Principle of superposition

Solution (displacement, internal forces/moments/stresses) at a point due to multiple external loads on a structure is equal to sum (superposition) of solution due to each individual ext. load applied. For this to be valid, must have

a) Linear elastic material, ie., Hooke's law valid

b) Small displacements/rotations, so that forces dont undergo significant change in direction as a result of deformation and also dimensions that are used to compute moment equilibrium do not change appreciably.

Equilibrium.

$$\sum \underset{\sim}{F} = 0 \; , \; \sum \underset{\sim}{M} = 0$$

3 eqns each, ie 6 eqns for 3-D structure

$$\sum F_x = 0, \; \sum F_y = 0 \; , \; \sum M = 0$$

3 eqns for planar (2-D) structure.

choose x, y axes and O to yield simplest soln.

Static Determinacy, Stability.

r = nos. of unknown reactions (i.e., ext & int forces, moments)
n = nos of parts/components of structure.

For planar structure,

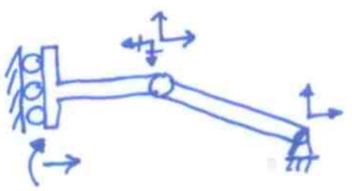
$r = 3n$ is Necessary (not sufficient) condt for SD.

$r > 3n \Rightarrow$ SID to degree $(r - 3n)$, Necessary condt.

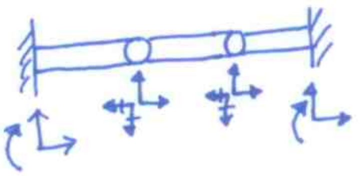
General definition of determinacy: It is the ability to solve for all reactions internal forces/moments by rigid body statics alone. May have to dismember structure into its components and solve for the interconnection forces during the process.

Indeterminate structure: requires additional (compatibility) equations relating applied loads and reactions to displacements/rotations. These equations involve geometric & material properties of structure.

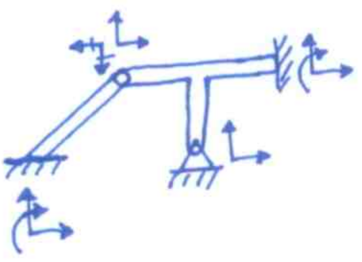
Ex 1



$r = 6$, $3n = 6$, SD.

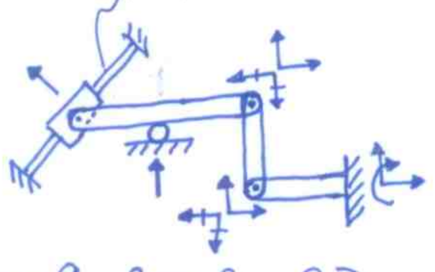


$r = 10$, $3n = 9$, SID to degree 1.



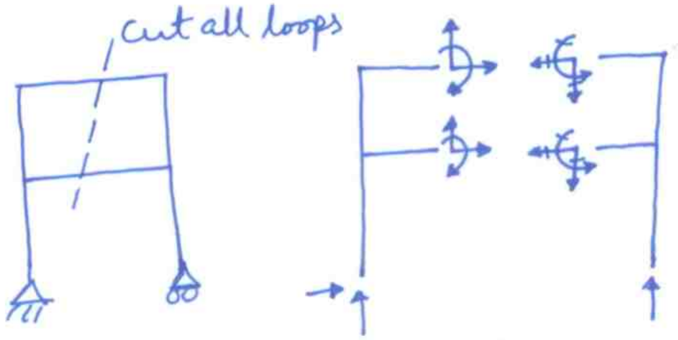
$r = 10$, $3n = 6$, $\Rightarrow$ SID to degree 4

a support - not part of structure.



$r = 9$, $3n = 9$, SD.

Ex 2



$r=9$, $3n=6$,
<u>SID to degree 3</u>

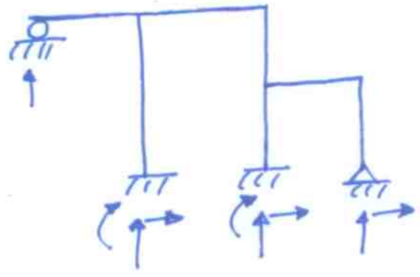
Cut all loops, draw FBD's and proseed as shown above.

equivalent explanation
- EID = Deg of ext indeterminacy = 0
- IID = Deg of int indeterminacy = 3 (assuming ext reactions are somehow known even if not ext. det)
- DOI = Deg of indet = EID + IID = 3, i.e, same result as above.

- Ext determinate means ext reactions solvable w/o dismembering, ie from ext equil.
- In general each loop adds '3' to deg of IID.
- IID = nos of int. reactions − 3(n−1)

Note: The '−1' in (n−1) above is due to fact that one set of 3 eqns is removed when considering the set of dismembered FBD's, since ext. equil eqns are not independent of this set of dismembered FBD's eqns.

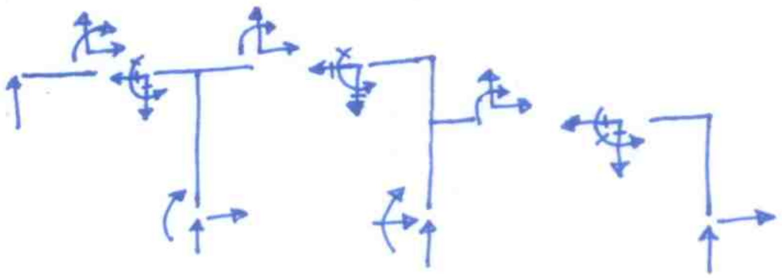
Ex 3



$r=9$, $3n=3$, <u>SID to degree 6 = DOI</u>

equivalent explanation
- EID = 9 − 3 = 6
- IID = 0 (∵ all int ~~forces~~ reactions known once ext reactions known).
- Deg of indet = 6 + 0 = 6 → same as above

or another alternative FBD breakup is,
(yields same result)



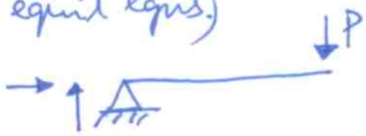
$r=18$, $3n=12$,
SID to degree 6 = DOI

eqv. expl.
- EID = 9 − 3 = 6
- IID = 9 − 3(4 − 1) = 0
- deg of indet = 6 + 0 = 6

Stability

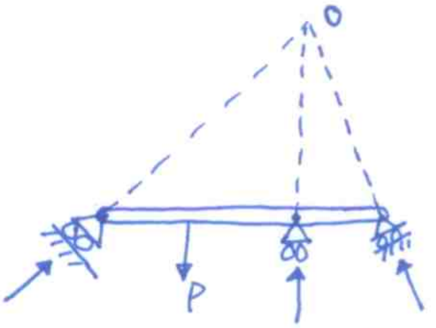
Partial constraints: $r < 3n$

(i.e., fewer reactions than equil eqns)



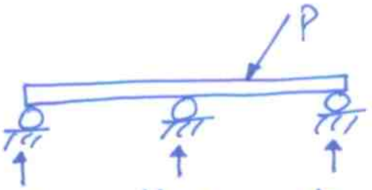
$r = 2$, $3n = 3$
so Equil can't be maintained.

Improper constraints: support reactions are either concurrent or parallel.



Concurrent reactions

$r = 3$, $3n = 3$, yet we cannot find 'r' reactions since unstable due to concurrent support reactions, i.e., $\sum M_o = 0$ not satisfied.
To satisfy $\sum M_o = 0$ we require either large (∞) reactions or displ/rot.



Parallel reactions

$r = 3$, $3n = 3$, yet cannot find 'r' reactions $\because$ $\sum F_x = 0$ not satisfied. It will translate horizontally.

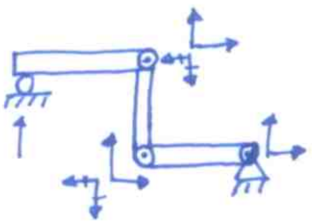
Summary/general definition of geometric instability: Structure moves/collapses due to partial/improper constraints, whence equilibrium violated. Thus,

$r < 3n \Rightarrow$ unstable (US).

$r \geq 3n \Rightarrow$ unstable iff reactions are concurrent or parallel or part of structure forms a collapsible mechanism.

- Once structure found to be unstable, question of SD or SID doesn't arise. $\boxed{ST \Leftrightarrow SD \text{ or } SID}$

Ex 4

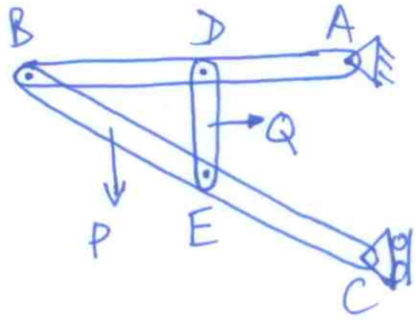


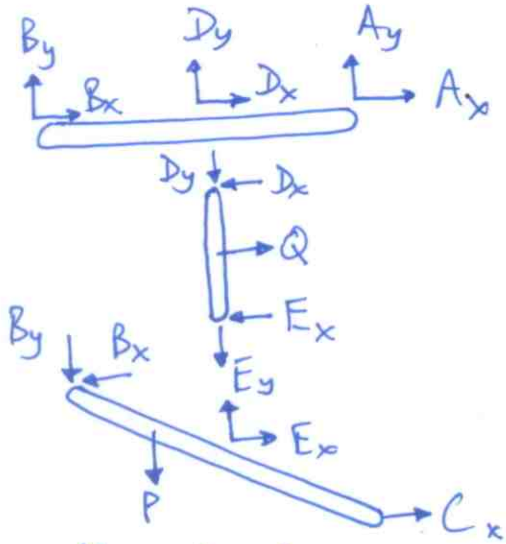
$r = 7$, $3n = 9$, $r < 3n$
$\Rightarrow$ unstable.
Structure will collapse is seen by inspection.

- In general if you can solve for all reactions (ext & int. forces) then SD & Stable.

Examples – Equilibrium.

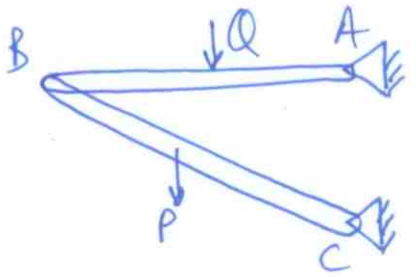
Ex 5

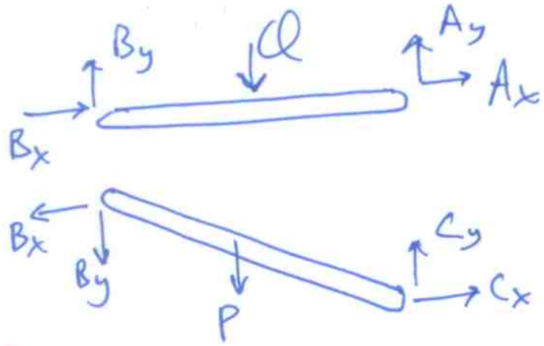




$r=9,\ 3n=9$, SD

Structure rigid (doesent collapse) when removed from supports. In that case ext reactions determined by ext equil applied to whole structure.





$r=6,\ 3n=6$, SD.

Structure non-rigid (collapses) when removed from supports. Thus, we have to dismember to find ext reactions.

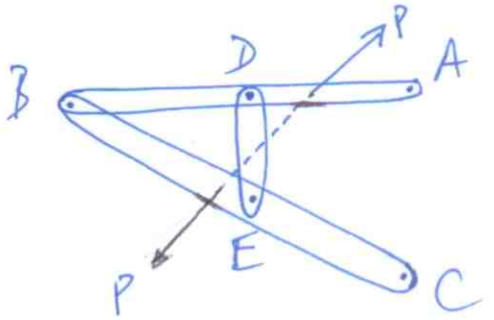
Conclusion from this example is that if system is SD and stays rigid when removed from supports, then ext. reactions solvable by ext equil (i.e., w/o dismembering). If SD but non-rigid when removed from supports then you must dismember to solve for ext reactions also. It is obvious since if it stays rigid, then we know that a planar rigid body requires exactly 3 ext reactions to maintain equil under general loading. Thus,

$$EID = 0$$

$$SD \Rightarrow \text{Deg of indet} = 0 = EID + IID$$

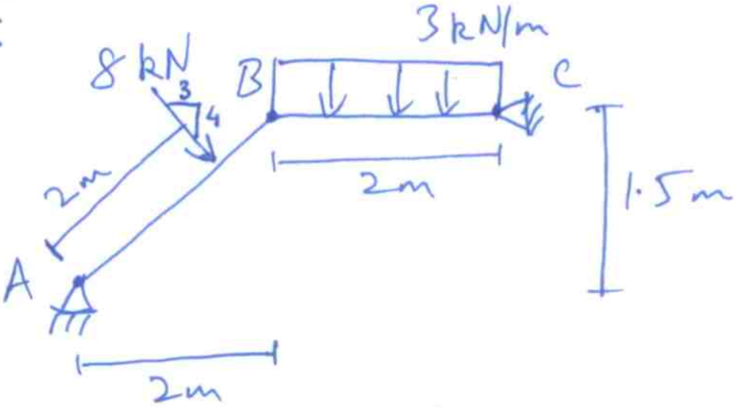
$$\Rightarrow IID = 0$$

This means that the rigid-without-supports structure maintains equilibrium when detached from supports and loaded in a special manner with equal & opp forces (P) applied at two points that lie along the line of action of the applied loads, as shown below

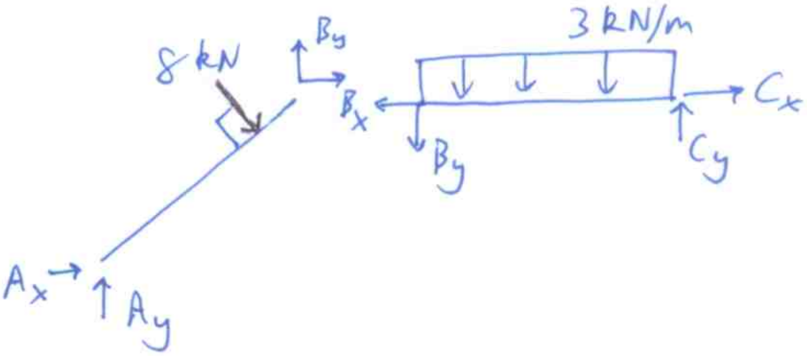


This yields int forces but requires no (ie zero) ext reactions to maintain equilibrium.
On the other hand, with link DE removed, ie nonrigid-without-supports structure, it wont maintain equil even for the above special loading. This implies nos of int unknowns is less than that for the rigid-without-supports structure. So for this case it is like $IID < 0$. But structure still SD, ie $EID + IID = 0$, ie $\underline{EID > 0}$, ie ext indet but SD so we need to dismember to solve.

Ex 6.



Find reactions at A, B, C pins.



$\Sigma M_B = 0 \Rightarrow C_y = \frac{(3)(2)(1)}{2} = 3\,kN$; $\Sigma F_y = 0 \Rightarrow B_y = 3 - 3(2) = -3\,kN$

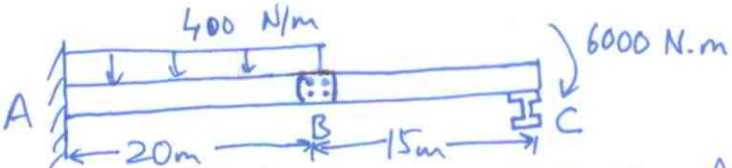
$\Sigma M_A = 0 \Rightarrow B_x = \frac{(-3)(2) - (8)(2)}{1.5} = -14.67\,kN$

$\Sigma F_y = 0 \Rightarrow A_y = 3 + 8\left(\frac{4}{5}\right) = 9.4$; $\Sigma F_x = 0 \Rightarrow C_x = -14.67\,kN$

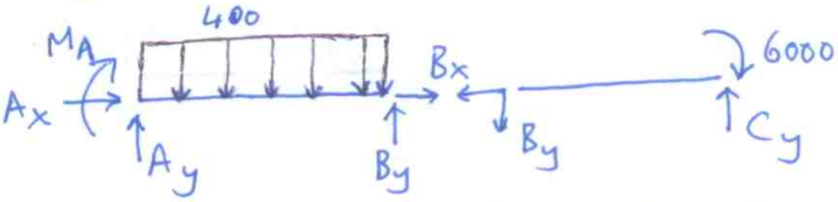
$\Sigma F_x = 0 \Rightarrow A_x = +14.67 - 8\left(\frac{3}{5}\right) = 9.87\,kN$

Note: as always the +ve, -ve signs are w.r.t. FBD's.

Ex 7



Find reactions at A, B, C. Assume B is pin, C is roller.



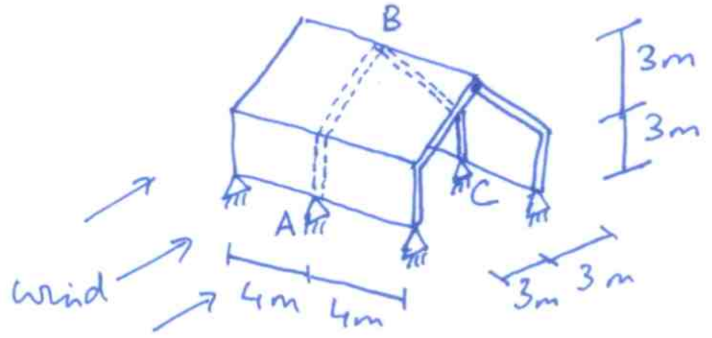
$C_y = \frac{6000}{15} = 400N = B_y$; $A_y = (400)(20) - 400 = 7600\,N$

$B_x = 0 = A_x$, $M_A = (400)(20)(10) - (7600)(20) = -72000$ N.m

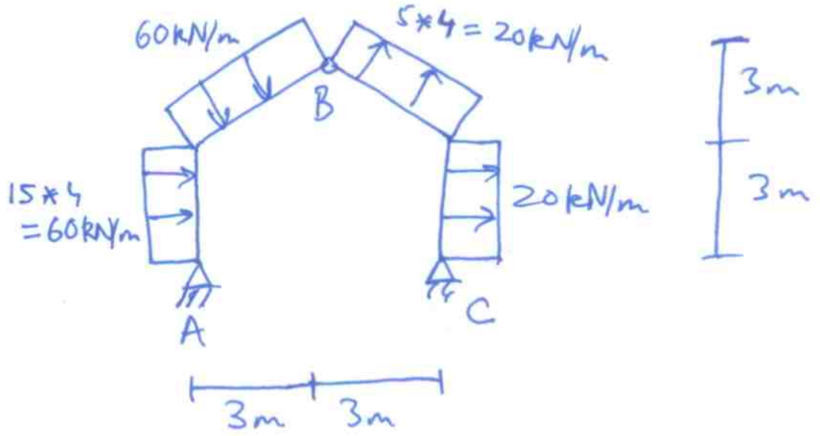
Ex 8

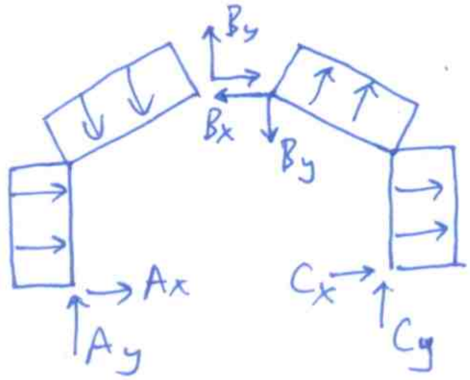
<u>Given</u>: Building <u>comprising</u> <u>3 gable arches</u> that are pin connected at their vertices (e.g. at B for central arch), <u>plus walls and roof.</u>

Windward pressure = 15 kPa
Leeward pressure = 5 kPa. (<u>suction</u>)



<u>Find</u>: Reactions at Pins A, B, C.





$$(\Sigma M_C)_{ext\,FBD} = 0 \Rightarrow A_y = \frac{\left\{(-60)(3)(1.5) + (-20)(3)(1.5) + (60)(3\sqrt{2})\left(\frac{1}{\sqrt{2}}\right)(4.5-4.5) + (20)(3\sqrt{2})\left(\frac{1}{\sqrt{2}}\right)(-1.5-4.5)\right\}}{6}$$

$$= \underline{-120\text{ kN}}$$

$$C_y = +120 + (60)(3\sqrt{2})\left(\frac{1}{\sqrt{2}}\right) - (20)(3\sqrt{2})\left(\frac{1}{\sqrt{2}}\right) = \underline{240\text{ kN}}$$

$$B_y = 240 + (20)(3\sqrt{2})\left(\frac{1}{\sqrt{2}}\right) = \underline{300\text{ kN}}$$

$$\Sigma M_B = 0 \Rightarrow -\left\{-(-120)(3) + (60)(3)(4.5) + (60)\frac{(3\sqrt{2})^2}{2}\right\}/6 = A_x$$

i.e, $\underline{A_x = -285\text{ kN}}$

$$C_x = -A_x - (60+20)(3) - (60+20)(3\sqrt{2})\left(\frac{1}{\sqrt{2}}\right)$$
$$= 285 - 480 = \underline{-195\text{ kN}}$$

$$B_x = C_x + (20)(3) + (20)(3\sqrt{2})\left(\frac{1}{\sqrt{2}}\right) = -195 + 120 = \underline{-75\text{ kN}}$$