

DEPARTMENT OF CIVIL ENGINEERING, IIT BOMBAY

CE 221 Solid Mechanics

Tutorial Sheet = 8

Instructor : A. Laskar/N. K. Chandiramani

1. For the engineering structures shown in Fig. 1 determine the normal and shear stresses at the point shown. Sketch an element in each case showing the magnitude and sense of the stresses on each face.
2. For the elements illustrated in Fig. 2 calculate the stress components on the inclined planes.
3. Find the principal normal and maximum shearing stresses and show their sense on a properly oriented element for the state of stress (in MPa) shown in Table 1. by (a) analytically and (b) Mohr's circle of stresses.
4. A chemical pressure vessel is to be manufactured from glass fibres in an epoxy matrix as shown in Fig. 3. If the optimum orientation of fibre is that in which the fibres are subjected to tensile stress with no transverse or shear stresses, determine the optimum value of α .
5. Draw Mohr's circle of strain and determine principal normal and shear strains and their directions for the elements having the strains as in Table 2. Verify your results analytically. Also, determine principal normal and shearing stresses. Take $E=200$ GPa and $\nu=0.3$.
6. Data from rectangular strain rosette glued to steel plate are as follows $\epsilon_{0^\circ} = -0.00022$, $\epsilon_{45^\circ} = 0.00012$ and $\epsilon_{90^\circ} = 0.00022$. What are the principal stresses and in which direction do they act? $E=200$ GPa and $\nu=0.3$.
7. Data from equiangular strain rosette attached to aluminium alloy are as follows $\epsilon_{0^\circ} = 0.0004$, $\epsilon_{60^\circ} = 0.0004$ and $\epsilon_{120^\circ} = -0.0006$. What are the principal stresses and their directions. $E=70$ GPa and $\nu=0.25$

Table - 1

Element.	σ_x	σ_y	τ_{xy}
1.	60	20	0
2.	-30	50	-40
3.	200	0	80
4.	20	30	20

Table - 2

Element.	ϵ_x	ϵ_y	γ_{xy}
1.	-0.00012	0.00112	-0.0002
2.	0.0008	0.0020	0.0008

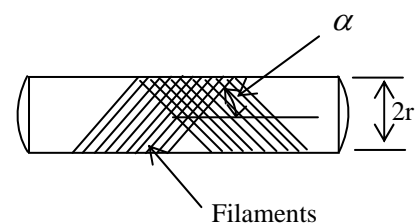
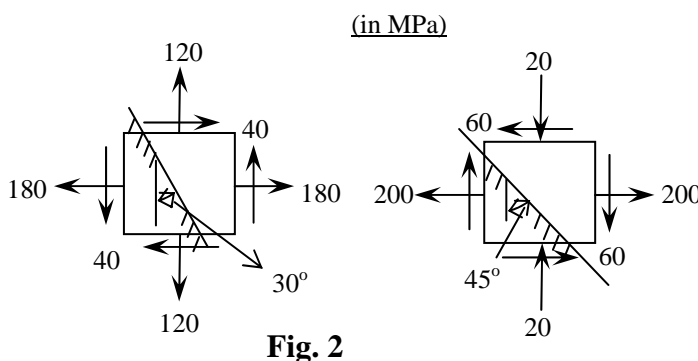
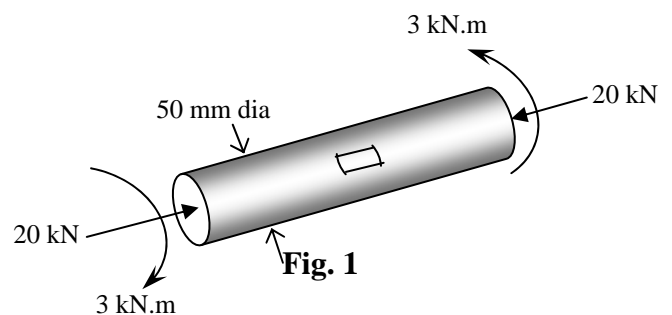
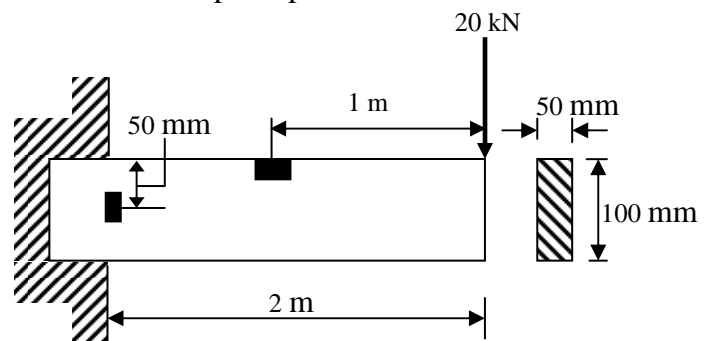
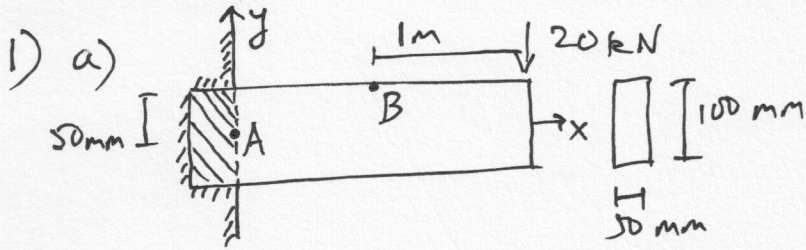


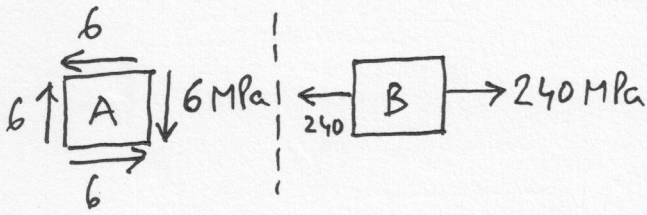
Fig. 3

CE221 TUTORIAL # 8.

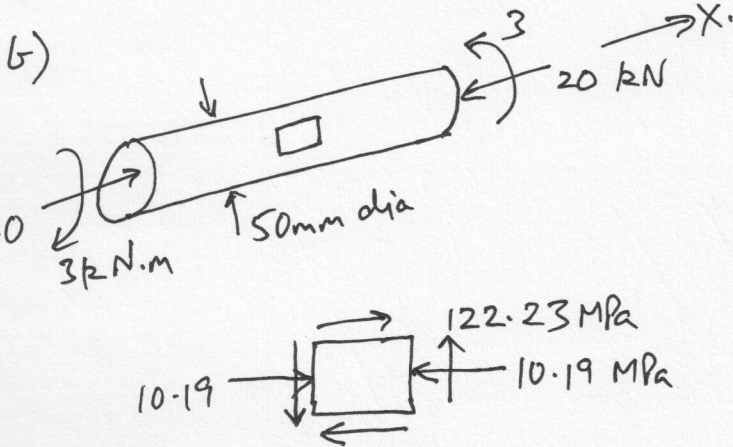
(1)



Point A: $\sigma_x = 0, \tau_{xy} = \frac{(20E3)(50)(50)(25)}{(50)(100)^2(50)}$
 $\sigma_y = 0$
 $= 6 \text{ MPa}$



Point B: $\tau_{xy} = 0, \sigma_x = -\frac{(-20E3)(50)}{(50)(100)^3}$
 $\sigma_y = 0$
 $= 240 \text{ MPa}$



$\tau = \frac{(3E6)(50)}{\frac{\pi}{32}(50^4)} = \frac{244.46}{2} = 122.23 \text{ MPa}$

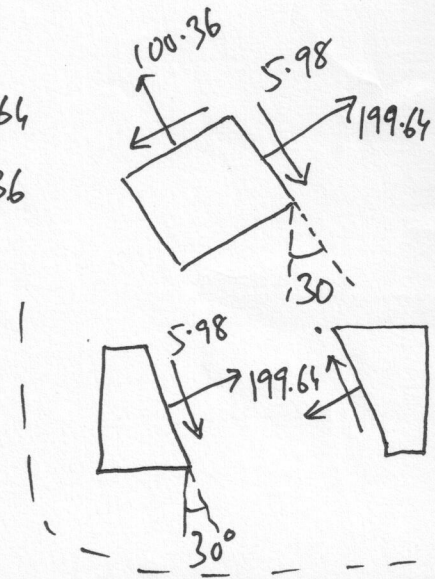
$\sigma_x = -\frac{20E3}{\frac{\pi}{4}50^2} = -10.19 \text{ MPa}$

2) a) $\sigma_x = 180, \sigma_y = 120, \tau_{xy} = 40, \theta = 30^\circ$

$\sigma_{x'} = \frac{180+120}{2} + \frac{180-120}{2} \cos 60 + 40 \sin 60 = 199.64$

(not reqd) $\sigma_{y'} = \frac{180+120}{2} - \frac{180-120}{2} \cos 60 - 40 \sin 60 = 100.36$

$\tau_{x'y'} = -\left(\frac{180-120}{2}\right) \sin 60 + 40 \cos 60 = -5.98$

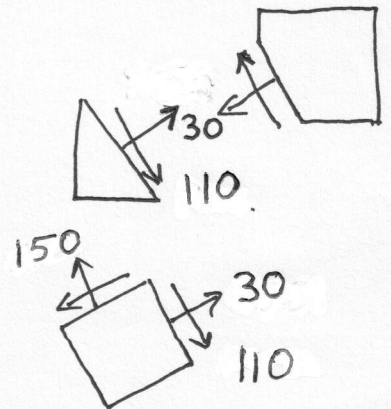


b) $\sigma_x = 200, \sigma_y = -20, \tau_{xy} = -60, \theta = 45^\circ$

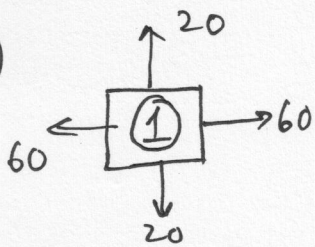
$\sigma_{x'} = \frac{200-20}{2} + \frac{200+20}{2} \cos 90 - 60 \sin 90 = 30$

(not reqd) $\sigma_{y'} = \frac{200-20}{2} - \frac{200+20}{2} \cos 90 + 60 \sin 90 = 150$

$\tau_{x'y'} = -\left(\frac{200+20}{2}\right) \sin 90 - 60 \cos 90 = -110$



3)



Principal element

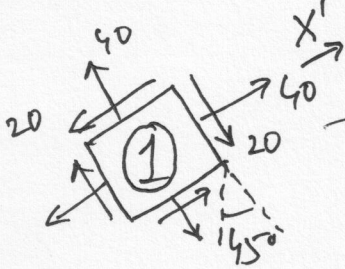
$\therefore \tau_{xy} = 0$, it is the principal element itself.

$\sigma_{max} = 60, \sigma_{min} = 20.$

$\tau_{max} = \left| \frac{60-20}{2} \right| = 20$

On max τ plane, $\sigma_{x'} = \sigma_{y'} = \sigma_{ave} = \frac{60+20}{2} = 40.$

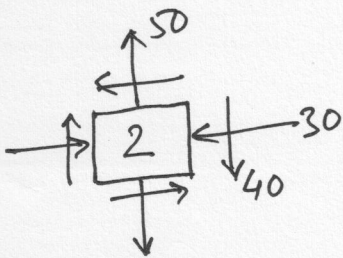
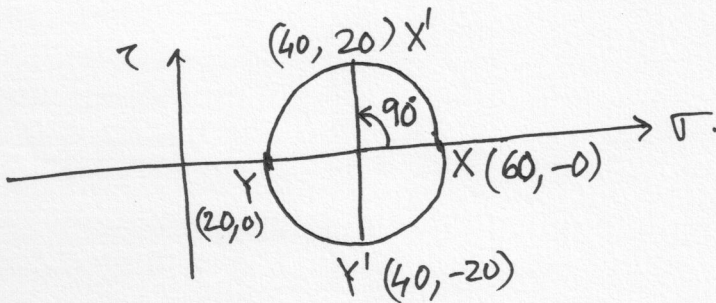
τ_{max} planes make 45° with principal planes



τ Max element

For direction of τ_{max} , use

$\tau_{x'y'} = -\left(\frac{60-20}{2}\right) \sin 2\theta + (0) \cos 2\theta = -20$
(with $\theta = +45^\circ$)



Principal element

$\sigma_{ave} = \frac{-30+50}{2} = 10, R = \sqrt{\left(\frac{-30-50}{2}\right)^2 + (-40)^2} = 40\sqrt{2}$

$\sigma_{max} = \sigma_{ave} + R = 66.57, \sigma_{min} = \sigma_{ave} - R = -46.57$

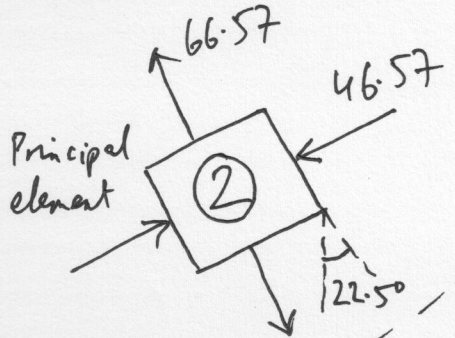
$R = \tau_{max} = 40\sqrt{2} = 56.57$

$\theta_p = \frac{1}{2} \tan^{-1} \left(\frac{2 \times (-40)}{-30-50} \right) = 22.5^\circ, 112.5^\circ.$

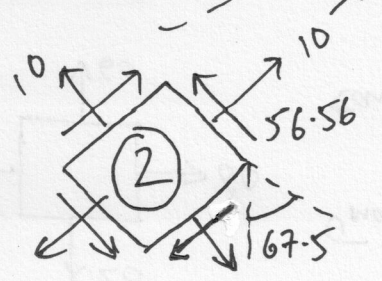
$\theta_s = 22.5 \pm 45 = 67.5^\circ, -22.5^\circ.$

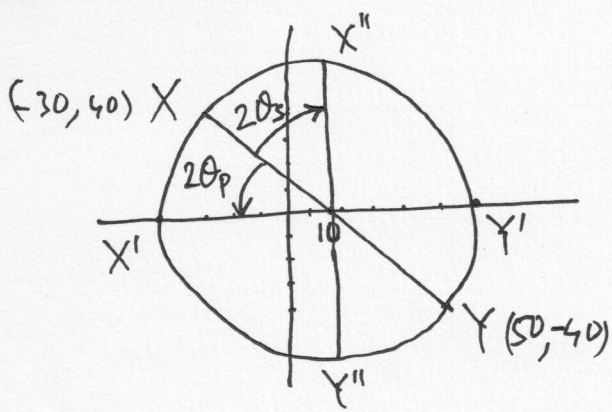
$\sigma_{x'} |_{\theta=\theta_p=22.5} = \left(\frac{-30+50}{2}\right) + \left(\frac{-30-50}{2}\right) \cos 45 - 40 \sin 45 = -46.57$

$\tau_{x'y'} |_{\theta=\theta_s=67.5} = -\left(\frac{30-50}{2}\right) \sin(2 \times 67.5) - 40 \cos(2 \times 67.5) = 56.56.$



τ_{max} element





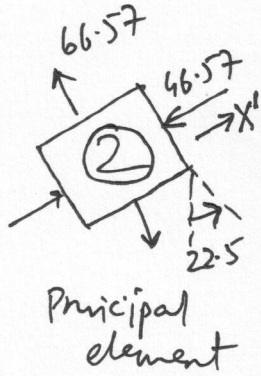
$$R = \frac{1}{2} \sqrt{(50+30)^2 + (40+40)^2} = 56.57 = 40\sqrt{2}$$

$$\sigma_{\min} (\equiv X') = 10 - R = -46.57$$

$$\sigma_{\max} (\equiv Y') = 10 + R = 66.57$$

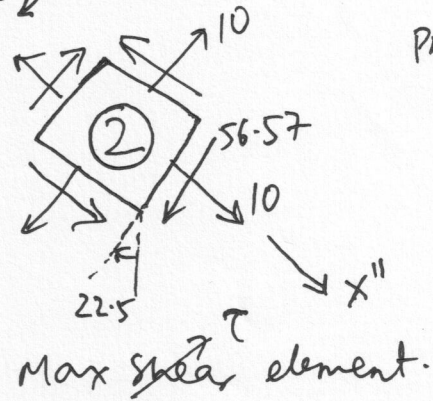
$$\theta_p = \frac{1}{2} \sin^{-1} \left(\frac{40}{R} \right) = 22.5^\circ$$

$$\theta_s = 22.5^\circ$$

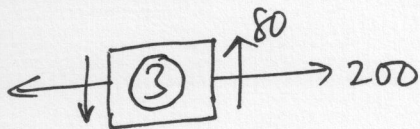


Principal element

(Same results as from formulae)



Max shear element.



$$\sigma_{\text{ave}} = \frac{200+0}{2} = 100, R = \sqrt{\left(\frac{200-0}{2}\right)^2 + 80^2} = 128.06$$

$$\sigma_{\max} = 228.06, \sigma_{\min} = 100 - 128.06 = -28.06$$

$$\tau_{\max} = 128.06$$

$$\theta_p = \frac{1}{2} \tan^{-1} \left(\frac{2 \times 80}{200-0} \right) = 19.33^\circ, 109.33^\circ$$

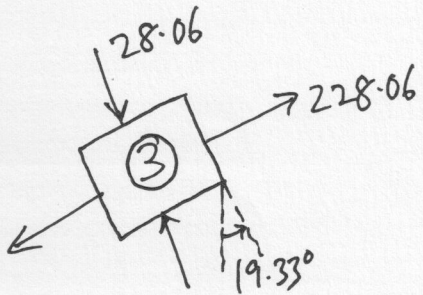
$$\theta_s = 19.33^\circ \pm 45^\circ = 64.33^\circ, -25.67^\circ$$

$$\sigma_{X'} = \frac{200+0}{2} + \left(\frac{200-0}{2}\right) \cos(2 \times 19.33) + 80 \sin(2 \times 19.33)$$

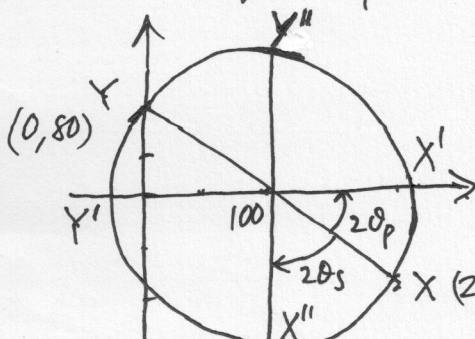
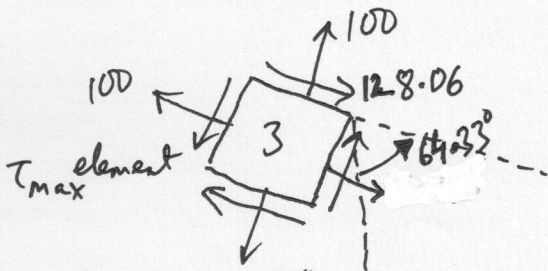
$$\theta = \theta_p = 19.33^\circ \quad \sigma = 228.06$$

$$\tau_{X'Y'} = -\left(\frac{200-0}{2}\right) \sin(2 \times 64.33) + 80 \cos(2 \times 64.33)$$

$$\theta_s = 64.33^\circ \quad \tau = -128.06$$



Principal element.



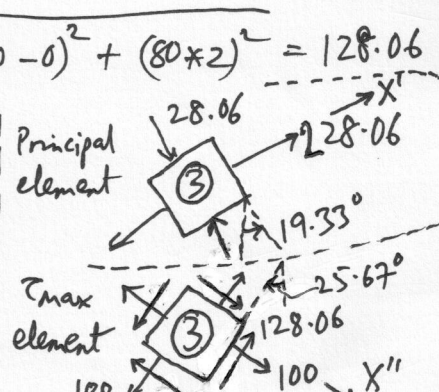
$$X' \equiv \sigma_{\max} = 100 + R, R = \frac{1}{2} \sqrt{(200-0)^2 + (80 \times 2)^2} = 128.06$$

$$= 228.06$$

$$Y' \equiv \sigma_{\min} = 100 - R = -28.06$$

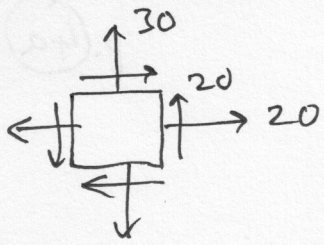
$$\theta_p = \frac{1}{2} \sin^{-1} \left(\frac{80}{R} \right) = 19.33^\circ$$

$$\theta_s = 45 - \theta_p = 25.67^\circ$$



Principal element

τ_{max} element



$$\sigma_{av} = \frac{20+30}{2} = 25, \quad R = \sqrt{\left(\frac{20-30}{2}\right)^2 + 20^2} = 20.62 \quad (4)$$

$$\sigma_{max} = 25 + 20.62 = 45.62, \quad \sigma_{min} = 25 - 20.62 = 4.38$$

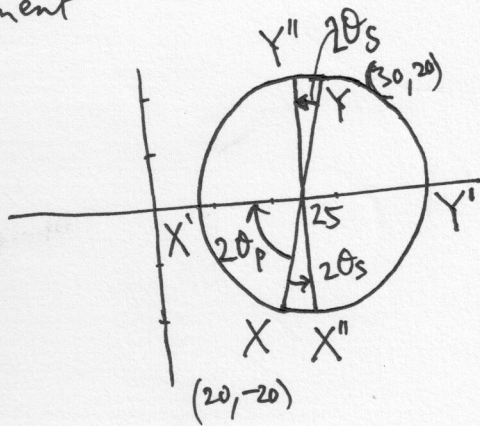
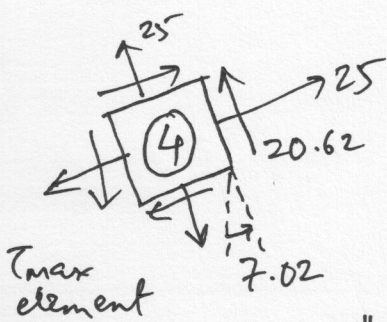
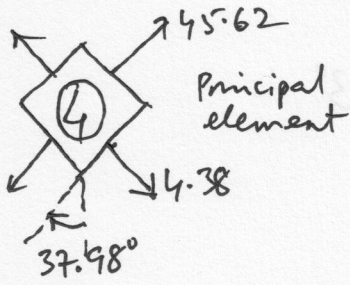
$$\tau_{max} = 20.62$$

$$\theta_p = \frac{1}{2} \tan^{-1} \left(\frac{2 \times 20}{20-30} \right) = -37.98^\circ, 52.02^\circ$$

$$\theta_s = -37.98 \pm 45 = 7.02^\circ, -82.98^\circ$$

$$\sigma_{x'} \Big|_{\theta = \theta_p = -37.98^\circ} = \frac{20+30}{2} + \frac{20-30}{2} \cos(2 \times -37.98) + 20 \sin(2 \times -37.98) = 4.38$$

$$\tau_{x'y'} \Big|_{\theta_s = 7.02^\circ} = -\left(\frac{20-30}{2}\right) \sin(2 \times 7.02) + 20 \cos(2 \times 7.02) = 20.62$$

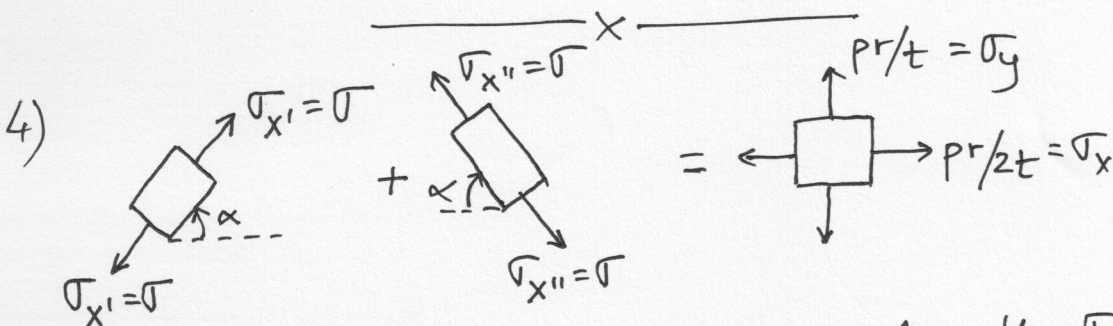
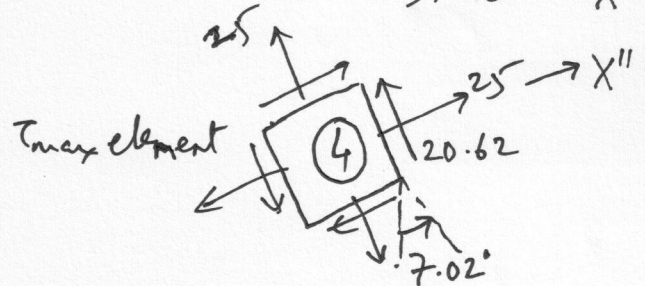
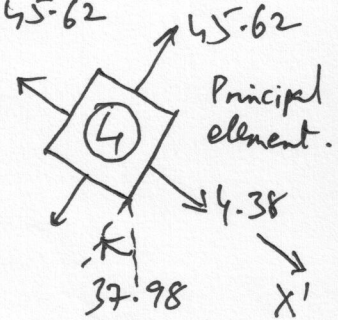


$$X' \equiv \sigma_{min} = 25 - \frac{1}{2} \sqrt{40^2 + 10^2} = 4.38; \quad R = \frac{1}{2} \sqrt{40^2 + 10^2} = 20.62$$

$$Y' \equiv \sigma_{max} = 25 + \frac{1}{2} \sqrt{40^2 + 10^2} = 45.62$$

$$\theta_p = \frac{1}{2} \sin^{-1} \left(\frac{20}{R} \right) = 37.98^\circ$$

$$\theta_s = \frac{1}{2} \cos^{-1} \left(\frac{20}{R} \right) = 7.02^\circ$$



Transform from x' & x'' systems to x system. Use $\sigma_y = \sigma_{y''} = \tau_{x'y'} = \tau_{x''y''} = 0$

$$\sigma_x = \frac{pr}{2t} = \frac{\sigma_{x'}}{2} + \frac{\sigma_{x'}}{2} \cos 2\alpha + \frac{\sigma_{x''}}{2} + \frac{\sigma_{x''}}{2} \cos 2\alpha = \sigma (1 + \cos 2\alpha) \rightarrow (1) \quad \tau_{xy} = 0$$

$$\sigma_y = \frac{pr}{t} = \frac{\sigma_{x'}}{2} - \frac{\sigma_{x'}}{2} \cos 2\alpha + \frac{\sigma_{x''}}{2} - \frac{\sigma_{x''}}{2} \cos 2\alpha = \sigma (1 - \cos 2\alpha) \rightarrow (2) \quad \sigma_x = \frac{pr}{2t}$$

$$\tau_{xy} = 0 = \frac{\sigma_{x'}}{2} \sin 2\alpha - \frac{\sigma_{x''}}{2} \sin 2\alpha = \frac{\sigma}{2} (\sin 2\alpha - \sin 2\alpha) = 0 \rightarrow (3) \quad \sigma_y = \frac{pr}{t}$$

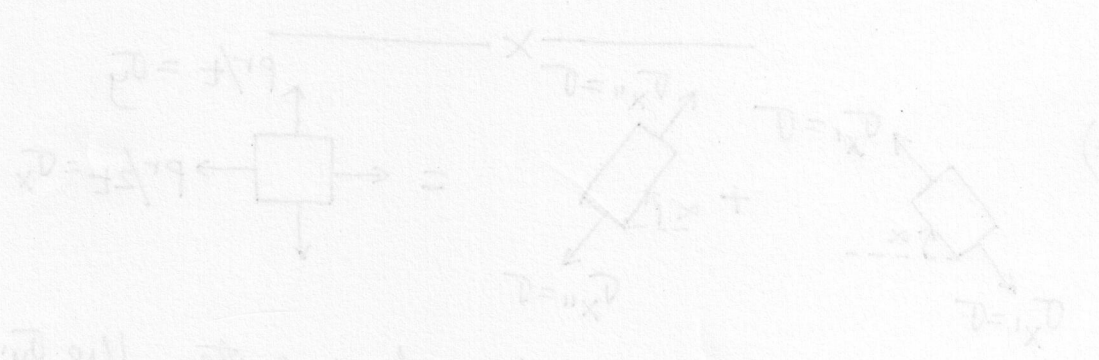
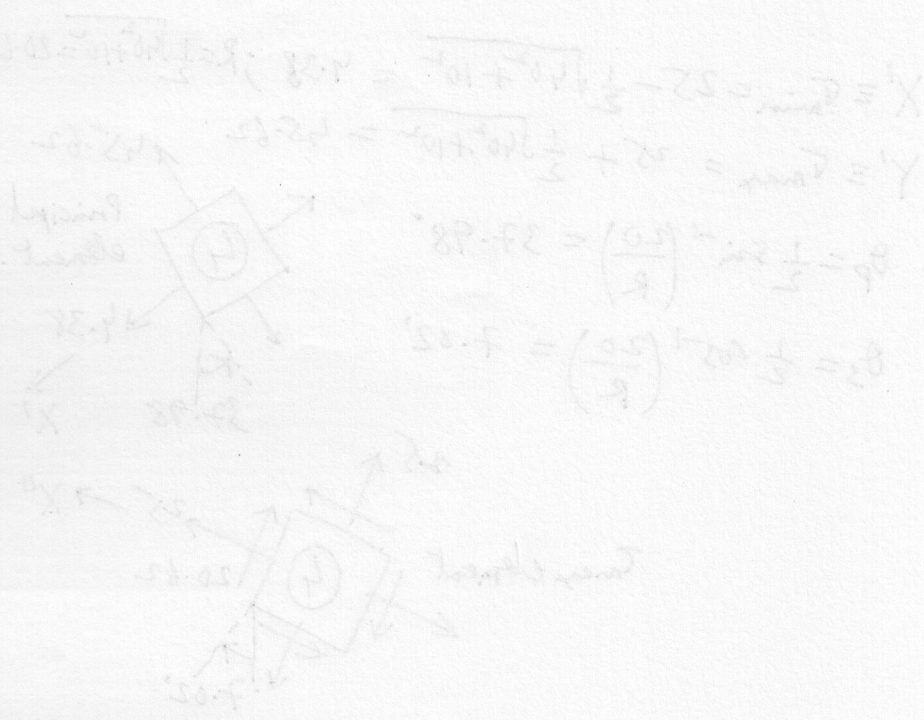
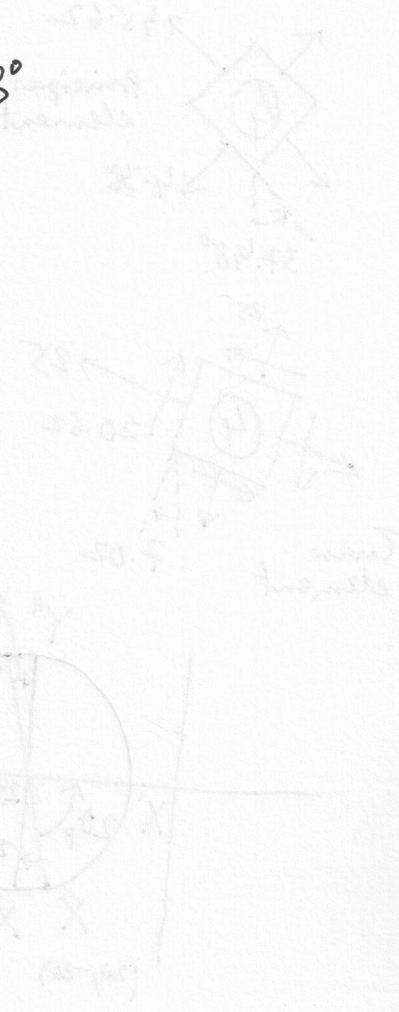
satisfied identically

4) contd.

solve ①, ②,

$$2\sigma = \frac{3}{2} \frac{pr}{t} \Rightarrow \sigma = \frac{3}{4} \frac{pr}{t}$$

$$\Rightarrow \cos 2\alpha = -1 + \frac{2}{3} = -\frac{1}{3} \Rightarrow \alpha = 54.73^\circ$$



transform for x, y systems to x', y' system. Use $\sigma_x = \sigma_{xx}$
 $\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\alpha + \tau_{xy} \sin 2\alpha$
 $\tau_{x'y'} = \frac{\sigma_y - \sigma_x}{2} \sin 2\alpha + \tau_{xy} \cos 2\alpha$
 $\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2} \cos 2\alpha - \tau_{xy} \sin 2\alpha$
 $\tau_{x'y'} = \frac{\sigma_y - \sigma_x}{2} \sin 2\alpha + \tau_{xy} \cos 2\alpha$

5)

$(\epsilon_x = -12, \epsilon_y = 112, \gamma_{xy} = -20) \times 10^{-5}$

(5)

$\epsilon_{ave} = \frac{112 - 12}{2} = 50$

$R = \frac{1}{2} \sqrt{(112 + 12)^2 + 20^2} = 62.80$

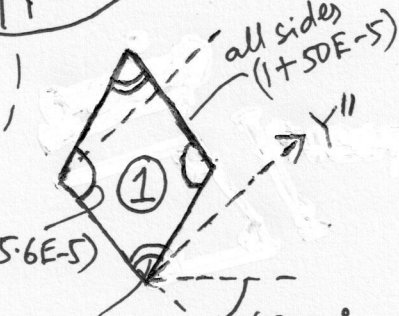
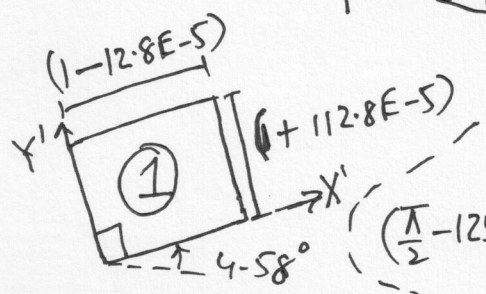
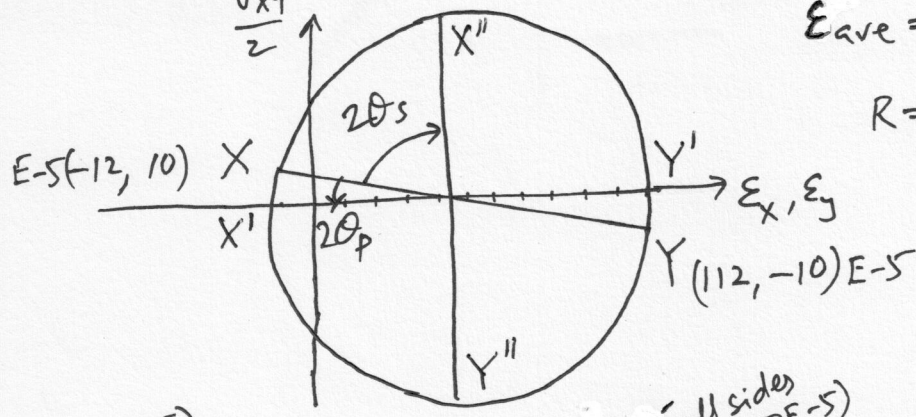
$\epsilon_{max} = 50 + 62.8 = 112.8 \times 10^{-5}$

$\epsilon_{min} = 50 - 62.8 = -12.8 \times 10^{-5}$

$(\gamma_{xy})_{max} = 2R = 125.6 \times 10^{-5}$

$\theta_p = \frac{1}{2} \sin^{-1} \left(\frac{10}{R} \right) = 4.58^\circ$

$\theta_s = \frac{1}{2} \cos^{-1} \left(\frac{10}{R} \right) = 40.42^\circ$



Principal element.

Max γ_{xy} element

$\epsilon_{max, min} = \left(\frac{-12 + 112}{2} \right) \pm \sqrt{\left(\frac{-12 - 112}{2} \right)^2 + \left(\frac{-20}{2} \right)^2} = (112.8, -12.8) \times 10^{-5}$

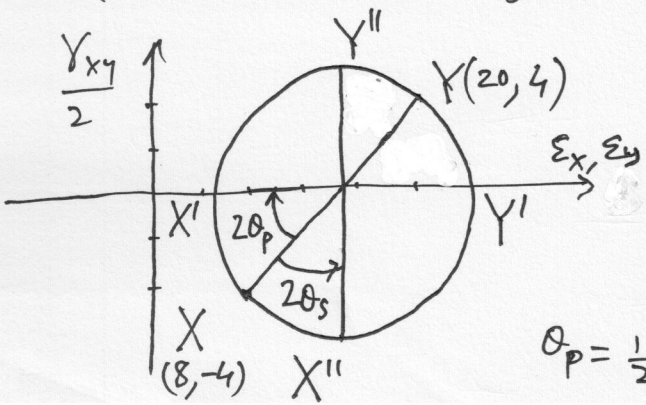
$(\gamma_{xy})_{max} = 2R = 125.6 \times 10^{-5}$

$\theta_p = \frac{1}{2} \tan^{-1} \left(\frac{-20}{-12 - 112} \right) = 4.58^\circ, 94.58^\circ$; $\theta_s = \theta_p \pm 45^\circ = 49.58^\circ, -40.42^\circ$

$\epsilon_{x'} \Big|_{\theta = \theta_p = 4.58} = \left(\frac{-12 + 112}{2} \right) + \left(\frac{-12 - 112}{2} \right) \cos(2 \times 4.58) + \left(\frac{-20}{2} \right) \sin(2 \times 4.58) = -12.8 \times 10^{-5}$. \checkmark so it checks out with Mohr circle

$\gamma_{x'y'} \Big|_{\theta_s = -40.42} = - \left(\frac{-12 - 112}{2} \right) \sin(2 \times -40.42) + (-20) \cos(2 \times -40.42) = -125.6 \times 10^{-5}$. \checkmark so it checks out with Mohr circle.

$(\epsilon_x = 8, \epsilon_y = 20, \gamma_{xy} = 8) \times 10^{-4}$



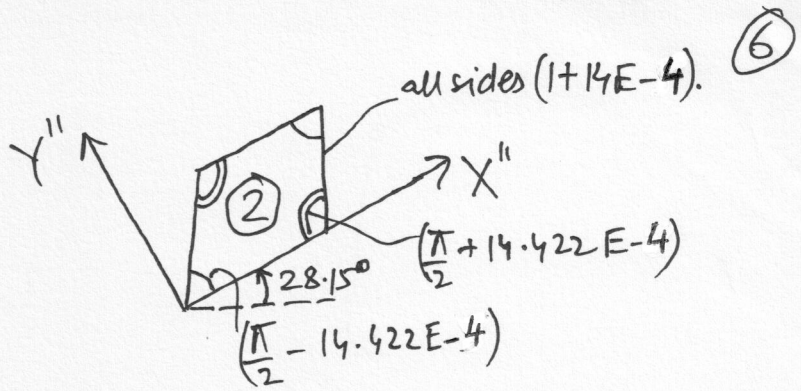
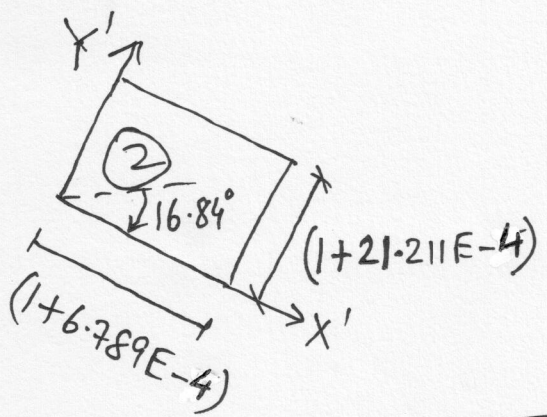
$\epsilon_{ave} = \frac{8 + 20}{2} = 14$

$R = \frac{1}{2} \sqrt{(20 - 8)^2 + 8^2} = 7.211$

$\epsilon_{max} = 14 + 7.211 = 21.211 \times 10^{-4}$, $\epsilon_{min} = 14 - 7.211 = 6.789 \times 10^{-4}$

$(\gamma_{xy})_{max} = 2R = 14.422 \times 10^{-4}$

$\theta_p = \frac{1}{2} \sin^{-1} \left(\frac{4}{R} \right) = 16.84^\circ$, $\theta_s = \left(\frac{\pi}{2} - 2\theta_p \right) \frac{1}{2} = 28.15^\circ$



$$E_{max, min} = \left(\frac{8+20}{2}\right) \pm \sqrt{\left(\frac{8-20}{2}\right)^2 + \left(\frac{8}{2}\right)^2} = (21.211, 6.789)E-4$$

$$(\gamma_{xy})_{max} = 2R = 2\sqrt{2} = 14.422E-4$$

$$\theta_p = \frac{1}{2} \tan^{-1}\left(\frac{8}{8-20}\right) = -16.84^\circ, 73.15^\circ, \theta_s = \pm 45 + \theta_p = 28.16^\circ, -61.84^\circ$$

$$\epsilon_{x'} \Big|_{\theta = \theta_p = -16.84^\circ} = \left(\frac{8+20}{2}\right) + \left(\frac{8-20}{2}\right) \cos(2 \times -16.84) + \frac{8}{2} \sin(2 \times -16.84) = 6.789E-4$$

checks out with Mohr circle ✓

$$\gamma_{x'y'} \Big|_{\theta = \theta_s = 28.16^\circ} = -(8-20) \sin(28.16 \times 2) + 8 \cos(28.16 \times 2) = 14.422E-4$$

checks out with Mohr circle ✓

THEORY.

Principal stresses, max shearing stresses form principal and max shearing strains.

From constitutive law $\tau_{xy} = G \gamma_{xy}$

So $\gamma_{xy} = 0 \Rightarrow \tau_{xy} = 0, \gamma_{xy} = \max \Rightarrow \tau_{xy} = \max.$

So principal planes & maximum shear planes of strain coincide with principal planes & max shear planes of stress.

$$\epsilon_{x'} = \frac{\sigma_{x'}}{E} - \frac{\nu}{E} \sigma_{y'} - \frac{\nu}{E} \sigma_{z'} = \frac{1}{E} [(1-\nu^2) \sigma_{x'} - \nu(1+\nu) \sigma_{y'}]$$

$$\epsilon_{y'} = \frac{\sigma_{y'}}{E} - \frac{\nu}{E} (\sigma_{x'} + \sigma_{z'}) = \frac{1}{E} [(1-\nu^2) \sigma_{y'} - \nu(1+\nu) \sigma_{x'}]$$

$$\epsilon_{z'} = 0 = \frac{\sigma_{z'}}{E} - \frac{\nu}{E} (\sigma_{x'} + \sigma_{y'})$$

∴ plane strain

$$\Rightarrow \sigma_{x'} = [(1-\nu^2) \epsilon_{x'} + \nu(1+\nu) \epsilon_{y'}] \frac{E}{(1-\nu^2)^2 - \nu^2(1+\nu)^2} = \frac{E}{(1+\nu)(1-2\nu)} [(1-\nu) \epsilon_{x'} + \nu \epsilon_{y'}]$$

$$\sigma_{x'} = \frac{E}{(1+\nu)(1-2\nu)} \left[(1-\nu)\epsilon_{x'} + \nu\epsilon_{y'} \right]$$

$$\sigma_{y'} = \frac{E}{(1+\nu)(1-2\nu)} \left[(1-\nu)\epsilon_{y'} + \nu\epsilon_{x'} \right]$$

$$\tau_{z'} = \frac{E\nu}{(1+\nu)(1-2\nu)} (\epsilon_{x'} + \epsilon_{y'})$$

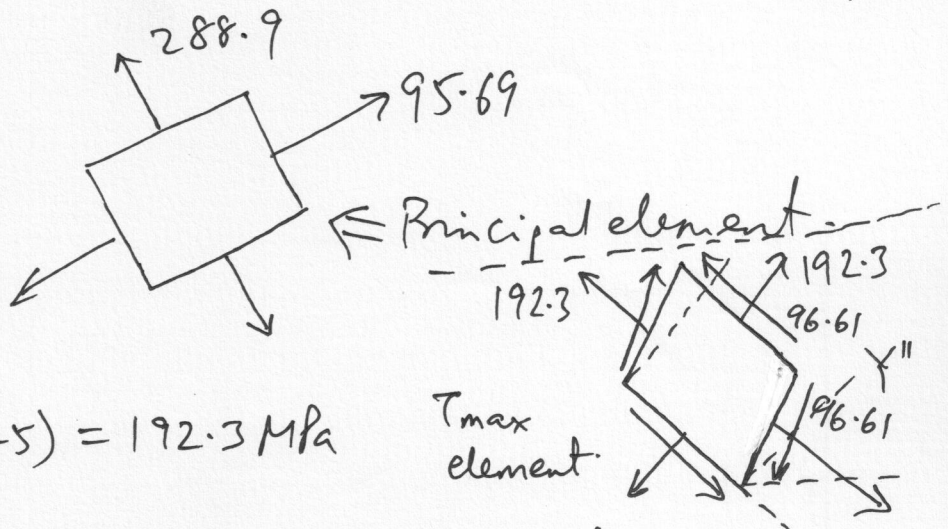
were derived in
L2 - axial loading
chapter (slide 34).

Case I For principal strains
 $\epsilon_{x'} = \epsilon_{min} = -12.8E-5$, $\epsilon_{y'} = \epsilon_{max} = 112.8E-5$, $\gamma_{x'y'} = 0$

$$\sigma_{x'} = \frac{200E3}{(1.3)(0.4)} \left[(0.7)(-12.8E-5) + (0.3)(112.8E-5) \right] = 95.69 \text{ MPa} = \sigma_{min}$$

$$\sigma_{y'} = \frac{200E3}{(1.3)(0.4)} \left[(0.7)(112.8E-5) + (0.3)(-12.8E-5) \right] = 288.9 \text{ MPa} = \sigma_{max}$$

$$\tau_{x'y'} = G\gamma_{x'y'} = 0$$



For max shear strain,
 $\epsilon_{x''} = \epsilon_{y''} = \epsilon_{ave} = 50E-5$
 $\gamma_{x''y''} = -125.6E-5$

$$\sigma_{y''} = \sigma_{x''} = \frac{200E3}{(1.3)(0.4)} (50E-5) = 192.3 \text{ MPa}$$

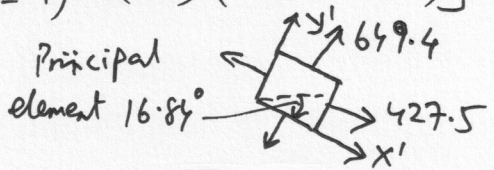
$$\tau_{x''y''} = G\gamma_{x''y''} = \frac{200E3}{2(1.3)} (-125.6E-5) = -96.61 \text{ MPa} = (\tau_{xy})_{max}$$

Case II For principal strains, $\epsilon_{x'} = \epsilon_{min} = 6.789E-4$, $\epsilon_{y'} = \epsilon_{max} = 21.211E-4$, $\gamma_{x'y'} = 0$

$$\sigma_{x'} = \frac{200E3}{(1.3)(0.4)} \left[(0.7)(6.789E-4) + (0.3)(21.211E-4) \right] = 427.5 \text{ MPa} = \sigma_{min}$$

$$\sigma_{y'} = \frac{200E3}{(1.3)(0.4)} \left[(0.7)(21.211E-4) + (0.3)(6.789E-4) \right] = 649.4 \text{ MPa} = \sigma_{max}$$

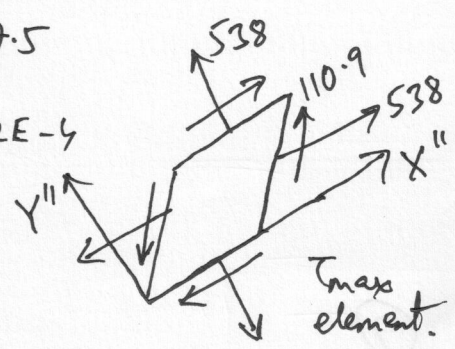
$$\tau_{x'y'} = G\gamma_{x'y'} = 0$$



For max shear strains, $\epsilon_{x''} = \epsilon_{y''} = 14E-4$, $\gamma_{x''y''} = 14.422E-4$

$$\sigma_{x''} = \sigma_{y''} = \frac{200E3}{(1.3)(0.4)} (14E-4) = 538 \text{ MPa}$$

$$\tau_{x''y''} = G\gamma_{x''y''} = \frac{200E3}{2(1.3)} (14.422E-4) = 110.9 \text{ MPa}$$



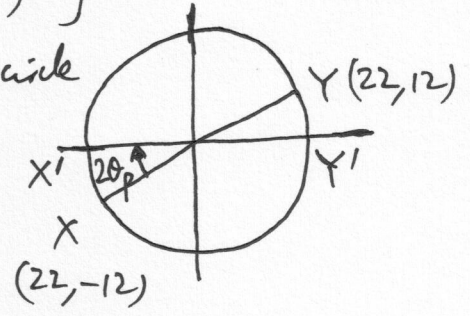
6) $\epsilon_x = \epsilon_0 = -22E-5$, $\epsilon_y = \epsilon_{90} = 22E-5$

$\epsilon_{45} = 12E-5 = \left(\frac{-22+22}{2}\right)E-5 + \frac{\gamma_{xy}}{2} \Rightarrow \gamma_{xy} = 24E-5$

$\epsilon_{max,min} = \left[\frac{(-22+22)}{2} \pm \sqrt{\left(\frac{-22-22}{2}\right)^2 + \left(\frac{24}{2}\right)^2}\right]E-5 = \pm 25.06E-5$

$\theta_p = \frac{1}{2} \sin^{-1}\left(\frac{12}{R}\right) = 14.31^\circ$

here used Mohr circle to determine planes of $\epsilon_{min}, \epsilon_{max}$

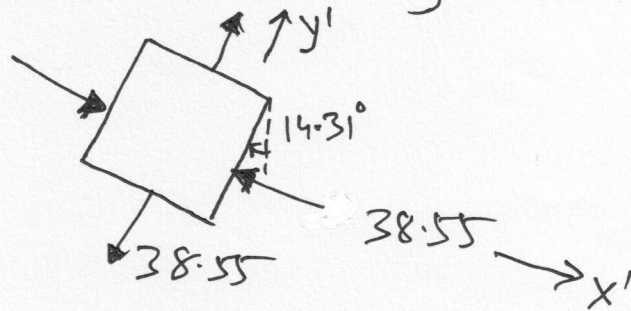


$\Rightarrow \epsilon_{x'} = \epsilon_{min} = -25.06E-5$, $\epsilon_{y'} = \epsilon_{max} = 25.06E-5$

$\gamma_{x'y'} = 0$

$\sigma_{x'} = \frac{200E3}{(1.3)(0.4)} [(0.7)(-25.06) + (0.3)(25.06)]E-5 = -38.55 \text{ MPa}$

$\sigma_{y'} = 38.55 \text{ MPa}$



7) $\epsilon_x = \epsilon_0 = 4E-4$

$\epsilon_{60} = 4E-4 = \epsilon_x \frac{1}{4} + \epsilon_y \frac{3}{4} + \gamma_{xy} \frac{\sqrt{3}}{2} \cdot \frac{1}{2} = 1E-4 + \frac{3}{4}\epsilon_y + \frac{\sqrt{3}}{4}\gamma_{xy}$

$-6E-4 = \epsilon_{120} = \epsilon_x \frac{1}{4} + \epsilon_y \frac{3}{4} + \gamma_{xy} \frac{\sqrt{3}}{2} \left(-\frac{1}{2}\right) = 1E-4 + \frac{3}{4}\epsilon_y - \frac{\sqrt{3}}{4}\gamma_{xy}$

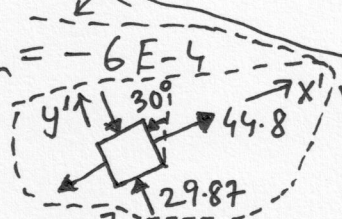
$\Rightarrow \epsilon_y = \frac{2}{3} \cdot (-4E-4) = -\frac{8}{3}E-4$, $\gamma_{xy} = \frac{2}{\sqrt{3}} \cdot (10E-4) = \frac{20}{\sqrt{3}}E-4$

$\epsilon_{max,min} = \frac{(4 - \frac{8}{3})}{2} \pm \sqrt{\left(\frac{4 + \frac{8}{3}}{2}\right)^2 + \left(\frac{20}{2\sqrt{3}}\right)^2} = (7.33, -6) * E-4$

$\theta_p = \frac{1}{2} \tan^{-1} \left[\frac{20/\sqrt{3}}{(4 + \frac{8}{3})} \right] = 30^\circ, 120^\circ$; $\epsilon_{30} = \left(4 \cdot \frac{3}{4} - \frac{8}{3} \cdot \frac{1}{4} + \frac{20}{\sqrt{3}} \cdot \frac{1}{2} \cdot \frac{1}{2}\right)E-4 = 7.33E-4$

$\epsilon_{x'} = \epsilon_{max} = 7.33E-4$; $\epsilon_{y'} = \epsilon_{min} = -6E-4$

$\gamma_{x'y'} = 0$



here used transformation formulae to determine planes of $\epsilon_{max}, \epsilon_{min}$ (as an alternative to using Mohr's circle, as done above).

$\sigma_{x'} = \frac{70E3}{(1.25)(0.5)} [(0.75)(7.33) + (0.25)(-6)]E-4 = 44.8 \text{ MPa}$

$\sigma_{y'} = \frac{70E3}{(1.25)(0.5)} [(0.75)(-6) + (0.25)(7.33)]E-4 = -29.87 \text{ MPa}$