DEPARTMENT OF CIVIL ENGINEERING, IIT BOMBAY

CE 221 Solid Mechanics

Tutorial Sheet = 7 Instructors : A. Laskar/N.K. Chandiramani

1. Three boards, each of 40 mm x 90 mm rectangular cross-section are nailed together to form a beam which is subjected to a vertical shear force of 1 kN. If the spacing between each pair of nails is 60 mm, determine the shearing force in each nail.

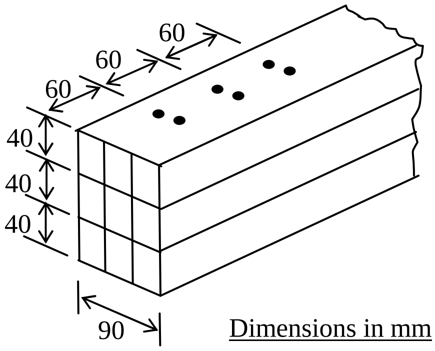
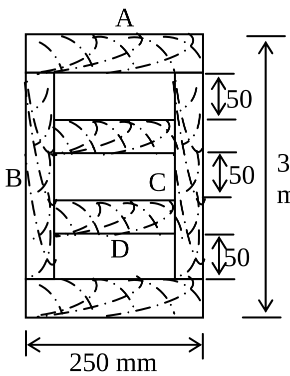


Fig. 1

2. A wooden box beam made up of 50 mm thick boards, has the dimensions shown in Fig. 2. If the beam transmits a shear of 3380 N, what should be the longitudinal spacing of the nails connecting (a) board A with boards B and C, (b) board D with boards B and C. The shear carrying capacity of each nail is 220 N.



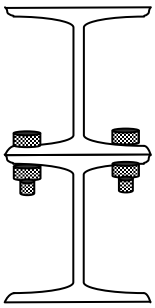


Fig. 2 **Fig. 3**

3. The composite beam shown in Fig. 3 has been formed by bolting two I sections using 16 mm diameter bolt spaced longitudinally every 150 mm. If the average allowable stress in the bolts is 75 MPa, determine the largest permissible vertical shear. For the I section, A=3790 mm^2, depth == 157 mm, and I_z=17.23 x 10^6 mm^4.

4 The three beam section shown in Fig. 4 are subjected to a shear force of 100 kN. Draw the shear stress and shear flow diagram along the depth.

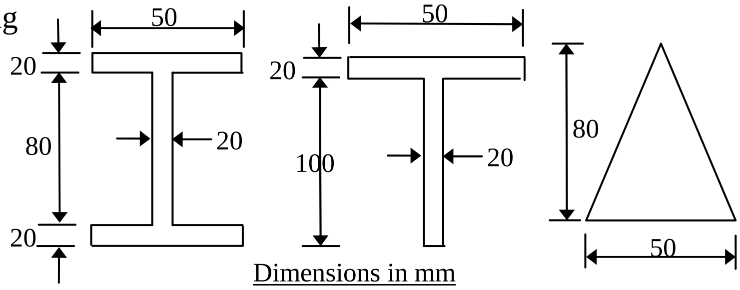
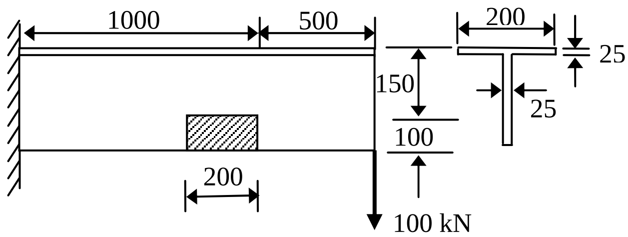


Fig. 4

5. A T-beam is loaded as shown in Fig. 5. From this beam isolate a segment 200 x 100 x 25 mm (shown by shaded portion). Draw the free body diagram of this segment indicating the location, magnitude and sense of resultant forces due to bending and shear stresses. Check the equilibrium of the isolated segment.



Dimensions in mm

Fig. 5

6. Determine the location of the shear centre for the sections shown in Fig. 6.

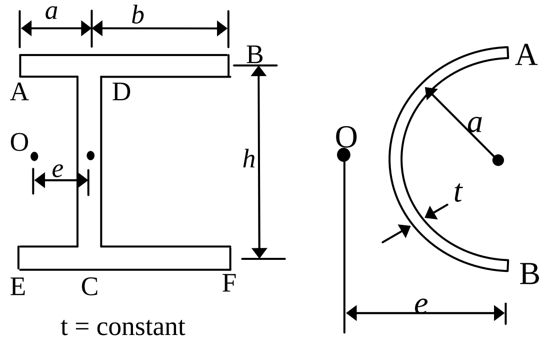


Fig. 6

P1 $I = \frac{90*120^3}{12}$, $t = 90$, $Q = 40*90*40$, $V = 1\,kN$

$$q = \frac{VQ}{I} = 11.11\ N/mm = \frac{\Delta H}{\Delta x}$$

$$SF \text{ in each nail} = \frac{11.11*60}{2} = 333.33\,N.$$

P2 $$I = 2*\left(\frac{50*250^3}{12} + \frac{150*50^3}{12} + 150*50*50^2 + \frac{250*50^3}{12} + 250*50*150^2\right)$$

$$= 738541.7*10^3\ mm^4$$

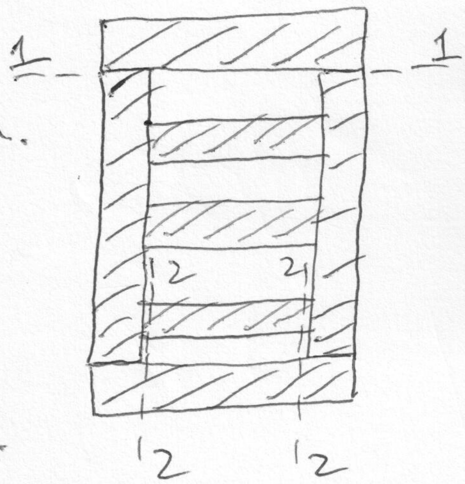
$$Q_1 = Q|_{\text{horizontal cut},\ y=125} = 250*50*150 = 1875000\ mm^3$$

$$Q_2 = Q|_{\text{vertical cut at ends of bar D}} = 150*50*50 = 375000\ mm^3$$

$V = 3380\,N.$

$$q_1 = \frac{\Delta H_1}{\Delta x} = \frac{VQ_1}{I} = 8.581\ N/mm$$

$$\text{spacing } s_1 = \frac{220*2}{q_1} = 51.28\ mm.$$



$$q_2 = \frac{\Delta H_2}{\Delta x} = \frac{VQ_2}{I} = 1.716\ N/mm.$$

$$\text{spacing } s_2 = \frac{2*220}{q_2} = 256.4\ mm.$$

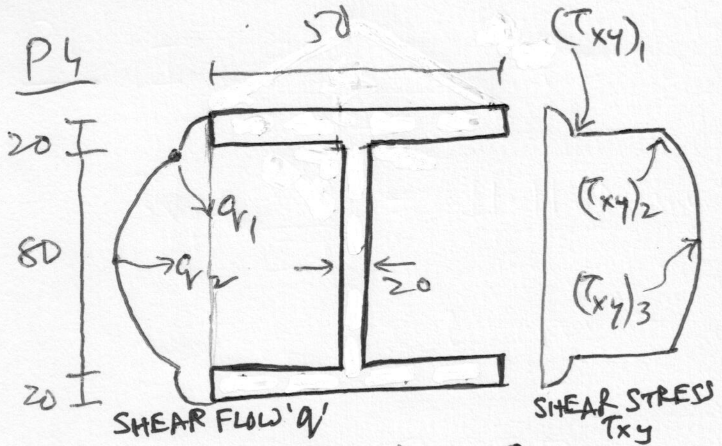
P3 $$I = 2\left[17.23*10^6 + 3790*\left(\frac{157}{2}\right)^2\right] = 81.17*10^6\ mm^4$$

$$q = \frac{\Delta H}{\Delta x} = \frac{2*75*\frac{\pi}{4}(16)^2}{150} = 201.06\ N/mm$$

$$Q = 3750*\frac{157}{2} = 294375\ mm^3$$

$$q = \frac{VQ}{I} = 201.06 \Rightarrow V = V_{max} = 55.44\,kN.$$

P4



$$I = \frac{20*80^3}{12} + 2\left(\frac{50*20^3}{12} + 50*20*50^2\right)$$

$$= 5920000 \text{ mm}^4$$

$V = 100$ kN.

$$q = \frac{VQ}{I}, \quad Q_F = (50)(60-y)\frac{(60+y)}{2} \quad \text{for } 60 \leq y \leq 40$$

$$Q_W = (50)(60-40)\left(\frac{60+40}{2}\right) + 20(40-y)\left(\frac{40+y}{2}\right) \quad \text{for } 40 \leq y \leq -40$$

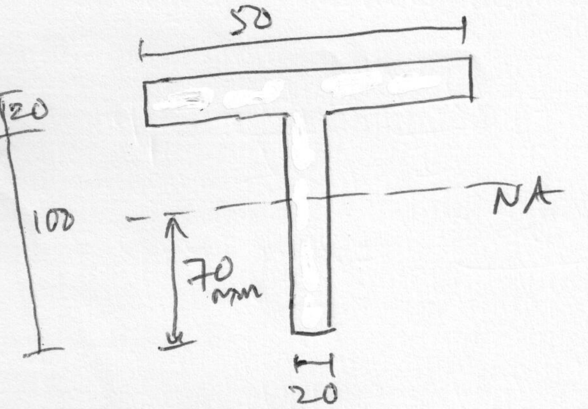
for horizontal cuts.

$q_1 = 844.6$ N/mm, $(\tau_{xy})_1 = \frac{q_1}{50} = 16.89$ N/mm^2

$\frac{V}{I}(Q_F)_{y=40}$

$$(\tau_{xy})_2 = \frac{q_1}{20} = 42.23 \text{ N/mm}^2$$

$$q_2 = \frac{V}{I}(Q_W)_{y=0} = 1115 \text{ N/mm} \qquad (\tau_{xy})_3 = \frac{q_2}{20} = 55.7 \text{ N/mm}^2 = \max$$



$$y_c = \frac{(50)(20)(110) + (100)(20)(50)}{(50)(20) + (100)(20)} = 70 \text{ mm}$$

$$I = \frac{1}{12}(50)(20)^3 + (50)(20)(40)^2 + \frac{1}{12}(20)(100)^3 + (20)(100)(20)^2$$

$$= 4.1*10^6 \text{ mm}^4$$

$$q_1 = \frac{V}{I}(Q_F)_{y=30} = 975.61 \text{ N/mm}$$

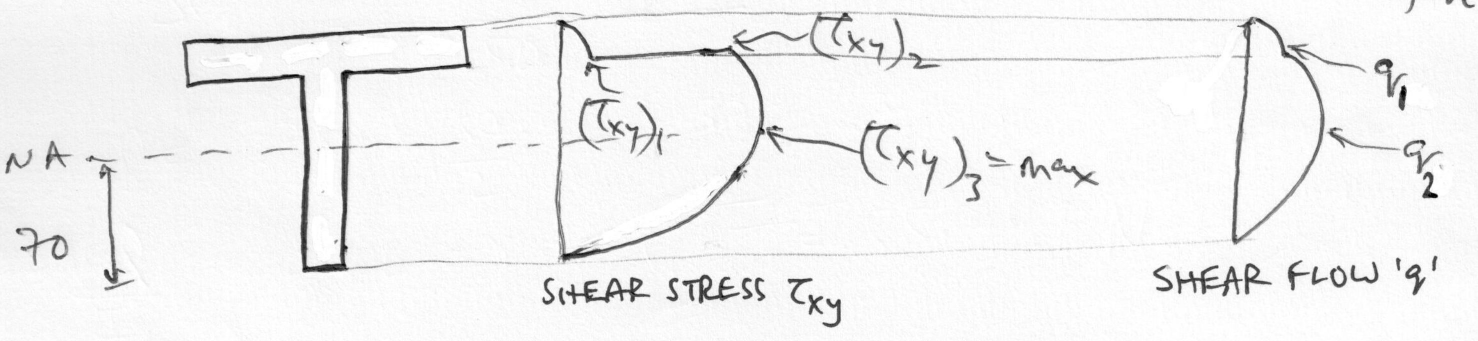
$$(\tau_{xy})_1 = \frac{q_1}{50} = 19.51 \text{ N/mm}^2$$

$$(\tau_{xy})_2 = \frac{q_1}{20} = 48.78 \text{ N/mm}^2$$

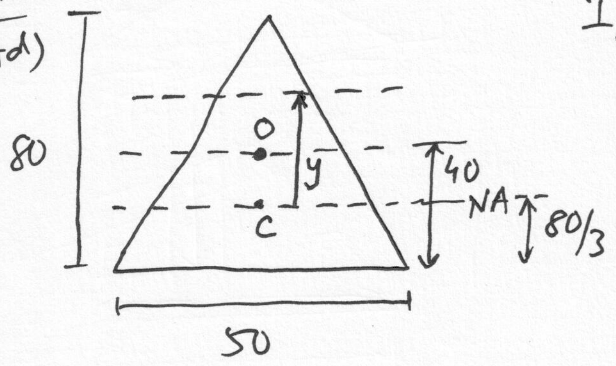
$$Q_F = (50)(50-y)\left(\frac{50+y}{2}\right), \quad 50 \leq y \leq 30$$

$$Q_W = (50)(50-30)\left(\frac{50+30}{2}\right) + (20)(30-y)\left(\frac{30+y}{2}\right), \quad 30 \leq y \leq -70$$

$$(\tau_{xy})_3 = \frac{q_2}{20}, \quad q_2 = \frac{V}{I}(Q_W)_{y=0} = 1195.12 \text{ N/mm}, \quad (\tau_{xy})_3 = 59.76 \text{ N/mm}^2$$



P4 (contd)



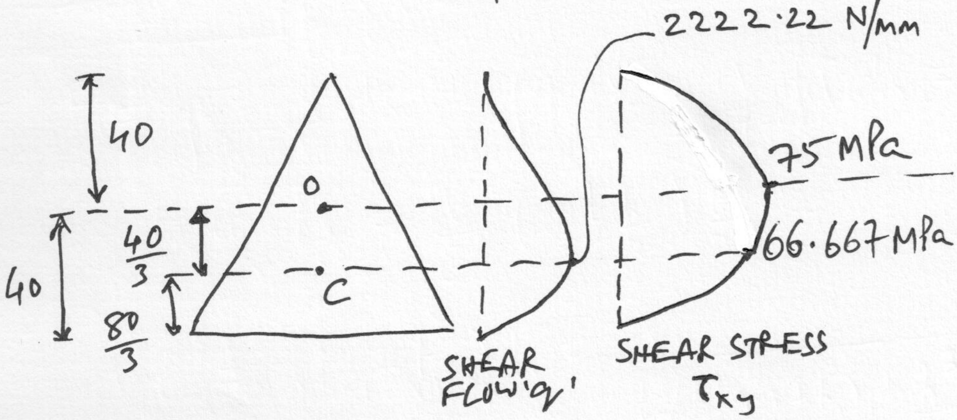
$$I_o = \frac{50 * 80^3}{12} \cdot \frac{1}{2} = I_c + \frac{1}{2} \cdot 50 \cdot 80 \cdot \left(\frac{80}{6}\right)^2$$

$$\Rightarrow I_c = \frac{64}{9} * 10^5 \text{ mm}^4.$$

$$Q = \frac{1}{2}\left(50 * \frac{2}{3} - \frac{5}{8}y\right)\left(80 \cdot \frac{2}{3} - y\right)\left(\frac{2}{3}y + 80 \cdot \frac{2}{9}\right)$$

$$t = 50 * \frac{2}{3} - \frac{5}{8}y$$

$$\tau_{xy} = \frac{VQ}{It} = \frac{100 * 10^3 \cdot \frac{1}{2}\left(\frac{160}{3} - y\right)\left(\frac{160}{9} + \frac{2}{3}y\right)}{\frac{64}{9} * 10^5}$$



$$\frac{d\tau_{xy}}{dy} = 0 \Rightarrow -\frac{160}{9} - \frac{2}{3}y + \frac{2}{3}\left(\frac{160}{3} - y\right) = 0$$

$$\Rightarrow y = \frac{40}{3}$$

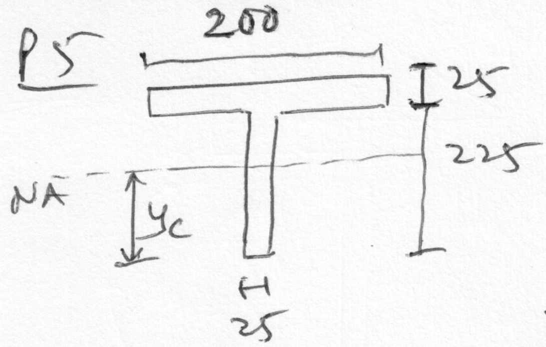
$$q = \frac{VQ}{I} = \frac{100E3 \cdot \frac{1}{2}}{\frac{64}{9}E5}\left(\frac{256E4}{81} - \frac{100}{3}y^2 + \frac{5}{12}y^3\right)$$

$$\frac{dq}{dy} = 0 \Rightarrow y = 0,\ y = \frac{160}{3}\ ; \quad \frac{d^2q}{dy^2} = \frac{-200}{3} + \frac{5}{2}y$$

$< 0,\ y = 0$

$> 0,\ y = 160/3$

P5



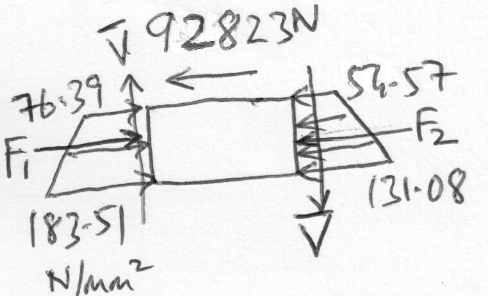
$$y_c = \frac{(200)(25)(237.5) + (25)(225)\left(\frac{225}{2}\right)}{(200)(25) + (25)(225)}$$

$= 171.32$ mm.

$$I = \frac{1}{12}(200)(25)^3 + (200)(25)(66.18)^2 + \frac{1}{12}(25)(225)^3 + (25)(225)(58.82)^2$$

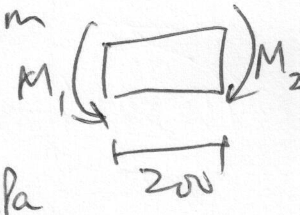
$= 65.35 * 10^6$ mm^4

$V = 100$ kN



$M_1 = 100 * 10^3 * 700 = 70 * 10^6$ N.mm

$M_2 = 100 * 10^3 * 500 = 50 * 10^6$ N.mm



$(\sigma_u)_1 = \frac{-(-70E6)(-71.32)}{I} = -76.39$ MPa

$(\sigma_u)_2 = \frac{-(-50E6)(-71.32)}{I} = -54.57$ MPa

$(\sigma_b)_1 = \frac{-(-70E6)(-171.32)}{I} = -183.51$ MPa

$(\sigma_b)_2 = \frac{-(-50E6)(-171.32)}{I} = -131.08$ MPa.

$F_1 = \frac{1}{2}(76.39 + 183.51)(100)(25)$
$= 324875$ N

$F_2 = \frac{1}{2}(54.57 + 131.08) * (100)(25)$
$= 232062.5$

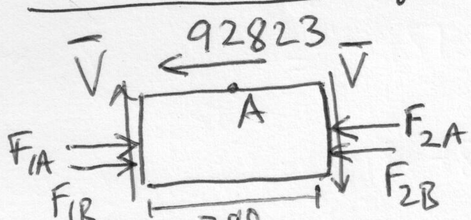
$\Delta H + F_2 - F_1 = 10.5 \simeq 0$ when compared with magn of $\Delta H, F_1, F_2$

So $\Sigma F_x = 0$ ✓ checks out

$$q = \frac{\Delta H}{\Delta x} = \frac{\Delta H}{200} = \frac{VQ}{I} = \frac{(100E3)(100 * 25 * [-121.32])}{I}$$

$= -464.12$ N/mm.

$\Delta H = -92823$ N.



split trapezium into rectangle & triangle.

$F_{1A} = (76.39)(100)(25) = 190975$ N

$F_{2A} = (54.57)(100)(25) = 136425$ N

$F_{1B} = \frac{1}{2}(183.51 - 76.39)(100)(25) = 133900$ N

$F_{2B} = \frac{1}{2}(131.08 - 54.57)(100)(25) = \frac{191275}{2}$ N.

$(F_1 = F_{1A} + F_{1B},\ F_2 = F_{2A} + F_{2B})$.

F_{1A}, F_{2A} act at 50 mm from bottom.

F_{1B}, F_{2B} act at $\frac{100}{3}$ mm from bottom.

$$\bar{V} = \int_0^{100} \tau_{xy}(25)\,d\bar{y} = \int_0^{100} \frac{q}{25}(25)\,d\bar{y}$$

$$= \frac{V}{I}\int_0^{100}(25)\,\bar{y}\left(y_c - \frac{\bar{y}}{2}\right)d\bar{y}$$

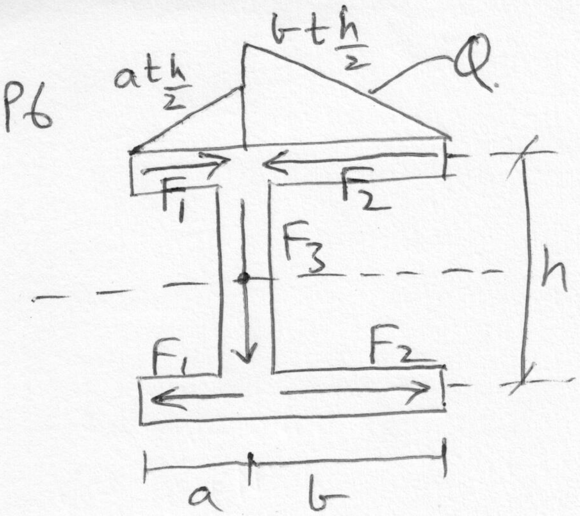
$\bar{V} = 26393.8$ N

$$\Sigma M_A = (F_{1A} - F_{2A})(50) + (F_{1B} - F_{2B})\left(\frac{2}{3}\cdot 100\right) - \bar{V}(200) = -426.67 \text{ Nmm} \simeq 0$$

when compared to M_1, M_2

$\Rightarrow \Sigma M_A = 0$ ✓ checks out

P6



Thickness 't' throughout.

$$q = \frac{VQ}{I}$$

$$I = 2\left[(a+b)\frac{t^3}{12} + (a+b)t\left(\frac{h}{2}\right)^2\right] + \frac{t(h-t)^3}{12}$$

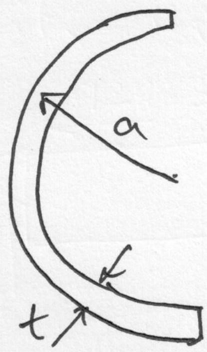
neglect t^2, t^3, t^4 terms $\rightarrow I \simeq t\left[\frac{h^2}{2}(a+b) + \frac{h^3}{12}\right]$

$$F_1 = \int \tau_{xz}\, t\, ds = \int q\, ds = \frac{V}{I}\int Q\, ds = \frac{V}{I}\left(a t \frac{h}{2}\right)\frac{a}{2}$$

Similarly $F_2 = \frac{V}{I}\left(b t \frac{h}{2}\right)\frac{b}{2}$.

$$\cancel{V}e = (F_2 - F_1)h = \frac{\cancel{V}}{\cancel{t}\left(\frac{h^2}{2}(a+b) + \frac{h^3}{12}\right)}\frac{\cancel{t}h}{4}(b^2 - a^2)h$$

$$e = \frac{b^2 - a^2}{\left(2(a+b) + \frac{h}{3}\right)}$$



Done in class notes.