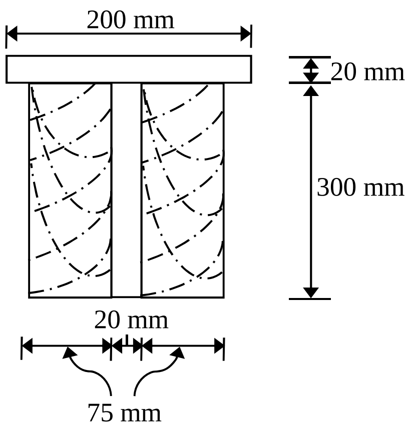
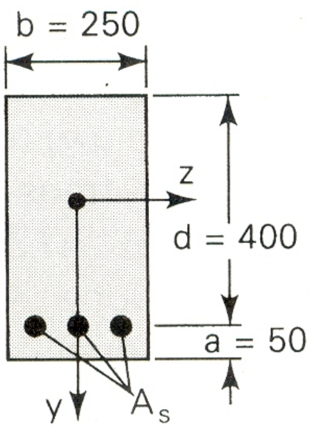


Fig. 2 **Fig. 3** **Fig. 4**

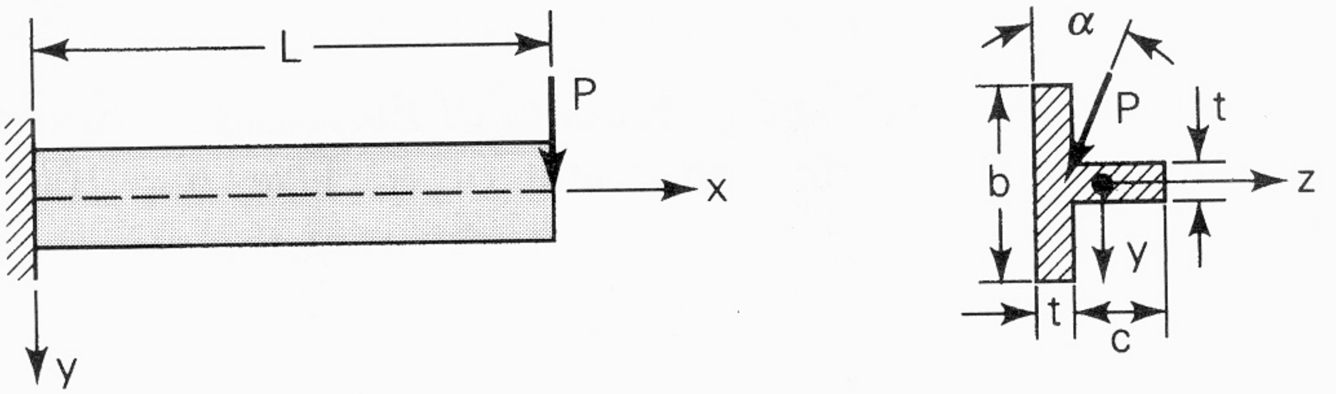
A concrete beam of width $b = 250$ mm and *effective depth* $d = 400$ mm is reinforced with three steel bars providing a total cross-sectional area $A_s = 1000\ \text{mm}^2$ (Fig. 5.14a). Dimensions are given in millimeters. Note that it is usual for an approximate *allowance* $a = 50$ mm to be used to protect the steel from corrosion and fire. Let $n = E_s/E_c = 10$. Calculate the maximum stresses in the materials produced by a *negative* bending moment of $M = 60\ \text{kN} \cdot \text{m}$.

Remark : As per our convention, use Y and Z axis are in opposite direction than what is shown in the Figure below, and therefore, assume positive bending moment



. A concentrated load P acts on a cantilever as shown in Fig. P5.6. The beam is constructed of a 2024-T4 aluminum alloy having a yield strength

$\sigma_{yp} = 290$ MPa, $L = 1.5$ m, $t = 20$ mm, $c = 60$ mm, and $b = 80$ mm. Based on a factor of safety $n = 1.2$ against initiation of yielding, calculate the magnitude of P for (a) $\alpha = 0°$ and (b) $\alpha = 15°$. Neglect the effect of shear in bending and assume that beam twisting is prevented.



Remark : As per our convention, use Y and Z axis are in opposite direction than what is shown in the Figure below, and therefore, assume positive bending moment

Answer: for 0^0 P=3.6 kN

For 15^o P=3.36 kN

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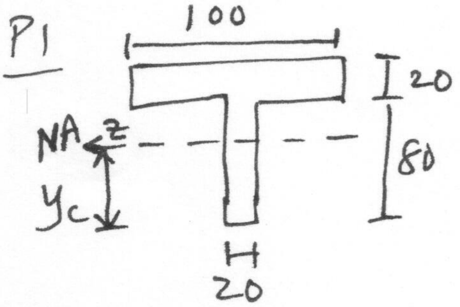
P1



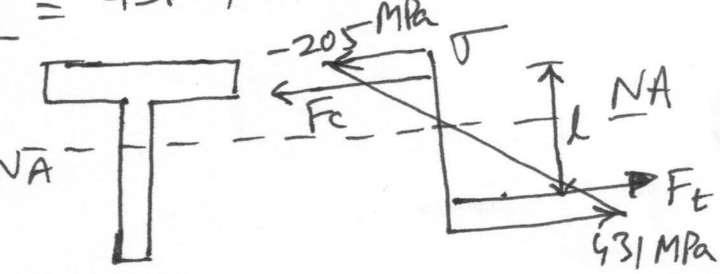
$$y_c = \frac{(80)(20)(40) + (100)(20)(90)}{(80)(20) + (100)(20)} = 67.78\,\text{mm}$$

$$I_z = \frac{(20)(80^3)}{12} + (20)(80)(y_c - 40)^2 + \frac{(100)(20^3)}{12} + (100)(20)(90 - y_c)^2$$

$$= 3142222\,\text{mm}^4$$

$$(\sigma_c)_{max} = \sigma_{top} = \frac{-(20E6)(100 - y_c)}{I_z} = -205.1\,\text{MPa} = -205.1\,\text{N/mm}^2$$

$$(\sigma_t)_{max} = \sigma_{bot} = \frac{-(20E6)(-y_c)}{I_z} = 431.4\,\text{MPa} = 431.4\,\text{N/mm}^2$$

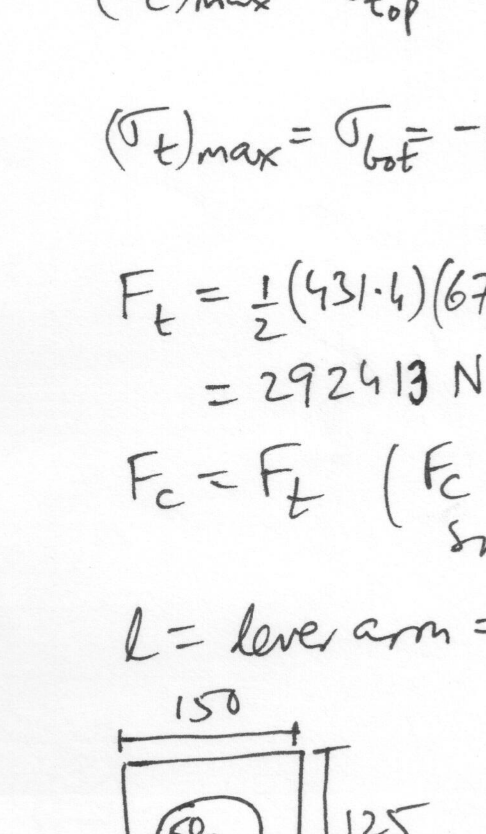


$$F_t = \tfrac{1}{2}(431.4)(67.78)(20)$$

$$= 292413\,\text{N}$$

$F_c = F_t$ (F_c direct calculation more cumbersome since width changes).

$$l = \text{lever arm} = \frac{M}{F_t} = \frac{M}{F_c} = 68.4\,\text{mm}$$



$$0 = \left\{(150)(250) - \frac{\pi}{4}(100^2)\right\}(-y_c) + \frac{\pi}{4}(100^2)(50)$$

$$y_c = 13.25\,\text{mm}$$

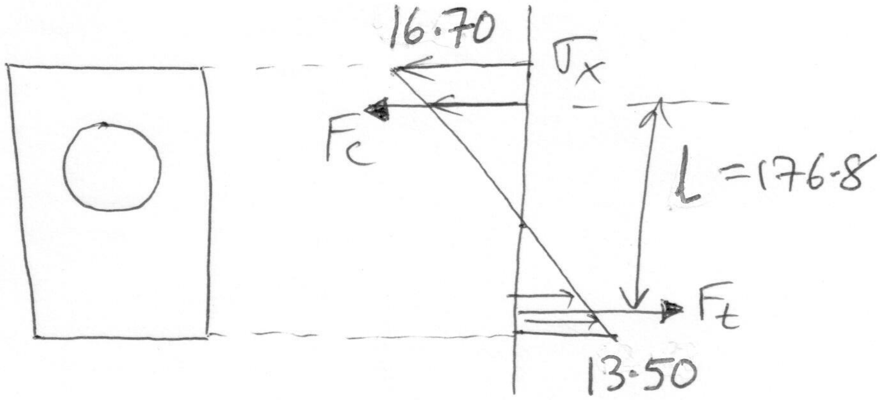
$$I_z = \frac{(150)(250^3)}{12} + (150)(250)\,y_c^2 - \left[\frac{\pi(100^4)}{64} + \frac{\pi(100^2)}{4}(50 + y_c)^2\right]$$

$$= 165567011\,\text{mm}^4$$

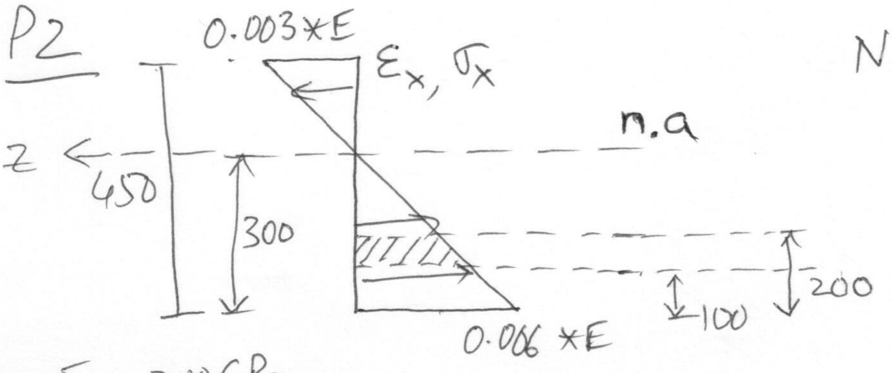
$$(\sigma_c)_{max} = \sigma_{top} = \frac{-(20E6)(125 + y_c)}{I_z} = -16.70\,\text{MPa}$$

$$(\sigma_t)_{max} = \sigma_{bot} = \frac{-(20E6)(-[125 - y_c])}{I_z} = 13.50\,\text{MPa}$$

$F_c = F_t$, $F_t = \frac{1}{2}(13.50)(125 - y_c)(150) = 113139\,\text{N}$; $l = \frac{M}{F_t} = \frac{M}{F_c} = 176.8\,\text{mm}$



P2



$E = 200 GPa$

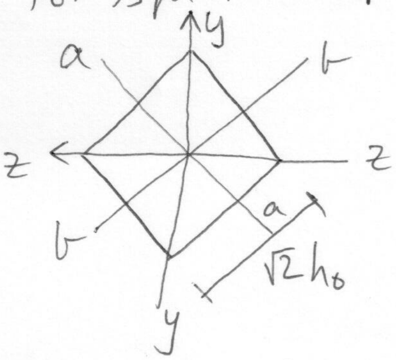
$N = F_t$ = area of trapezium $* b$, $b = 150$

$$= \frac{1}{2} E (0.002 + 0.004)(100) * (150)$$

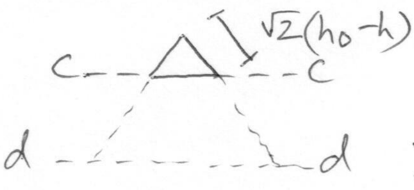
$$= 9 MN.$$

P3 $\sigma_{all} = \dfrac{M_{max}}{S_{max}}$, so we want S_{max} condition.

For square bar $I_a = I_b = I_y = I_z = \dfrac{(\sqrt{2}\, h_0)^4}{12}$



For milled triangular portion,



$$I_c = \frac{1}{2} \frac{[\sqrt{2}(h_0 - h)]^4}{12}$$

$$I_d = I_c - \frac{[\sqrt{2}(h_0-h)]^2}{2}\left[\frac{1}{3}(h_0 - h)\right]^2 + \frac{[\sqrt{2}(h_0-h)]^2}{2}\left[h + \frac{1}{3}(h_0 - h)\right]^2$$

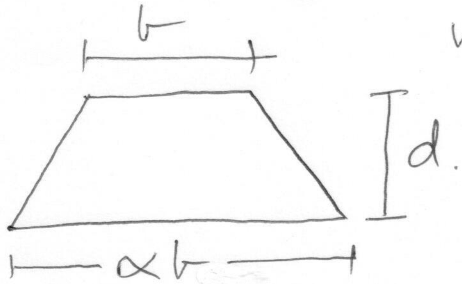
$$I = I_z - 2I_d$$

$$= \frac{4h_0^4}{12} - 2\Big[(h_0^4 - 4h_0^3 h + 6h_0^2 h^2 - 4h_0 h^3 + h^4)\left(\frac{1}{6}\right) + (h_0^2 h^2 - 2h_0 h^3 + h^4)\left(\frac{1}{3}\right) + (h_0^3 h - 2h_0^2 h^2 + h_0 h^3)\left(\frac{2}{3}\right)\Big] = +\frac{4}{3} h_0 h^3 - h^4$$

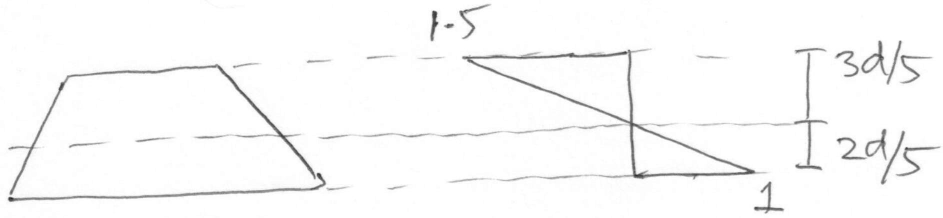
$$S = \frac{I}{h} = \frac{4}{3} h_0 h^2 - h^3, \quad \frac{dS}{dh} = 0 \Rightarrow \frac{8}{3} h_0 h - 3h^2 = 0 \Rightarrow h = \frac{8}{9} h_0$$

P4

③



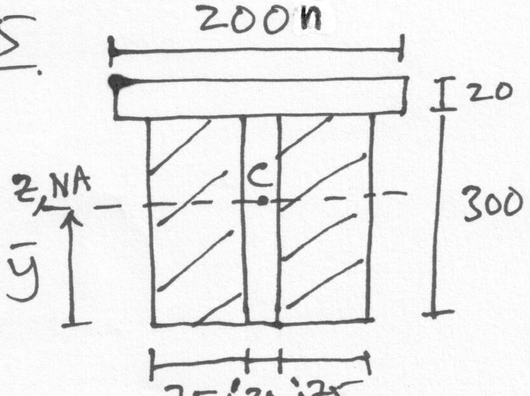
we know that $\alpha > 1$, $\because \frac{\sigma_{top}}{\sigma_{bot}} = 1.5 > 1$.



$$y_c = \frac{bd\frac{d}{2} + \frac{1}{2}b(\alpha-1)d.\frac{d}{3}}{\frac{b}{2}(\alpha+1)d} = \frac{d + (\alpha-1)\frac{d}{3}}{\alpha+1} = \frac{2d}{5}$$

$$\Rightarrow 2\alpha + 2 = 5 + \frac{5}{3}\alpha - \frac{5}{3} \Rightarrow \alpha = 4.$$

P5.



Transformed section. (full wood).

$$n = \frac{E_s}{E_w} = \frac{200}{12.5} = 16$$

For transformed section,

$$\bar{y} = \frac{\left\{(150)(300)(150) + (20n)(300)(150) + (200n)(20)(310)\right\}}{(150)(300) + (20n)(300) + (200n)(20)}$$

$$= 199.95 \text{ mm}$$

$$I_z = \frac{(150+20n)(300)^3}{12} + (150+20n)(300) * (199.95-150)^2 + \frac{(200n)(20)^3}{12} + (200n)(20)(310-199.95)^2$$

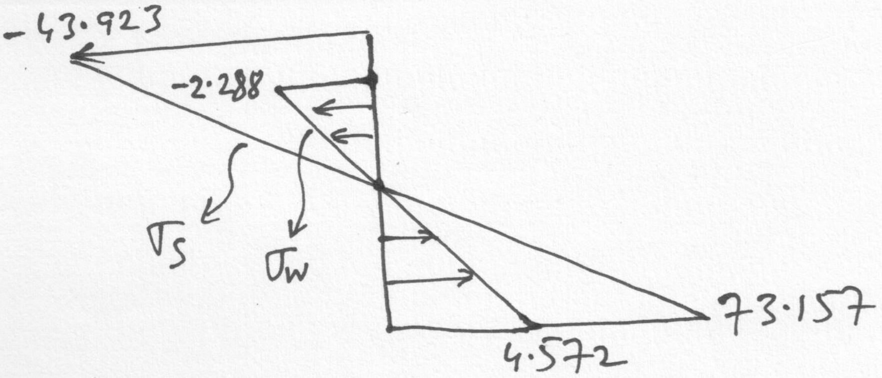
$$= 2186532846. \text{ mm}^4$$

$$(\sigma_b)_w = -\frac{50*10^6(-\bar{y})}{I_z} = 4.572 \text{ N/mm}^2$$

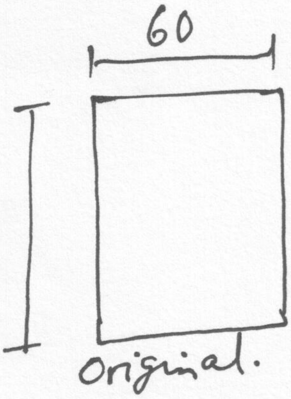
$$(\sigma_b)_s = n(\sigma_b)_w = 73.157 \text{ N/mm}^2$$

$$(\sigma_t)_w = -\frac{50*10^6(300-\bar{y})}{I_z} = -2.288 \text{ N/mm}^2$$

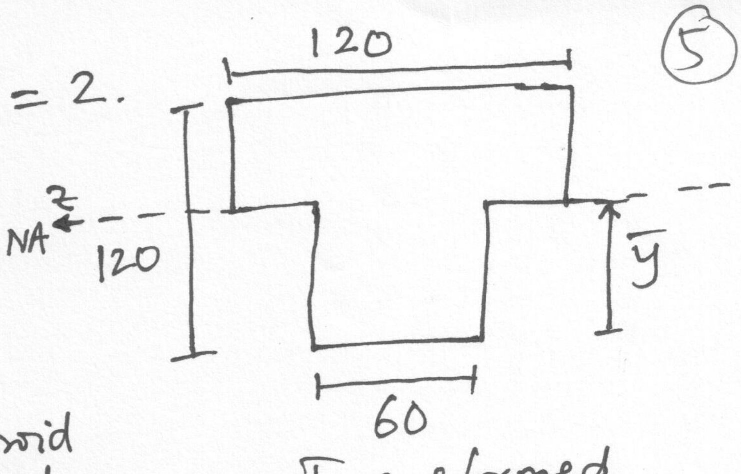
$$(\sigma_t)_s = -\frac{50*10^6(320-\bar{y})}{I_z} * n = -43.923 \text{ N/mm}^2$$



P6



$n = \frac{E_c}{E_T} = 2.$



$F_c = F_T \Rightarrow$ NA is at centroid of transformed section

$$\Rightarrow 60\,\bar{y}\,\frac{\bar{y}}{2} = 120\,\frac{(120-\bar{y})^2}{2} \Rightarrow \frac{\bar{y}^2}{2} - 240\bar{y} + 120^2 = 0$$

$\bar{y} = 70.29\,\text{mm}$, $\cancel{409.7}$ mm discard

$$(\sigma_t)_{max} = -\frac{5*10^6(-70.29)}{I_z} = 29.63\ \text{MPa}$$

$$(\sigma_c)_{max} = -\frac{5*10^6(120-70.29)}{I_z}*n = -41.92\ \text{MPa}$$

$$I_z = \frac{(60)(70.29)^3}{12} + (60)\frac{(70.29)^3}{4} + \frac{(120)(120-\bar{y})^3}{12} + (120)\frac{(120-\bar{y})^3}{4} = 1185\,9117\ \text{mm}^4$$

Check: $F_c = \frac{1}{2}*41.92*(120-70.29)*60 = 62515.3\,N$
$F_T = \frac{1}{2}*29.63*70.29*60 = 62480.8\,N$ $\Big\}\ \simeq$

P7 Kinematics remains same, ie $\varepsilon_x = -\frac{y}{\rho}$

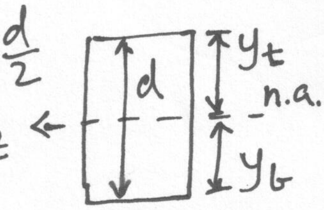
Constitutive law $\rightarrow \varepsilon_x = k\sigma^3 = -\frac{y}{\rho}$

not reqd

$$F_x = 0 = \int_A \sigma\, dA = -\frac{1}{(k\rho)^{1/3}}\, b \int_{-y_b}^{y_t} y^{1/3}\, dy \Rightarrow y_t^{4/3} - (-y_b)^{4/3} = 0$$

$$\Rightarrow y_t = y_b = \frac{d}{2}$$

✓ can get this by inspection ∵ stress distribution is antisymmetric about z-axis.

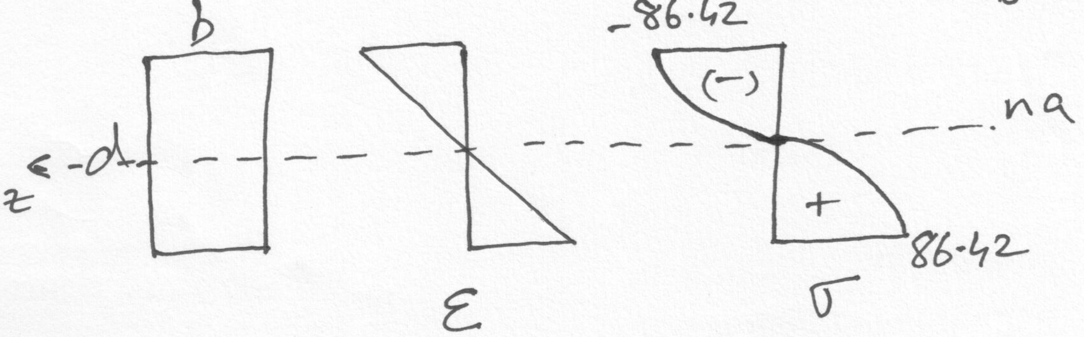


$$M_z = \int_A -\sigma y\, dA = +\frac{b}{(k\rho)^{1/3}}\int_{-y_b}^{y_t} y^{4/3}\, dy = +\frac{b}{(k\rho)^{1/3}}\left(\frac{y_t^{7/3} - (-y_b)^{7/3}}{7/3}\right)$$

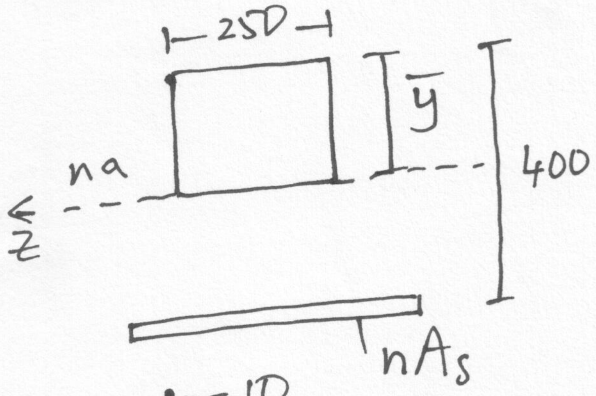
$$M_z = \frac{6}{7}\frac{b}{(k\rho)^{1/3}}\, y_t^{7/3} = \frac{6}{7}\, b \left(\frac{d}{2}\right)^{7/3}\left(-\frac{\sigma}{y^{1/3}}\right)$$

$$\sigma = -\frac{M_z\, y^{1/3}}{\frac{6}{7}\, b \left(\frac{d}{2}\right)^{7/3}} \Rightarrow \sigma_{max} = \mp \frac{M_z \left(\frac{d}{2}\right)^{1/3}}{\frac{6}{7}\, b\left(\frac{d}{2}\right)^{7/3}} = \mp\frac{28}{6}\frac{M_z}{bd^2}$$

$$= \mp \frac{28}{6}\frac{60 * 10^6}{(100)(180)^2} = 86.42 \text{ MPa.}$$



Extra P1 – Tute 6



$n = 10$

(Transformed section full concrete).

Locate n.a ($F_c = F_T$) :

$$250\frac{\bar{y}^2}{2} = (10)*(1000)(400-\bar{y})$$

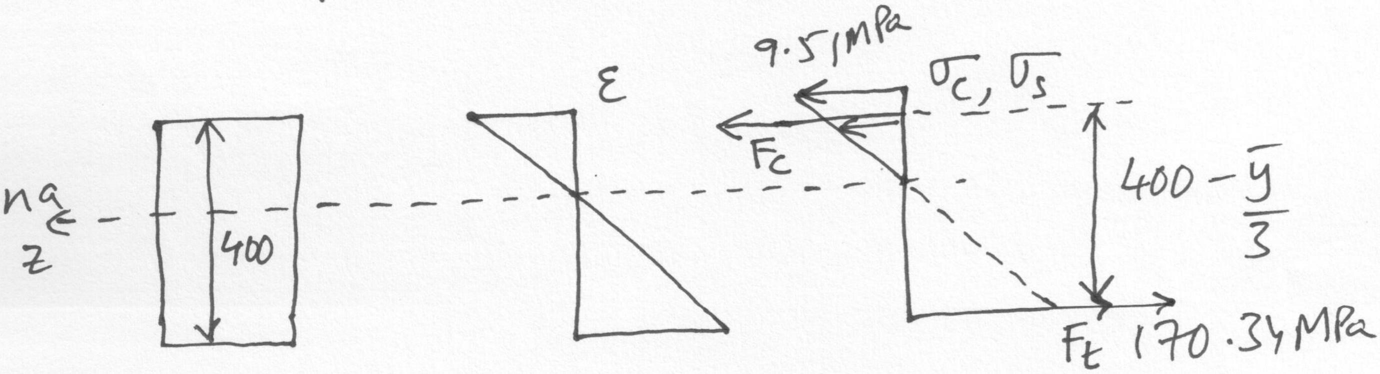
$\bar{y} = 143.3$, $\cancel{-223.3}$ discard

$$I_z = \frac{(250)(\bar{y}^3)}{3} + (10)(1000)(400-\bar{y})^2$$

$$= 904169711 \text{ mm}^4$$

$$(\sigma_c)_{max} = \frac{-60*10^6\,\bar{y}}{I_z} = 9.51 \text{ MPa}$$

$$\sigma_s = n.\left(\frac{-60*10^6[-(400-\bar{y})]}{I_z}\right) = 170.34 \text{ MPa.}$$



Alternate way to find max stresses without using I_z : $F_c = F_t$

$$\underset{60E6}{M} = F_c\left(400-\frac{\bar{y}}{3}\right) = \frac{1}{2}(\sigma_c)_{max}\,\bar{y}\,(250)\left(400-\frac{\bar{y}}{3}\right)$$

$$= F_t\left(400-\frac{\bar{y}}{3}\right) = \sigma_{st}\,\underset{1000}{A_s}\left(400-\frac{\bar{y}}{3}\right)$$

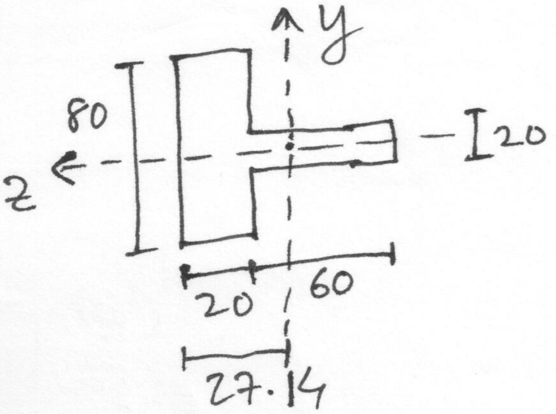
Solve for $(\sigma_c)_{max}$, σ_{st},

$(\sigma_c)_{max} = 9.51$ MPa , $\sigma_{st} = 170.34$ MPa , as before.

Extra Problem 2 — Tute 6

We will see later that if load applied through shear center then no twist occurs. In this case it is applied through shear center, so no twist will occur.

M_{max} at fixed end, so max stresses occur there. section symmetric so y, z are principal axes ($\because z$ is axis of symmetry). So no need to find p-axes, they are found by inspection.



$$z_c = \frac{(80)(20)(10) + (60)(20)(50)}{(80)(20) + (60)(20)}$$

$$= 27.14 \text{ mm}$$

$$M_z = -LP\cos\alpha = -1.5\, P\cos\alpha$$

$$M_y = -LP\sin\alpha = -1.5\, P\sin\alpha$$

$$\sigma_x = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y} \quad ; \quad I_z = \frac{(20)(80^3)}{12} + \frac{(60)(20^3)}{12}$$

$$= 893333.33 \text{ mm}^4$$

$$I_y = \frac{(20^3)(80)}{12} + (20)(80)(27.14-10)^2 + \frac{(60^3)(20)}{12} + (60)(20)(50-27.14)^2$$

$$= 1510476 \text{ mm}^4$$

$\alpha = 0$: z is neutral axis, $M_y = 0$!

$$\sigma_{yp} \rightarrow \frac{290 * 10^{-3}}{1.2} = \frac{(10^3)(1.5)P(80/2)}{I_z}$$

f.s. $\nearrow$ 1.2

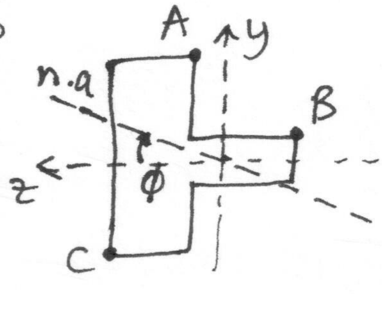
$$\Rightarrow P = 3.6 \text{ kN}.$$

$\alpha \neq 0$: Find n.a.

$$\sigma_x = 0 \Rightarrow \frac{y}{z} = \frac{M_y}{M_z}\frac{I_z}{I_y} = \tan\alpha\,(0.5914) = \tan\phi$$

$$\Rightarrow \phi = 9^\circ$$

For $(\sigma_x)_{max}$ consider points at farthest perpendicular distance from n.a., ie pts A, B, C.



By inspection C farther than A from n.a.
By inspection, for $\phi = 9^\circ$, A farther than B from n.a.
So only need to consider pt. C.

$$(\sigma_x)_C = \frac{-290 * 10^{-3}}{1.2} = (1.5)(10^3)P\left(\frac{[\cos\alpha][-40]}{I_z} + \frac{[-\sin\alpha][27.14]}{I_y}\right)$$

$\Rightarrow P = 3.36$ kN.