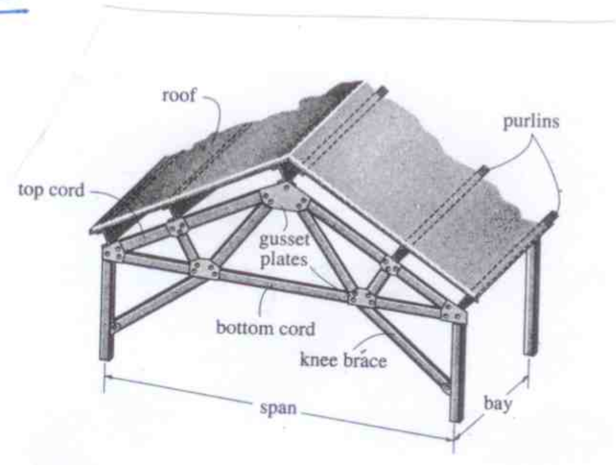
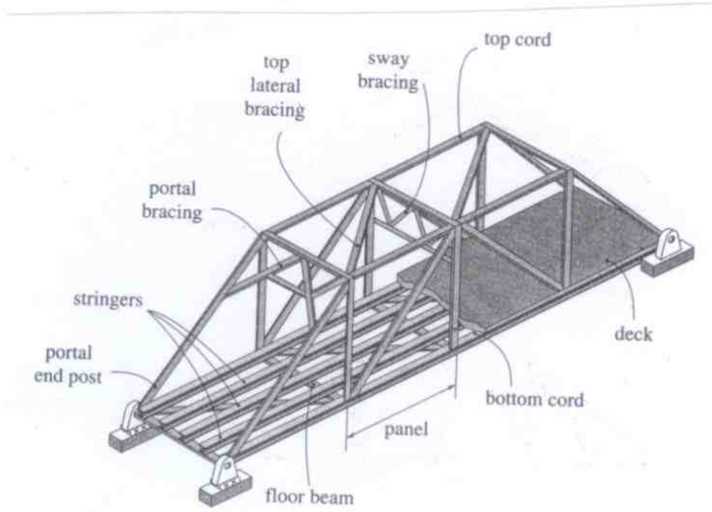
TRUSSES. (Topic - 1).



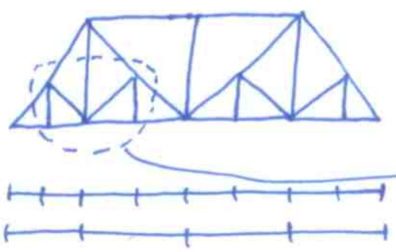
Roof Truss



Bridge Truss

Bottom (Top) Lateral bracings connect Bot. (top) chords. These, along with sway bracings and portal bracings help resist lateral forces caused by wind, earthquake, or vehicle - sidesway.

Optimal inclination of members is 45°–60° for economy. Thus, as span ↑, depth ↑, panel length ↑, and we go for sub-divided truss.



→ sub-divided due to additional "M" members.

} Panel lengths reduce after sub-division ⇒ floor beam inter-spacing reduces, so load per floor beam within limits.

Assumptions.

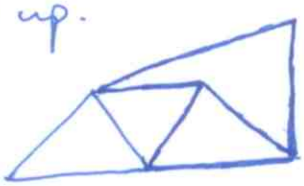
1) Joints are smooth pins. Valid if members are concurrent at joint. Actually not so, which yields bending stresses but these are secondary in value.

2) Loaded at joints. This is generally true, ie., achieved by purlins or stringer-floorbeam system in roofs & bridges, respectively. When member self wt considered, apply half at each end (joint).

(1) & (2) ⇒ only AF in members.

Classification (Planar trusses)

1) Simple Truss: Built up by triangulation algorithm. Start with basic △, add 2 members and 1 joint and build up.

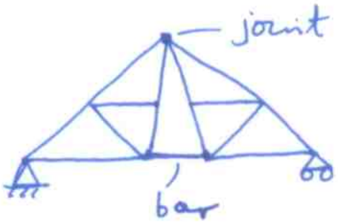


→ doesnt necessarily comprise triangles only.

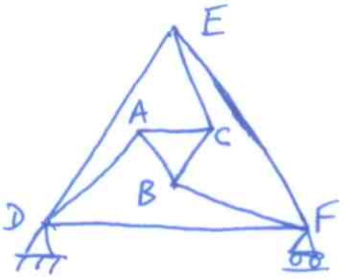
2) Compound Truss.

Formed by connecting simple trusses.

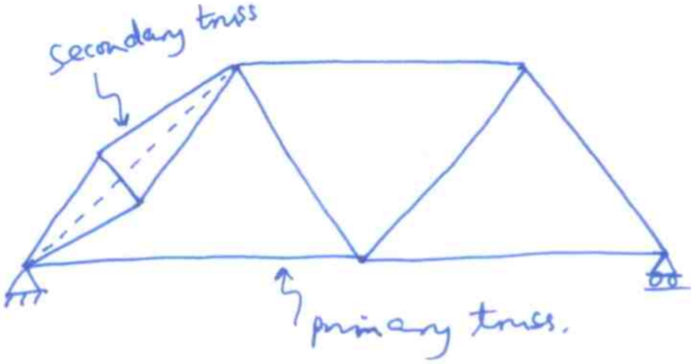
Type 1: Trusses connected by common joint & bar.



Type 2: Trusses joined by three bars.

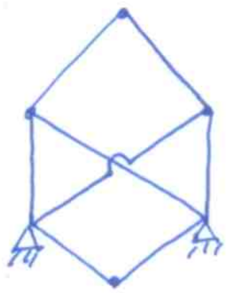


Type 3. Bars of large simple truss (called primary truss) substituted by smaller simple trusses (called secondary truss).



3) Complex Truss

Neither simple nor compound.



Determinacy, Stability

Determinacy

r = nos of ext. reactions.

b = nos of bars (members)

j = nos of joints.

$b + r = 2j$ → necessary (not sufficient) for being statically determinate

$b + r > 2j$ → statically indeterminate if stable (i.e. only necessary condt. for SID).

Stability

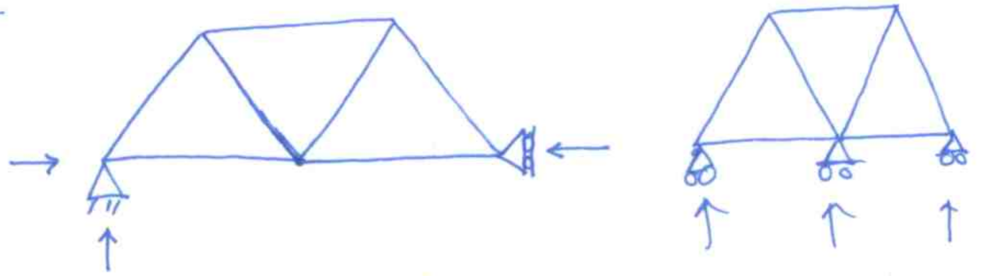
$b + r < 2j \rightarrow$ unstable ($\because$ all equil eqns can't be satisfied).

$\left.\begin{array}{l} b+r=2j \\ b+r>2j \end{array}\right\} \rightarrow$ could still be unstable. This happens if truss is externally or internally unstable. Then, we won't be able to solve all $(b+r)$ unknowns, ie, here also all equil eqns can't be satisfied.

External instability: Concurrent/parallel reactions.

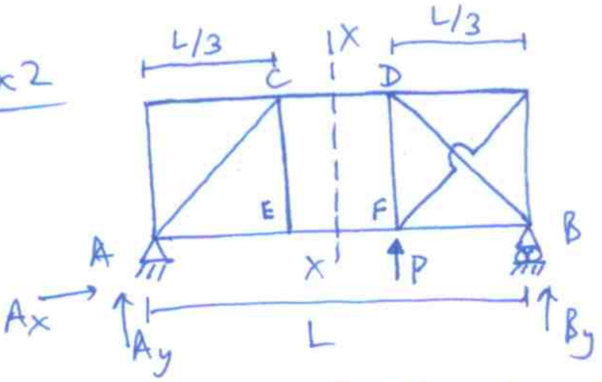
Internal instability: Faulty arrangement of members (bars).

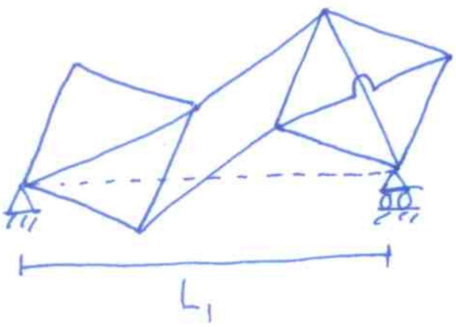
Ex1



Ext US.

Ex2





Int US.

$L_1 < L$

$b + r = 13 + 3 = 16, \quad 2j = 16.$

However, faulty arrangement of bars allows roller to move left, as shown, and middle ~~rectangle~~ square becomes parallelogram, ie relative motion of bars/joints occurs.

Another way to see it is by applying arbitrary load, say P. Then 3 ext reactions (A_x, A_y, B_y) determined from ext equil (ie, $0, -\frac{P}{3}, -\frac{2P}{3}$, respectively). Consider section XX.

$\Sigma M_A = 0 \Rightarrow F_{CD} = 0, \ \Sigma M_B = 0 \Rightarrow P\left(\frac{L}{3}\right) = 0 \rightarrow$ contradiction

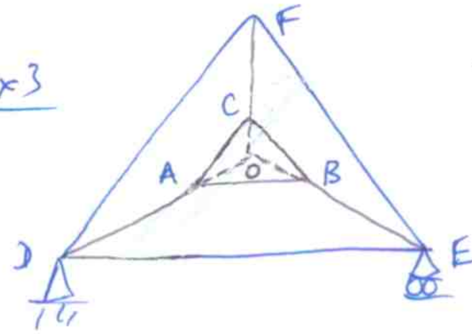
ie, equil can't be satisfied.

Valid even if support at B is pinned, ie $b+r=17$. In that case the parallelogramming, when truss is

still on supports, wont occur. However, still <u>Int US</u>.
$\therefore$ equilibrium unsatisfiable.

In both cases, detach truss from supports. Then, relative displacement (parallelogramming) can occur. This is another way of testing trusses for internal instability, i.e, detach from supports and check for rel. displ. This is possible only for trusses, $\because$ supports cant be fixed else members wont carry only AF, and hence $P\left(\frac{L}{3}\right)$ cannot be non-zero thus implying equil unsatisfiable.

<u>Ex3</u>



Type 2 compound truss.
Isolate inner truss. <u>Reactions</u> (R_{CF}, R_{DA}, R_{BE}) <u>concurrent</u> at O, so equil not possible. <u>Int US</u>.

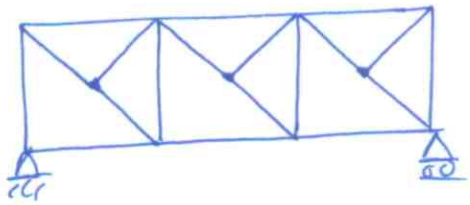
<u>General approach to check instability</u>: If you can solve for all member forces of SD or SID truss then it is stable. For general SD truss, solve $2j$ equations (computer soln if necessary), and if soln possible then stable.

Thus a SD simple truss is always stable $\because$ soln is always possible due to triangulation algorithm of building the truss and the solution algorithm whereby we start from a support joint (whose reactions are known due to SD) and work inwards using M.O.J. (method of joints).

If unstable, we dont discuss SD or SID as it is meaningless.

Note: For simple truss, $b = 2j-3$; for SD simple truss $r=3$ $\Rightarrow b+r=2j$.

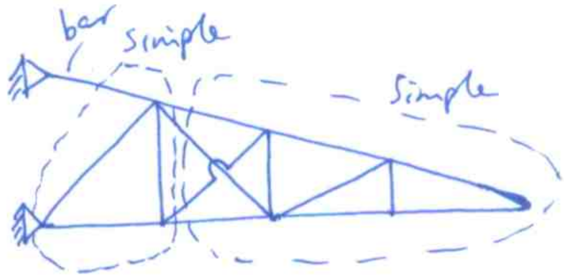
Ex 4. Classify SD, SID, ST, US.



Compound, Type 1.

$b = 19$, $j = 11$, $r = 3$, $b + r = 2j$

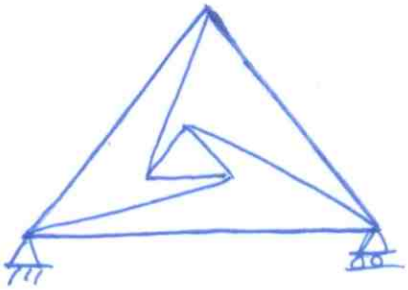
ST, SD.



Complex truss.

$b = 15$, $r = 4$, $j = 9$, $b + r > 2j$

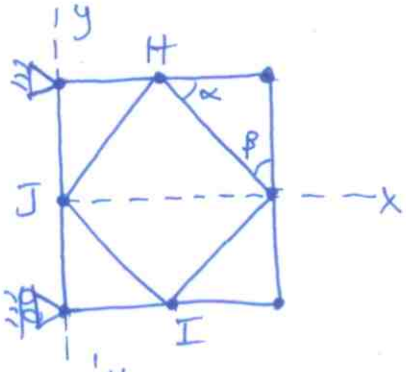
ST, SID



Compound, Type 2

$b = 9$, $r = 3$, $j = 6$, $b + r = 2j$

ST, SD.

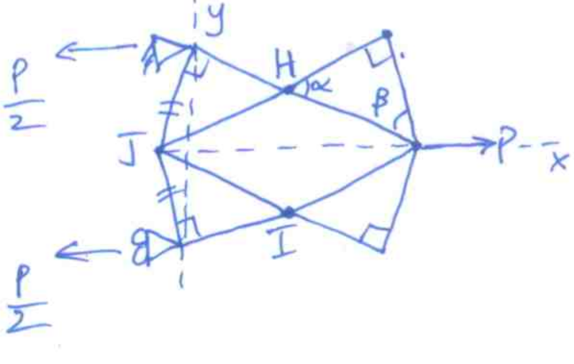


$b = 12$, $r = 3$, $j = 8$, $b + r < 2j$

US.

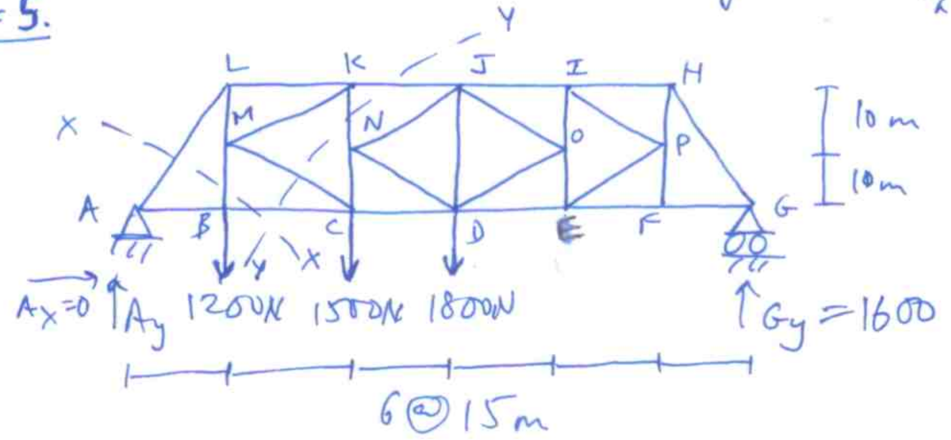
$\because$ ext ST (ie non-parallel, non-concurrent reactions)

hence int U.S.



Inner rhombus flattens, outer triangles rotate maintaining their original shape, roller moves up. Zero force members shown for P applied as shown. Joint J equilibrium in y-dir requires F_{HJ}, F_{HI} to be both (T) or (C), whence x-equil of jt J is violated.

Ex 5. Problem to review method of sections, M.O. joints, & zero for zero force members.



Find: Forces in BC, MC, zero force members, force in HI, HP

section XX $\Rightarrow \Sigma M_L = 0 = G_y(5 * 15) - F_{BC}(20) - 1500(15) - 1800(30)$

overall truss $\Rightarrow \Sigma M_A = 0 \Rightarrow G_y = \dfrac{(1200 + 1500(2) + 1800(3)) * \cancel{15}}{6 * \cancel{15}}$

$= 1600$

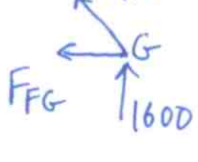
$\Rightarrow F_{BC} = 2175 (T)$

section YY $\Rightarrow \Sigma M_K = 0 = 1600(60) - 2175(20) - F_{MC}\left(\frac{15}{\sqrt{325}}\right)(20)$

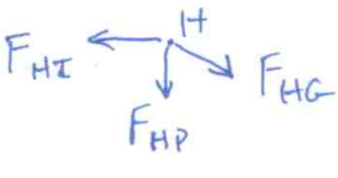
$- 1800(15)$

$F_{MC} = 1532.3 (T)$

zero force mem: FP



$F_{HG} = -1600\left(\frac{25}{20}\right) = -2000 (C)$



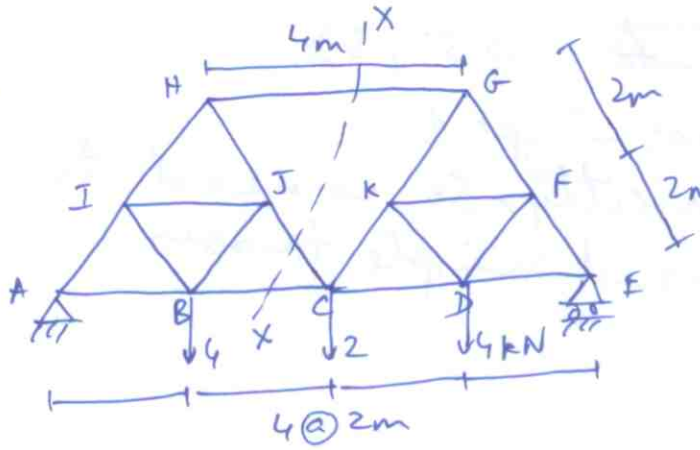
$F_{HI} = F_{HG}\left(\frac{15}{25}\right) = -1200 (C)$

$F_{HP} = -F_{HG}\left(\frac{20}{25}\right) = 1600 (T)$

M.O.S. guidelines (reviewed from CE102): (1) Pass section thru at most 3 members or if more than 3, then others zero-force mem's.
(2) Sum moments about point of intersection of unknowns to yield remaining unknown directly

MOS guidelines (contd): (3) If all but one unknowns are parallel, sum forces along direction $\perp^{ar}$ to them to directly obtain remaining unknown.

Ex 6.



Compound – Type 1

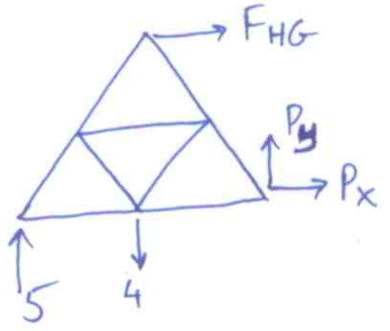
Simple trusses joined at jt. C and by link HG

$j = 11$, $r = 3$, $b = 19$, ST, SD.

Solve entire truss.

Isolate simple trusses and proceed.

$A_x = 0$, $A_y = E_y = 5$ kN.



LHFBD

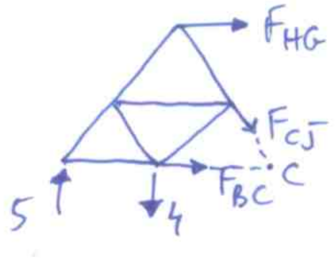
Does not matter whether we give 2kN load at pin C to LHFBD or RHFBD, results will be consistent (ie it is like deciding arbitrarily to give pin C to LHFBD or RHFBD, which is allowed & consistent. In this case pin C (where 2kN applied) given to RHFBD.

$$\Sigma M_c:\ F_{HG} = -\frac{(5)(4) + (4)(2)}{(4)\left(\frac{\sqrt{3}}{2}\right)} = -3.46 \text{ (C)}$$

$P_y = 4 - 5 = -1$; $P_x = -F_{HG} = 3.46$.

Now use these and MOJ to solve internal forces in the simple truss shown in LHFBD

Alternatively, use section XX.

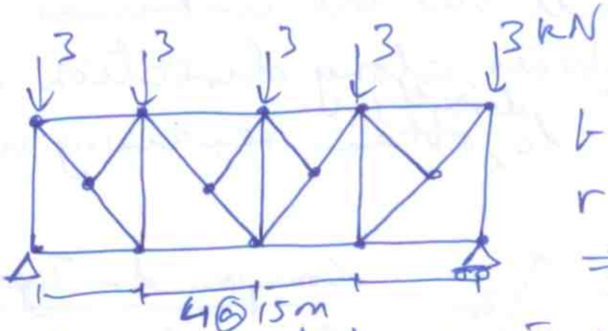


$\Sigma M_c = 0$ gives F_{HG} as above.

$\Sigma F_x = \Sigma F_y = 0$ gives F_{CJ}, F_{BC}.

Then proceed by MOJ.

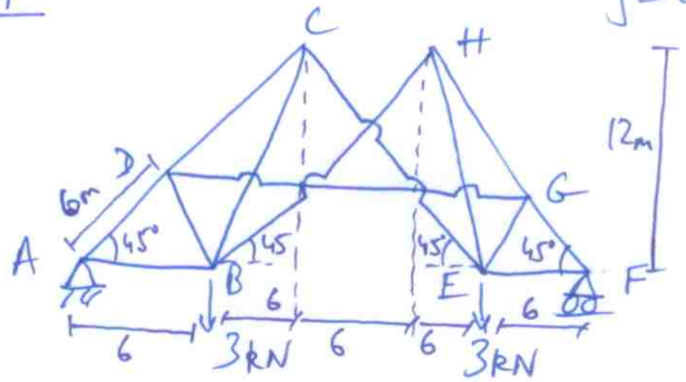
Ex 6a



$b = 25$, $j = 14$
$r = 3$
$\Rightarrow$ ST, SD.

Compound truss – Type 1.
M.O.J. works directly, so no need to isolate component simple trusses.

Ex 7



$j=6,\ r=3,\ b=9$, SD, ST

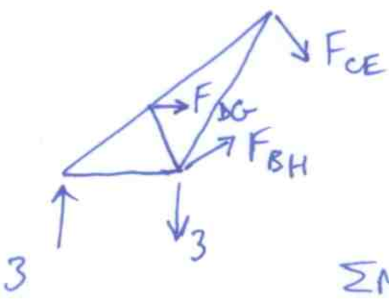
DG, CE, BH are connecting links.

Compound - Type 2.

Isolate simple trusses and proceed.

(NOTE: This is same as M.O.S. for this problem with section passing thru CE, BH, DG)

$$A_y = F_y = 3,\quad F_x = 0$$



$$F_{CE}\left(\frac{1}{\sqrt{2}}\right) = F_{BH}\left(\frac{1}{\sqrt{2}}\right) + 3 - 3 \Rightarrow F_{CE} = F_{BH}$$

$$F_{CE}\left(\frac{1}{\sqrt{2}}\right) + F_{BH}\left(\frac{1}{\sqrt{2}}\right) + F_{DG} = 0$$

$$\Sigma M_B = 0 \Rightarrow F_{DG}\left(\frac{6}{\sqrt{2}}\right) + 3(6) + F_{CE}\left(\frac{1}{\sqrt{2}}\right)[(6)+(12)] = 0$$

$$\Rightarrow \left[-F_{DG}\left(\frac{6}{\sqrt{2}}\right) - 18\right]\left(\frac{\sqrt{2}}{18}\right)\left(\frac{2}{\sqrt{2}}\right) + F_{DG} = 0$$

$$F_{DG} = 3.783\ (T)$$

$$F_{CE} = F_{BH} = -2.675\ (C)$$

Now proceed with MOJ for the simple truss to get F_{AB}, F_{BC}, F_{AD}, F_{DB}, F_{DC}.

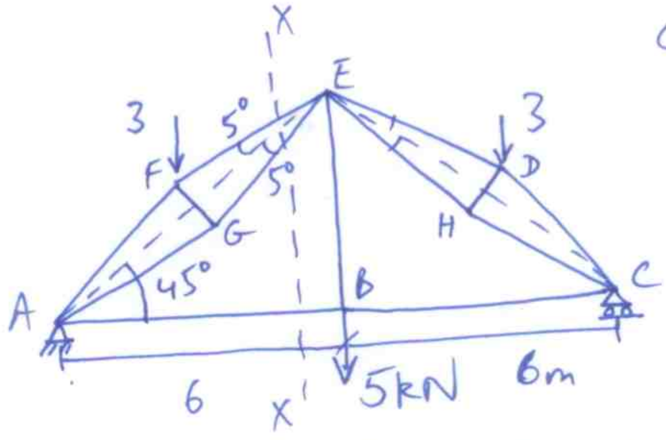
———— X ————

NOTE: In all SD trusses, MOJ gives $2j$ eqns, $\because 2j = b + r$ for SD, we can solve these eqns simultaneously (may require computer).

For all SD trusses (whether simple, compound, complex), first try MOJ. If hand-solution *not possible by MOJ, then do solution by isolating the component simple trusses as explained in Ex 6-9 in order to get soln by hand instead of computer.

* in case of compound or complex truss.

Ex 8.



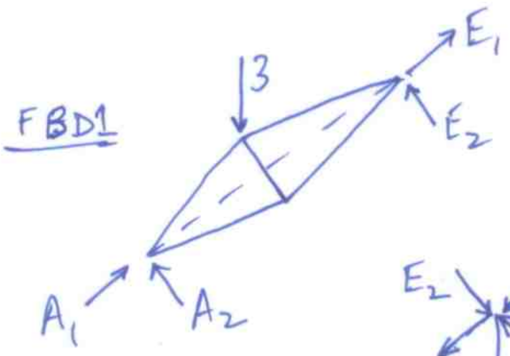
Compound - Type 3

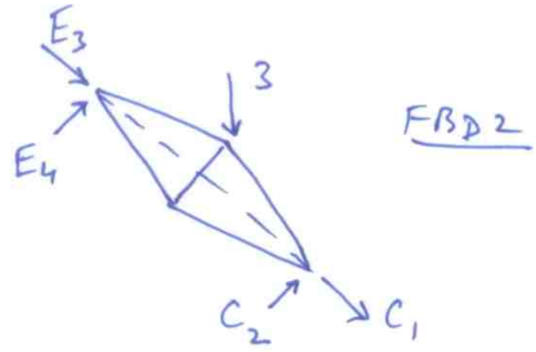
$r=3,\ b=13,\ j=8$, SD, ST

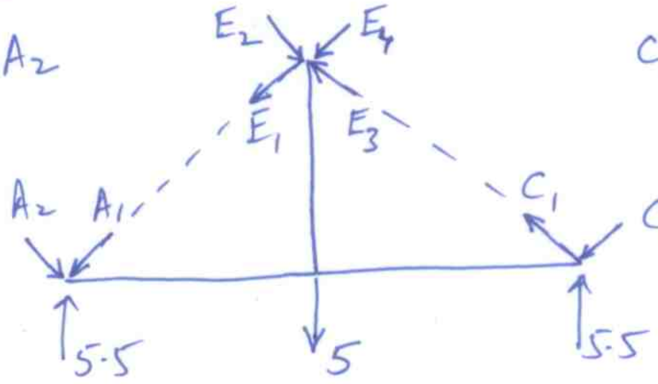
$A_x=0,\ A_y=B_y=5.5$ kN

NOTE: Problem done in class was without EB and 5kN, ie AC is one bar.

Isolate secondary truss & proceed.







FBD3

FBD1 & 2 provide 3 indep equations each.

FBD3 does not provide any indep equation.

Joints A and C provide 2 equations (indep).

From FBD 1&2

$$\Sigma M_E=0\quad A_2=\frac{1}{6\sqrt{2}}\left[\frac{3}{\cancel{\sqrt{2}}}\cdot 3\cancel{\sqrt{2}}+\frac{3}{\cancel{\sqrt{2}}}\cdot 3\cancel{\sqrt{2}}\tan 5^\circ\right]=\frac{3}{2\sqrt{2}}(1+\tan 5^\circ)$$

$$E_2=3/\sqrt{2}-A_2=\frac{3}{2\sqrt{2}}(1-\tan 5^\circ);\quad A_2=C_2;\quad E_2=E_4$$

$$A_1+E_1=\frac{3}{\sqrt{2}};\quad E_3+C_1=\frac{-3}{\sqrt{2}}$$

Of no use

$$\frac{1}{\sqrt{2}}(A_1+A_2+E_1+E_2+E_4-E_3-C_1+C_2)-11+5=0$$

$$\frac{1}{\sqrt{2}}(\cancel{A_2}-A_1-E_1-E_3-\cancel{E_4}+\cancel{E_2}-C_1-\cancel{C_2})=0$$

FBD3. Reduce to identities when subst above eqns

even moment eqn (say about E) is identity as seen by visual inspection.

Joint A: $(A_1+A_2)\frac{1}{\sqrt{2}}=5.5\Rightarrow A_1=6.62$

Joint C: $(C_2-C_1)\frac{1}{\sqrt{2}}=5.5\Rightarrow C_1=-6.62$ (expected due to symmetry).

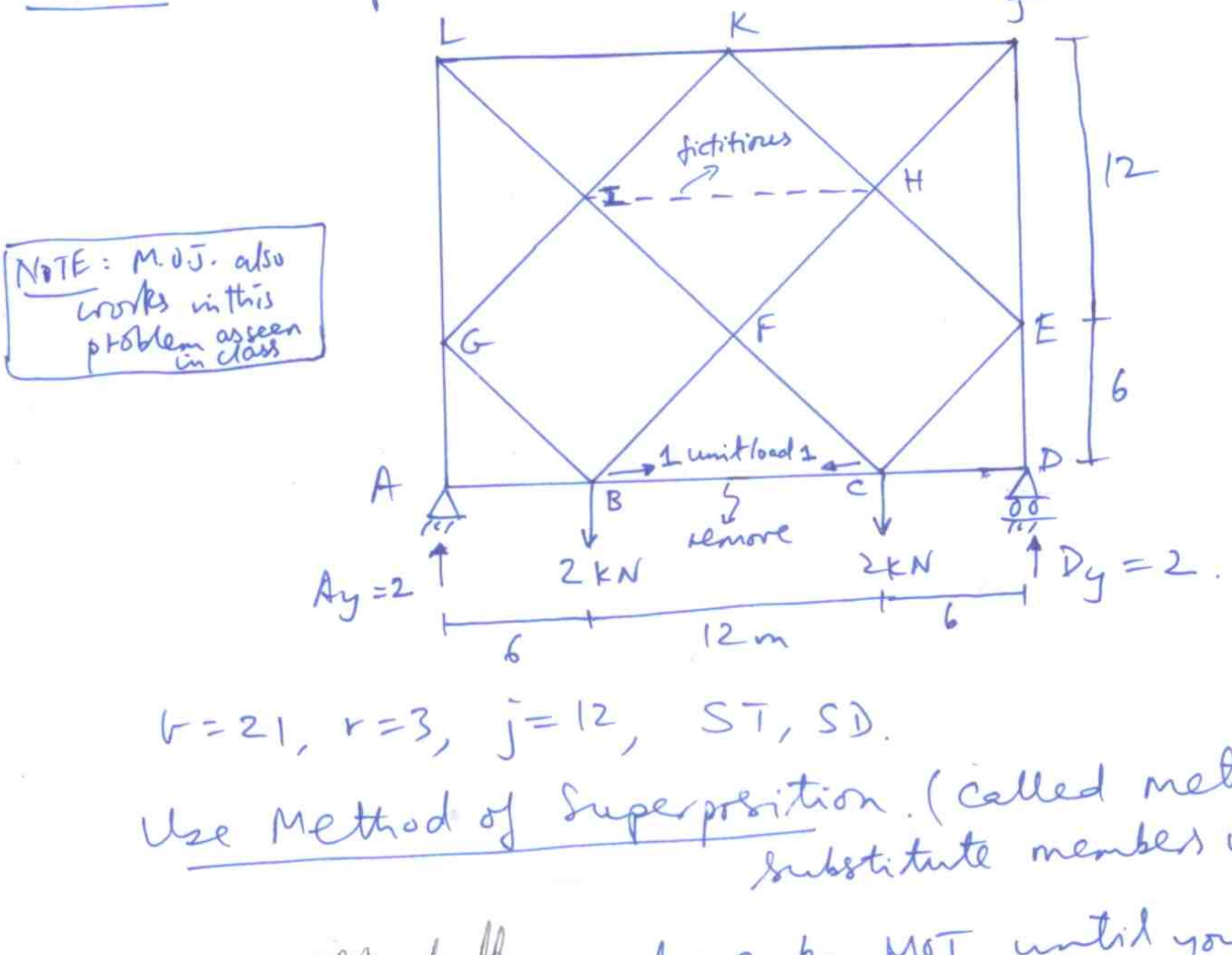
(11)

$E_7 = -4.5$, $E_3 = 4.5$. So $A_1, A_2, E_1 - E_4, C_1, C_2$ determined

Now Apply $A_1, A_2, E_1 - E_4, C_1, C_2$ to FBD's 1 & 2, use MOJ (also at jts A, B, C), get all internal forces.

Alternative: Section xx gives F_{FE}, F_{GE}, F_{AB}. Then use MOJ and proceed.

Ex 9. Complex Truss. (neither simple nor compound)



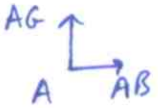
$b = 21$, $r = 3$, $j = 12$, ST, SD.

Use Method of Superposition. (called method of substitute members in this application).

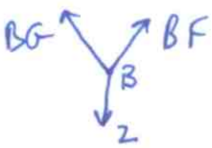
Step 1. Mentally analyze by MOJ until you reach joint where >2 unknowns present. Remove corresponding member and substitute it somewhere else so that resulting truss is simple.

$\Rightarrow$ Remove BC and replace with IH. Result is Simple truss. BC is actual member, IH is fictitious member

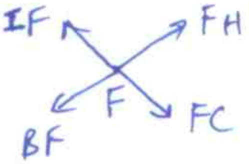
Step 2. Analyze resulting simple truss in step 1 for applied loads.



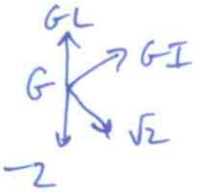
$AG = -2, \quad AB = 0$



$BG = \sqrt{2} = BF$

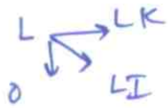


$FC = BF = FH = IF = \sqrt{2}$ (symmetry also used for $FC = BF$)

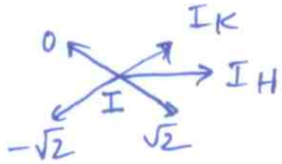


$GI = -\sqrt{2}$

$GL - \frac{\sqrt{2}}{\sqrt{2}} - \frac{\sqrt{2}}{\sqrt{2}} + 2 = 0 \Rightarrow GL = 0$



$\Rightarrow LI = LK = 0$



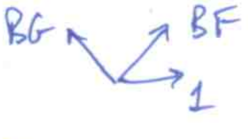
$\Rightarrow IK = 0$

$IH = -\frac{\sqrt{2}}{\sqrt{2}} - \frac{\sqrt{2}}{\sqrt{2}} = -2$

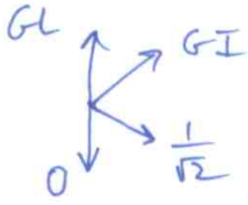
Step 3. Apply equal opp unit load at joints where member was removed, in direction of member removed. (see Fig). Analyze simple truss obtained for these unit loads, without applied ext loads.

$BC = 1$ applied equal opp at jts B and C. Hence no ext reactions develop (since no other loads applied).

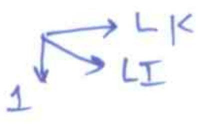
$\Rightarrow AB = AG = 0$



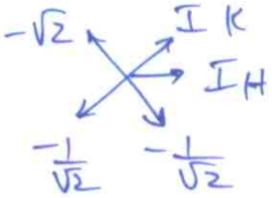
$BG = -BF = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}} = -FH = -FC = -IF$



$GI = -\frac{1}{\sqrt{2}}, \quad GL = 1.$



$LI = -\sqrt{2}, \quad LK = 1$



$$\frac{IK}{\sqrt{2}} - \frac{\sqrt{2}}{\sqrt{2}} + \frac{1}{2} + \frac{1}{2} = 0 \Rightarrow IK = 0$$

$$IH = -1$$

<u>Step 4.</u> <u>Superposition</u>

Need $IH = 0$ ($\because$ fictitious member).

$$\Rightarrow (IH)_{\text{Step 2}} + r(IH)_{\text{step 3}} = 0$$

$$r = \frac{2}{-1} = -2.$$

member	Step 2	Step 3	Step 4 = Step 2 + r (Step 3)
AG	-2	0	-2 (C)
AB	0	0	0
BG	$\sqrt{2}$	$1/\sqrt{2}$	$\sqrt{2} + (-2)(\frac{1}{\sqrt{2}}) = 0$
BF	$\sqrt{2}$	$-1/\sqrt{2}$	$\sqrt{2} + (-2)(-\frac{1}{\sqrt{2}}) = 2.83$ (T)
FC	$\sqrt{2}$	$-1/\sqrt{2}$	2.83 (T)
FH	$\sqrt{2}$	$-1/\sqrt{2}$	2.83 (T)
IF	$\sqrt{2}$	$-1/\sqrt{2}$	2.83 (T)
GI	$-\sqrt{2}$	$-1/\sqrt{2}$	0
GL	0	1	-2 (C)
LI	0	$-\sqrt{2}$	2.83 (T)
LK	0	1	-2 (C)
IK	0	0	0
IH	-2	-1	$0 \rightarrow$ (serves as a check.)

Other forces obtained by symmetry.

Space Truss

Determinacy, Stability.

$b + r < 3j \longrightarrow$ US.

$b + r = 3j \longrightarrow$ either SD, ST or US

$b + r > 3j \longrightarrow$ either SID, ST or US

} need to check as in plane truss.

Ext unstable either if reactions concurrent along a line (then moment equil along line wont be maintained) or reactions are parallel.

Int unstable if joints displ/rotate wrt each other, i.e., mechanism formed.

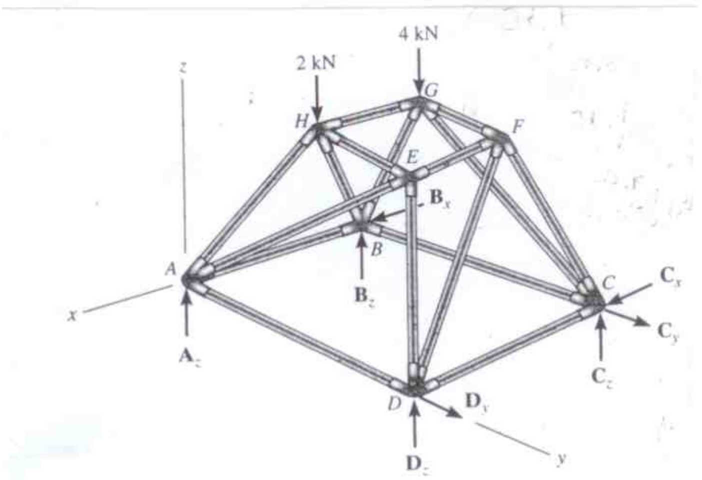
If we attempt a soln. and obtain consistent soln, then ST.

Zero-force members.

(i) If all but one members at an unloaded joint lie in same plane, then the one not in that plane is zero force member. or

(ii) If all members except two at an unloaded joint are zero force, and those two members that are not collinear are zero force members.

Ex 10



Find zero force members. The supports exert reaction components as shown.

proceed in this sequence

Jt. F $\Rightarrow$ GF = 0

Jt. E $\Rightarrow$ EF = 0

Jt. F $\Rightarrow$ FD = FC = 0

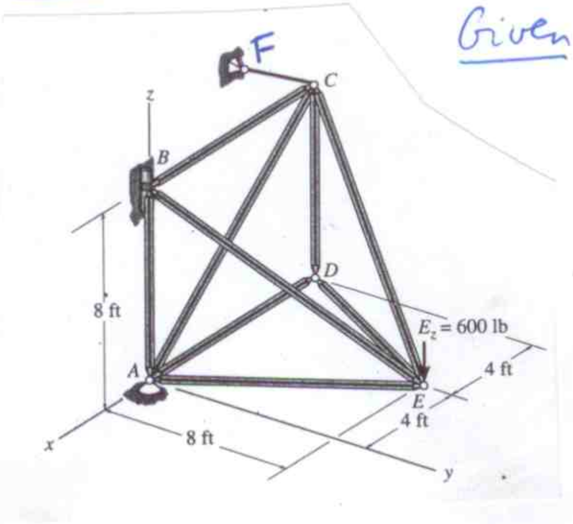
$b = 16$, $r = 8$, $3j = 3 * 8 = 24$

$b + r = 3j$, reactions dont permit rigid body motion; no mechanisms possible, so SD.

Use M.O.J at jt. G → get HG, BG, CG.
jt. H → get AH, BH, EH
jt E → get AE, DE
jt A → get A_z, AB, AD
jt B → get B_z, B_x, BC
jt D → get DC, D_z, D_y
jt C → get C_x, C_y, C_z

proceed in this sequence

Ex 11



Given: Ball and socket at A, slotted roller at B, cable at C.
Find: all member forces.

Notice its a Simple Space Truss (ie start with triangle, add 3 members at 3 district existing joints to meet at new joint, and so on).

$\Rightarrow b = 3j - 6 \rightarrow$ for simple space truss.

Now $r = 6 \Rightarrow b + r = 3j$, no mechanisms or rigid body motion possible $\Rightarrow$ SD.

jt D $\Rightarrow$ DC = 0, DA = DE = 0
jt E → get EC, EB, EA
jt C → get CB, CA, CF
jt B → get B_x, B_y, BA
jt A → get A_x, A_y, A_z.

proceed in this sequence.

CABLES.

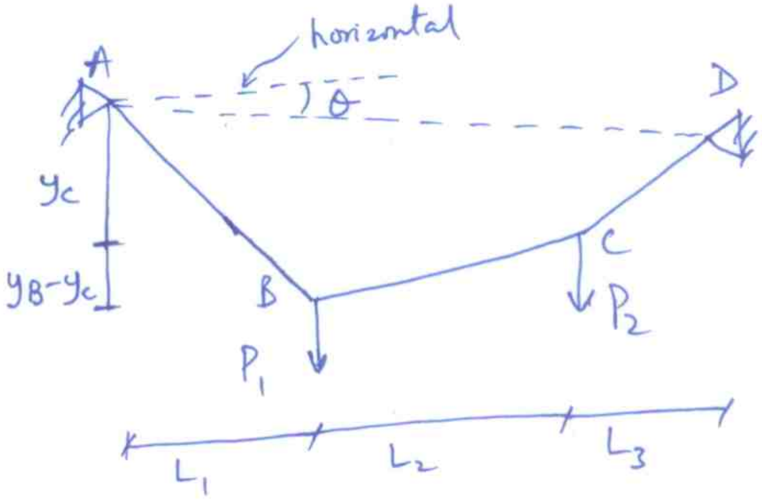
Assumptions:

(i) Completely flexible (BM = 0, SF = 0), carry only tension (tangential).

(ii) Inextensible.

(iii) Negligible self wt → valid when they support heavy loads, e.g. bridges. Not valid when minimal load supported, e.g. transmission lines, guys for electrical poles.

Cable with concentrated loads.

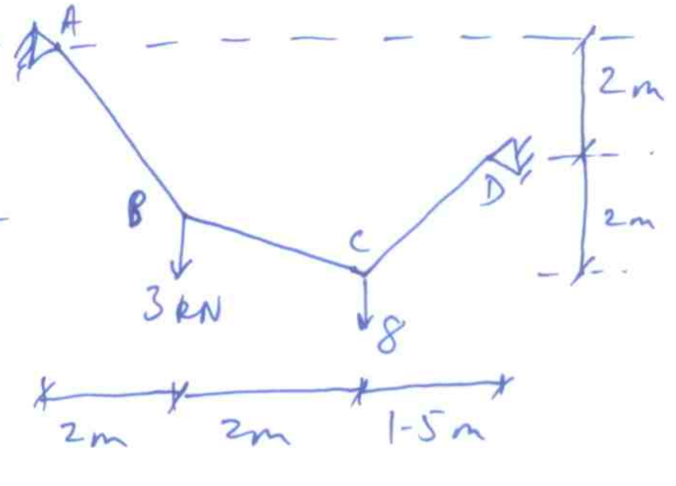


Given: $\theta, L_1, L_2, L_3, P_1, P_2$

Find: $A_x, A_y, B_x, B_y, y_c, y_B, T_{AB}, T_{BC}, T_{CD}$ → 9 unknowns

Eqns: 2 × 4 eqns for 4 jts A–D, plus one geometric condition (either total length of cable or one of the sags y_B or y_c).

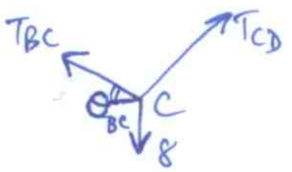
Ex 1.



Find: $h, T_{AB}, T_{BC}, T_{CD}$

$$\Sigma M_A = 0 = (3)(2) + (8)(4) - T_{CD}\left(\frac{1.5}{2.5}\cdot 4 + \frac{2}{2.5}\cdot 4\right)$$

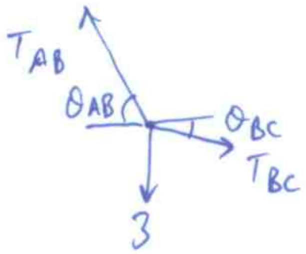
$$\Rightarrow T_{CD} = 6.786 \text{ kN}$$



$$T_{BC}\sin\theta_{BC} + T_{CD}\cdot\frac{2}{2.5} = 8$$

$$T_{BC}\cos\theta_{BC} = T_{CD}\cdot\frac{1.5}{2.5}$$

$$\Rightarrow \theta_{BC} = 32.27^\circ,\ T_{BC} = 4.815 \text{ kN}$$



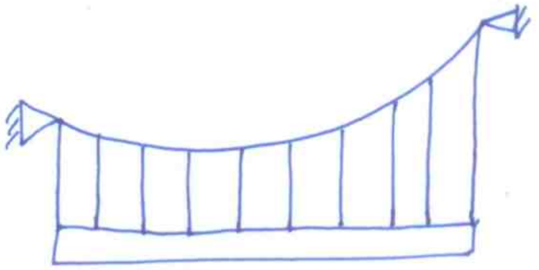
$$T_{AB}\cos\theta_{AB} = T_{BC}\cos\theta_{BC}$$

$$T_{AB}\sin\theta_{AB} = 3 + T_{BC}\sin\theta_{BC}$$

$$\theta_{AB} = 53.84^\circ,\ T_{AB} = 6.90$$

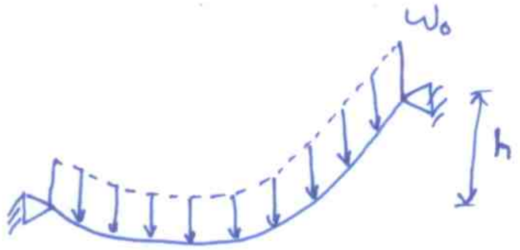
$$h = (2)(\tan 53.84) = 2.74 \text{ m}.$$

Cable with uniform distributed load.



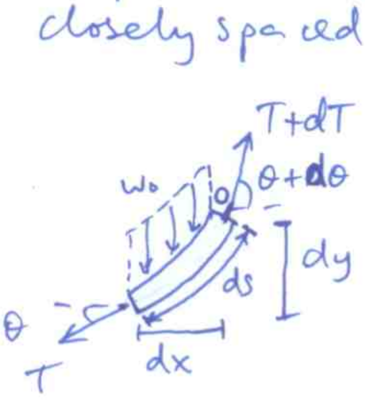
Suspension bridge with closely spaced hangars

$\equiv$



$w_o(x) = w_o = \text{const}$

$\because$ closely spaced hangars.



$$\Sigma F_x = 0 = (T+dT)\cos(\theta+d\theta) - T\cos\theta = 0$$

$$\Rightarrow d(T\cos\theta) = 0$$

$$T\cos\theta = F_H = \text{const} \longrightarrow (1)$$

$$\Sigma F_y = 0 = (T+dT)\sin(\theta+d\theta) - T\sin\theta - w_o dx = 0$$

$$\Rightarrow T\sin\theta\cos d\theta + T\cos\theta\sin d\theta + dT\sin\theta\cos d\theta$$

$$+ dT\cos\theta\sin d\theta - T\sin\theta - w_o dx = 0$$

$\sin d\theta \doteq d\theta$, $\cos d\theta \simeq 1$, neglect Higher order terms (HOT's) like $dT\,d\theta$ etc,

$$\Rightarrow T\cos\theta\, d\theta + dT\sin\theta - w_0\, dx = 0$$

$$\Rightarrow d(T\sin\theta) = w_0\, dx$$

$$T\sin\theta = w_0 x + c_1 \longrightarrow (2)$$

$$\Sigma M_0 = 0 = \underbrace{\cancel{w_0\, dx\left(\frac{dx}{2}\right)}}_{H.O.T.} + T\cos\theta\, dy - T\sin\theta\, dx = 0$$

$$\tan\theta = \frac{dy}{dx} \longrightarrow (3)$$

$$\Rightarrow \frac{dy}{dx} = \frac{T\sin\theta}{T\cos\theta} = \frac{w_0 x + c_1}{F_H} \longrightarrow (5)$$

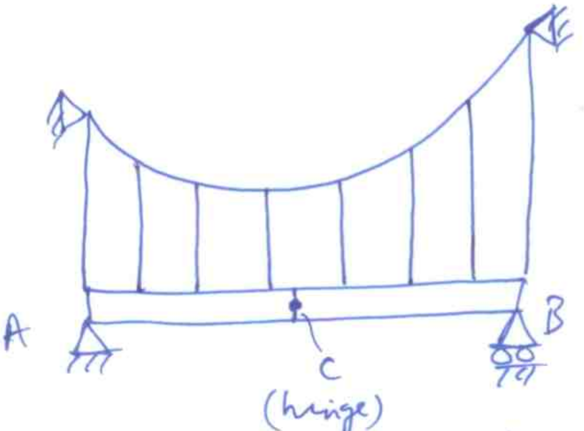
$$y = \frac{w_0 x^2}{2F_H} + \frac{c_1 x}{F_H} + C_2 \longrightarrow (4)$$ PARABOLIC SHAPE.

If origin $(y=0, x=0)$ at lowest pt and lowest pt has horizontal slope $(\theta = 0^\circ)$, i.e. lowest pt in the interior, then $c_1 = c_2 = 0$.

$$T_{max} = \frac{F_H}{\cos\theta_{max}} \longrightarrow (6)$$

In this case, find F_H from (4) using given geometric condition (e.g. sag of lowest pt.). Else we need two more geometric conditions to find c_1, c_2.

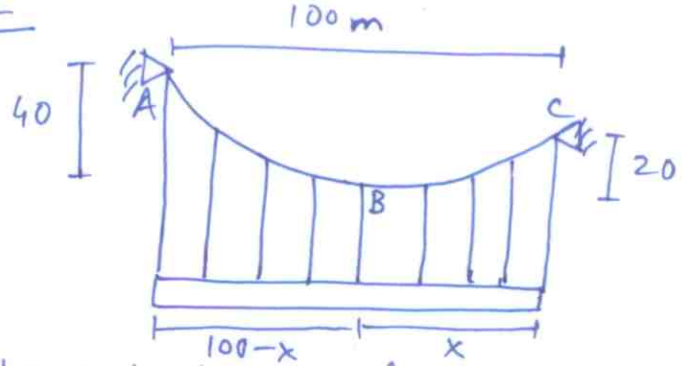
If self-wt of cable considered, we have $w_0(s)$ uniform (i.e. over length of cable it is uniform). You get a catenary shape (power lines is e.g.). Ref Shames (Engg. mechanics).



Without hinge at C, given lowest pt & given sag at lowest pt, Indet to 1st degree, ∵ $w_0(x) = w_0$ is unknown, reactions at deck supports obtained from deck FBD. Put hinge at C and it becomes determinate. See Ex 3

- Here the cable reduces BM in deck AB.

Ex 2



Given: Girder weighs $850\ N/m = w_0$.

Find: Tension at A, B, C.

Lowest pt in interior $\Rightarrow c_1 = c_2 = 0$

From (4) p. 18 $\rightarrow$ $$20 = \frac{850x^2}{2F_H} \rightarrow (1)$$

$$40 = \frac{850(100-x)^2}{2F_H} \rightarrow (2)$$

$$\Rightarrow (100-x)^2 = 2x^2 \Rightarrow x^2 + 200x - 10^4 = 0.$$

$$x = 41.42\,m.$$

$$T\cos\theta = F_H = 36459.2\ N \ (\text{from } (1))$$

From (5), p. 18,
$$\left.\frac{dy}{dx}\right|_{x_A=-58.58} = \frac{(850)(-58.58)}{36459.2} = -1.3657 = \tan\theta_A$$

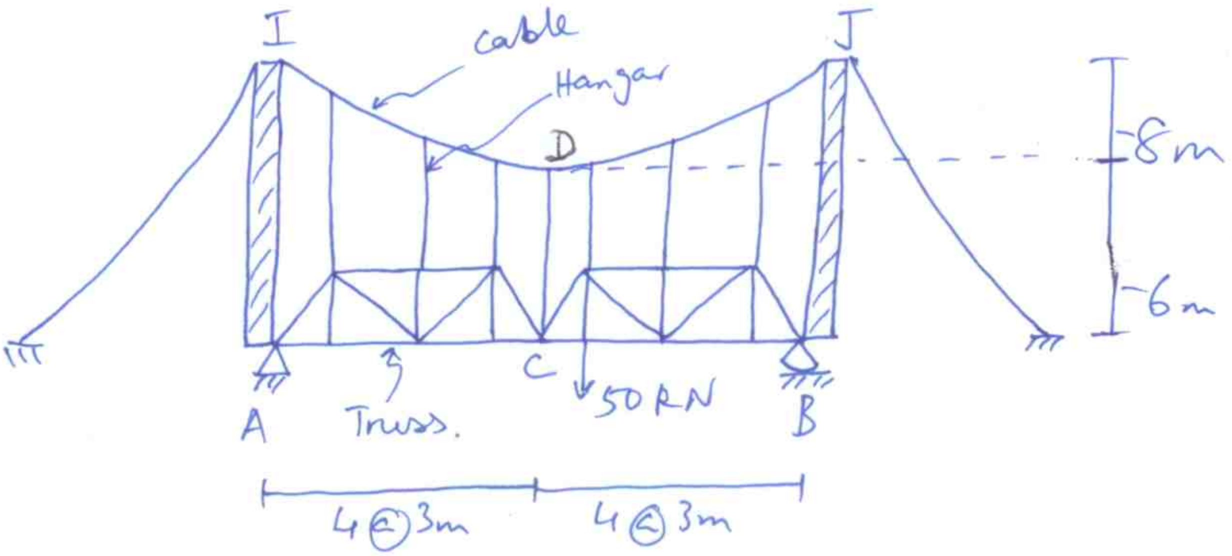
$$\theta_A = -53.78^\circ$$

$$T_A = \frac{36459.2}{\cos\theta_A} = 61714\ N = \underline{61.7\,kN}$$

$$T_B = 36459.2\,N = \underline{36.45\ kN}$$

$$\tan\theta_C = \frac{(850)(41.42)}{36459.2} \Rightarrow \theta_C = 44^\circ, \quad T_C = \frac{36459.2}{\cos\theta_C} = \underline{50.68\,kN}$$

Ex 3.



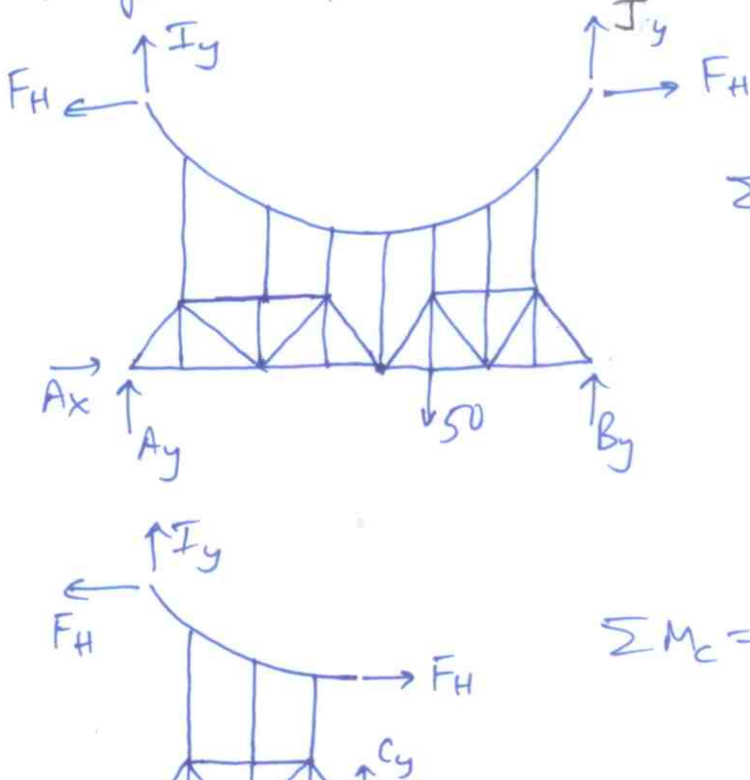
(Not to scale)

Given: Cable has parabolic shape. Bridge has 50 kN load.

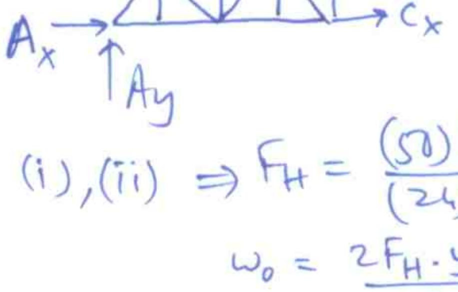
Find: Max tension.

Here $c_1 = c_2 = 0$, origin at D.

Hinge at C, so problem is determinate. Need to find w_0 also.



$\Sigma M_B = 0 = (A_y + I_y)\cdot 24 - (50)(9)$ → (i)



$\Sigma M_C = 0 = (I_y + A_y)\cdot 12 - F_H \cdot 8 = 0$ → (ii)

(i), (ii)

$(i),(ii) \Rightarrow F_H = \frac{(50)(9)}{(24)} \cdot \frac{(12)}{8} = 28.125$ kN.

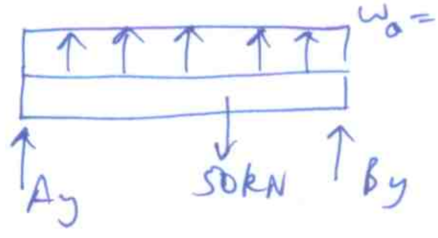
$w_0 = \frac{2 F_H \cdot y}{x^2} = \frac{2(28.125)(8)}{12^2} = 3.125$ kN/m.

$$\tan\theta_J = \left.\frac{dy}{dx}\right|_{x=12} = \frac{w_0 x}{F_H} = \frac{3.125 * 12}{28.125} = 1.333$$

$$T_{max} = T_J = \frac{F_H}{\cos\theta_J} = \frac{28.125}{\cos 53.13} = \underline{46.875 \text{ kN.}}$$

Extra: Find A_x, A_y, B_y.

$A_x = 0$



$B_y = \frac{1}{24}\left(50*16 - 3.125 * \frac{24^2}{2}\right) = -\frac{100}{24}$

$A_y = 50 + \frac{100}{24} - 3.125 * 24$

$A_y = \frac{125}{6}$

Check: Find $I_y \rightarrow I_y = F_H \tan\theta_I = 28.125 * 1.333 = 37.5$

$\Rightarrow A_y = 18.75 - 37.5 = -18.75$ (from (i)).

So results for A_y dont match. Reason is that 50 kN load is eccentric so cable cannot be exactly symmetric shape about origin D.

If instead you change problem and take 50kN applied at C and re-work, you get

$I_y + A_y = 25$, $F_H = 37.5$, $w_0 = \frac{25}{6}$

$A_y = B_y = \left(50 - \frac{25}{6} * 24\right)/2 = -25$

check: $I_y = F_H \tan\theta_I$ $\tan\theta_I = \frac{25/6}{37.5} * 12 = \frac{4}{3}$

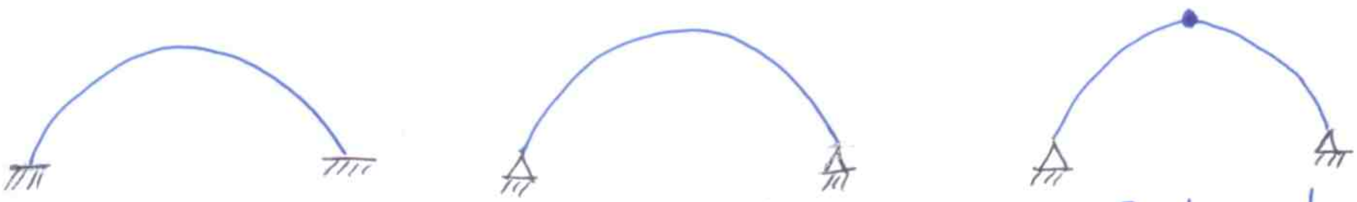
$I_y = 50$

$A_y = 25 - 50 = -25 \rightarrow$ checks out.

ARCHES.

1) Acts as inverted cable.
2) Has rigidity, so carries compression (NF/AF), BM, SF.
3) Like cable it reduces BM in the deck of a bridge.

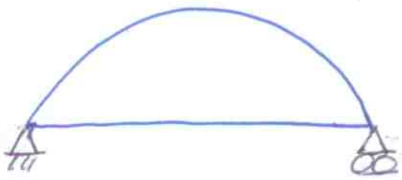
1), 2) ⇒ Parabolic arch with u.d.l along span carries only compression (no BM, SF). → see Ex 1.



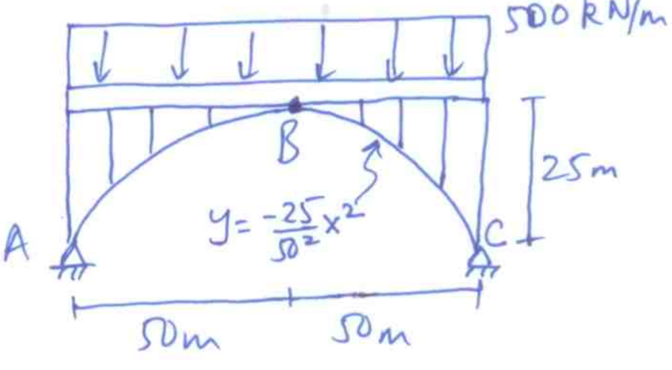
Fixed | Pinned / 2-hinged | 3-hinged

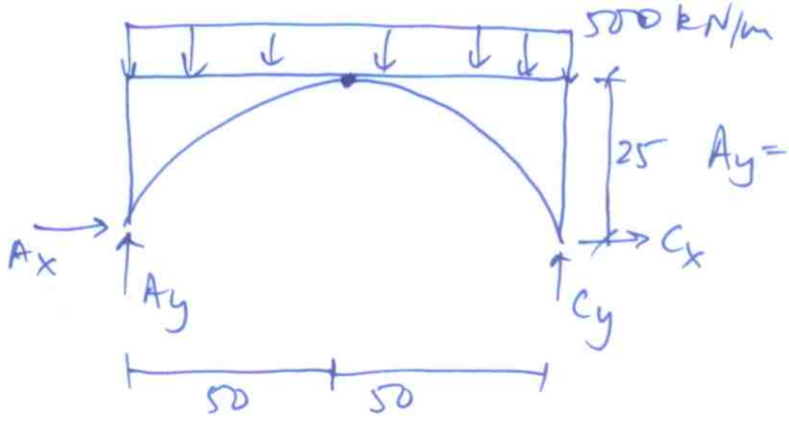
DOI = 3 (deg of indet) | DOI = 1 | SD, DOI = 0.

Fixed arch affected by settlement of supports & temperature, 2-hinged arch less affected, 3-hinged arch not affected.

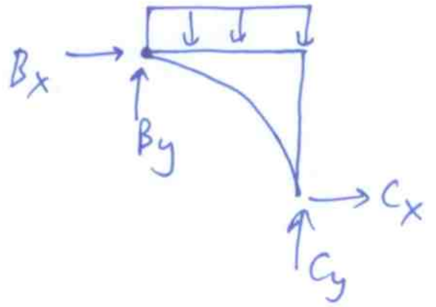


Ex 1 Show that BM = SF = 0 in a uniformly loaded parabolic 3-hinged arch, $w_0 = 500$ kN/m, as shown below.





$$A_y = C_y = \frac{1}{100} * 500 * \frac{100^2}{2} = 25000 \text{ kN.}$$

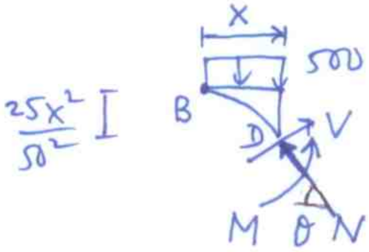


$$C_x = -\frac{1}{25}\left(25000 * 50 - 500 * \frac{50^2}{2}\right)$$

$$= -25000 \text{ kN.}$$

$$B_y = 500 * 50 - 25000 = 0$$

$$B_x = -C_x = 25000 \text{ kN}$$



$$\Sigma F_x = 0 \Rightarrow B_x - N\cos\theta + V\sin\theta = 0$$

$$\Sigma F_y = 0 \Rightarrow \underbrace{B_y}_{0} - 500x + N\sin\theta + V\cos\theta = 0$$

$$\Rightarrow B_x \sin\theta - 500x\cos\theta + V = 0$$

$$-B_x\cos\theta + N - 500x\sin\theta = 0$$

$$\tan\theta = \frac{dy}{dx} = -\frac{1}{50}x \quad , \quad \cos\theta = \sqrt{\frac{2500}{2500+x^2}} \quad , \quad \sin\theta = \sqrt{\frac{x^2}{x^2+2500}}$$

$$\Rightarrow \underline{V} = 500x\sqrt{\frac{2500}{x^2+2500}} - 25000\sqrt{\frac{x^2}{x^2+2500}} = \underline{0}.$$

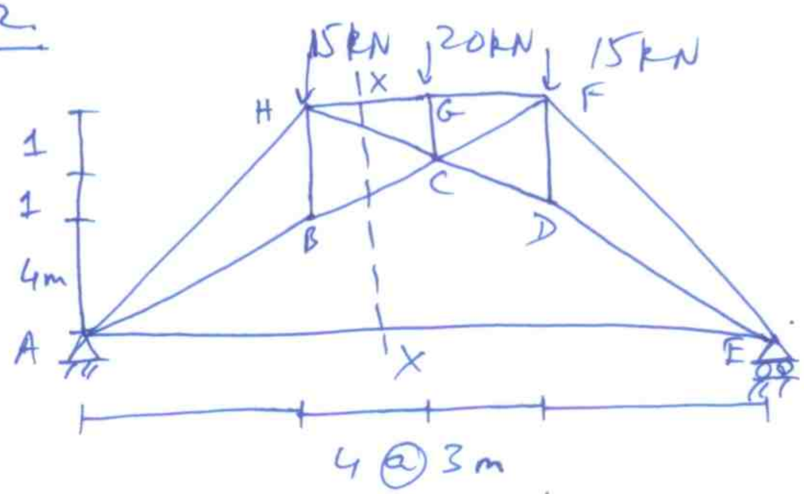
$$N = 25000\sqrt{\frac{2500}{x^2+2500}} + 500x\sqrt{\frac{x^2}{x^2+2500}} = \frac{125*10^4 + 500x^2}{\sqrt{x^2+2500}}$$

$$\Sigma M_D = 0 \Rightarrow M + 500\frac{x^2}{2} - 25000 * \frac{25x^2}{50^2} = 0$$

$$\Rightarrow \underline{M = 0}$$

Note: Structurally more efficient to resist applied load in compression than in bending (if SS beam used), but we must take care of buckling.

Ex2.



Given: 3-hinged tied arch. GF designed to carry no load.
Find forces in CH, CB.

$\Rightarrow$ HG = 0 (from jt. G equil).

Use section XX, get HC, BC, AE using MOS.