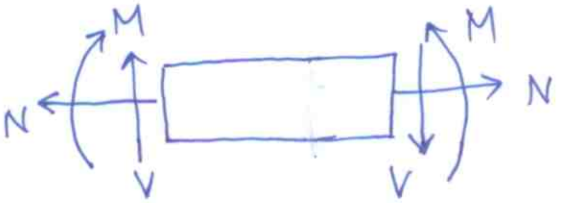
TOPIC – 2
BENDING MOMENT, SHEAR FORCE, AXIAL FORCE

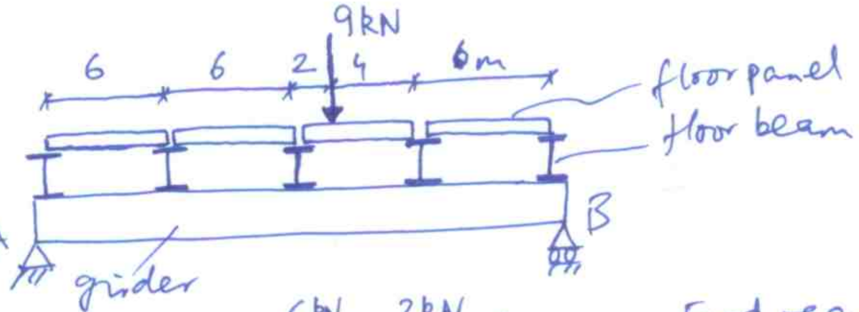
BM, SF, AF are internal forces. They represent resultants (vector sum) of distributed stresses acting on internal cross section of member.



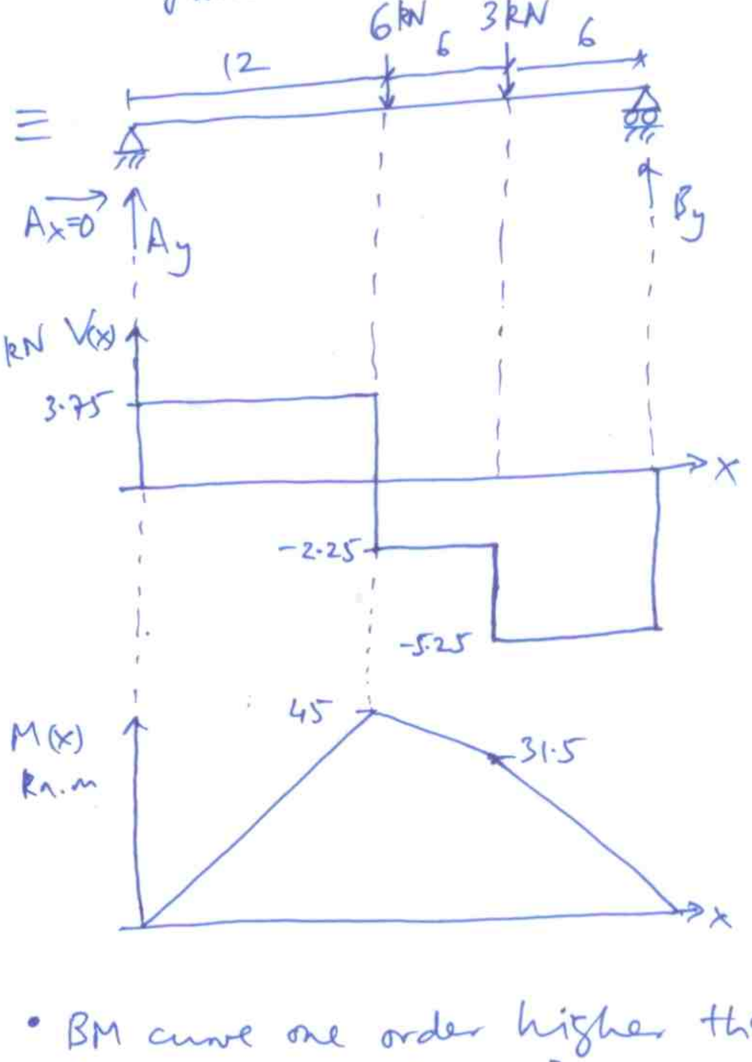
N: elongates element
V: rotates element CW
M: bends element concave upward.

Positive convention.

Ex 1.



Draw SFD, BMD.



Find reactions on loaded floor panel by mom't equil. These become loads on girder.

Girder reactions:

$A_y = \dfrac{6 \times 12 + 3 \times 6}{24} = 3.75$, $B_y = 5.25$

(or directly from whole structure, $A_y = \dfrac{10}{24} \times 9 = 3.75$).

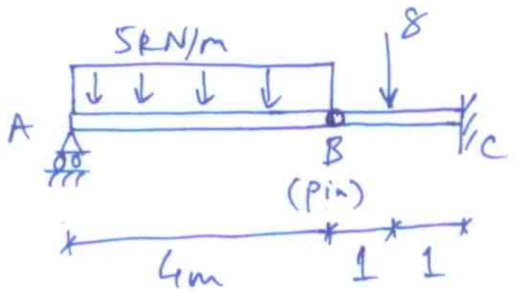
$3.75 \times 12 = 45$

$45 - 2.25 \times 6 = 31.5$.

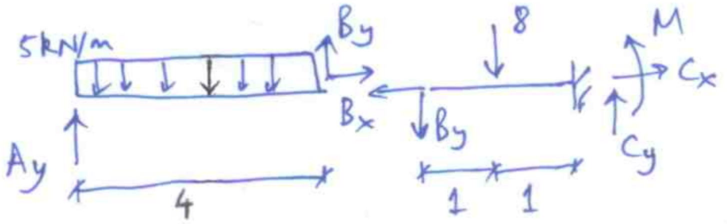
- BM curve one order higher than SF curve (SF const, BM linear).
- SF discontinuous where point load applied. Thus slope of BM discontinuous there ($\because V = M'$ → later).

- SF, BM, or their slopes, are discontinuous where abrupt change in distributed load or point load, point moment (couple), respectively, applied. So choose different coordinate system between consecutive discontinuities of loading.

Ex 2. (4.38 Hibbeler)



Find SF, BM functions.



$\Sigma M_B = 0 \Rightarrow \frac{(5)(4)(2)}{4} = 10 = A_y = B_y$.

SF, BM functions.

$0 \le x \le 4$ $\quad V(x) = 10 - 5x$; $M(x) = 10x - \frac{5x^2}{2}$

$4 \le x \le 5$ $\quad V(x) = 10 - 20 = -10$;

$M(x) = 10x - 20(x-2) = -10x + 40$

$5 \le x \le 6$ $\quad V(x) = -10 - 8 = -18$;

$M(x) = -10x + 40 - 8(x-5) = -18x + 80$.

Check: Ext equil of BC gives,

$C_y = B_y + 8 = 18$; $M = -(10 \times 2 + 8 \times 1) = -28$ kN.m

From SF, BM functions,

$M_B = 10 * 4 - \frac{5 * 4^2}{2} = 0$ ✓

$M_C = -18 * 6 + 80 = -28$ ✓

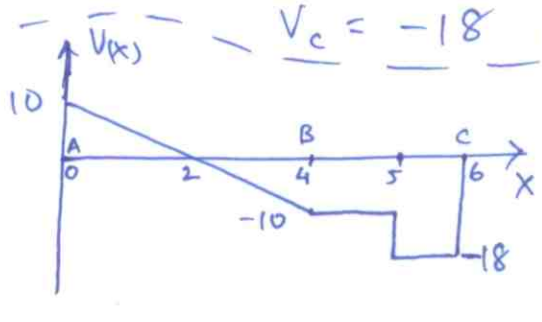
$V_C = -18$ ✓

$M(2) = \underbrace{M(0)}_{0} + \frac{(10)(2)}{2} = 10$

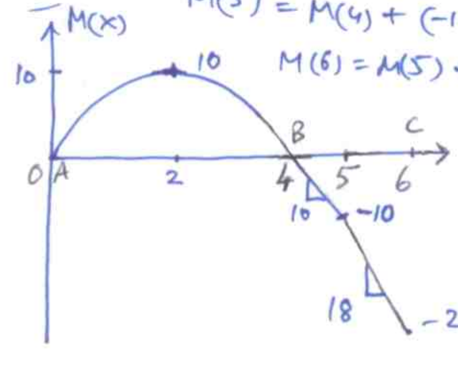
$M(4) = M(2) + \frac{(-10)(2)}{2} = 0$

$M(5) = M(4) + (-10)(1) = -10$

$M(6) = M(5) + (-18)(1) = -28$



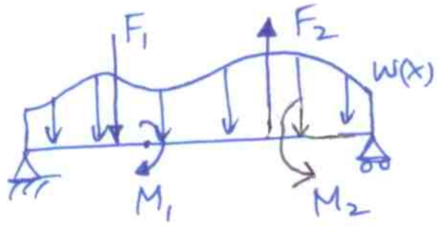
SFD



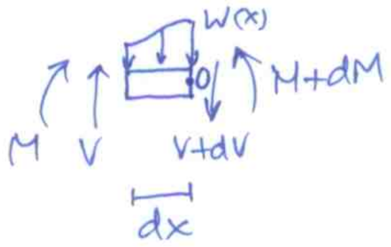
BMD

SFD, BMD for Beams.

Consider beam with distributed load and concentrated loads.



Consider segment w/o conc. loads.



$$\Sigma F_y = 0 = -dV - w\,dx$$

$$\Sigma M_0 = 0 = V\,dx + M - (M+dM) - \underbrace{\cancel{w\,dx\,\alpha\,dx}}_{\text{H.O.T.}}$$

(here $\alpha\,dx$ represents dist of resultant $w\,dx$ from 0). Neglect higher order term (H.O.T.) containing dx^2 compared to dx terms.

$\Rightarrow$ $$\boxed{\frac{dV}{dx} = -w(x) \quad ; \quad \frac{dM}{dx} = V(x)}$$ (known from CE201).

1

SLOPE OF SFD IS −ve DISTR LOAD

SLOPE OF BMD IS SHEAR

$\Rightarrow$ $$\boxed{\Delta V = -\int w(x)\,dx \quad ; \quad \Delta M = \int V\,dx}$$

2

CHANGE IN SF IS −ve AREA UNDER DISTR LOAD DIAGRAM BETWEEN RESPECTIVE POINTS ON BEAM

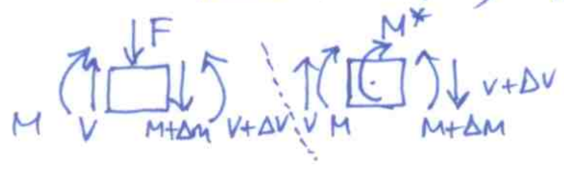
CHANGE IN BM IS AREA UNDER SFD BETWEEN RESPECTIVE POINTS ON BEAM

- The above is valid only when the region between the two points on the beam does not contain a point load/couple.
- If region contains point load/couple, <u>additional</u> changes in SF, BM are,

$$\boxed{\Delta V = -F \quad ; \quad \Delta M = M^*}$$

3

$F(\downarrow)$, $M^*(\curvearrowright)$ are (positive) applied point load & couple, resply.



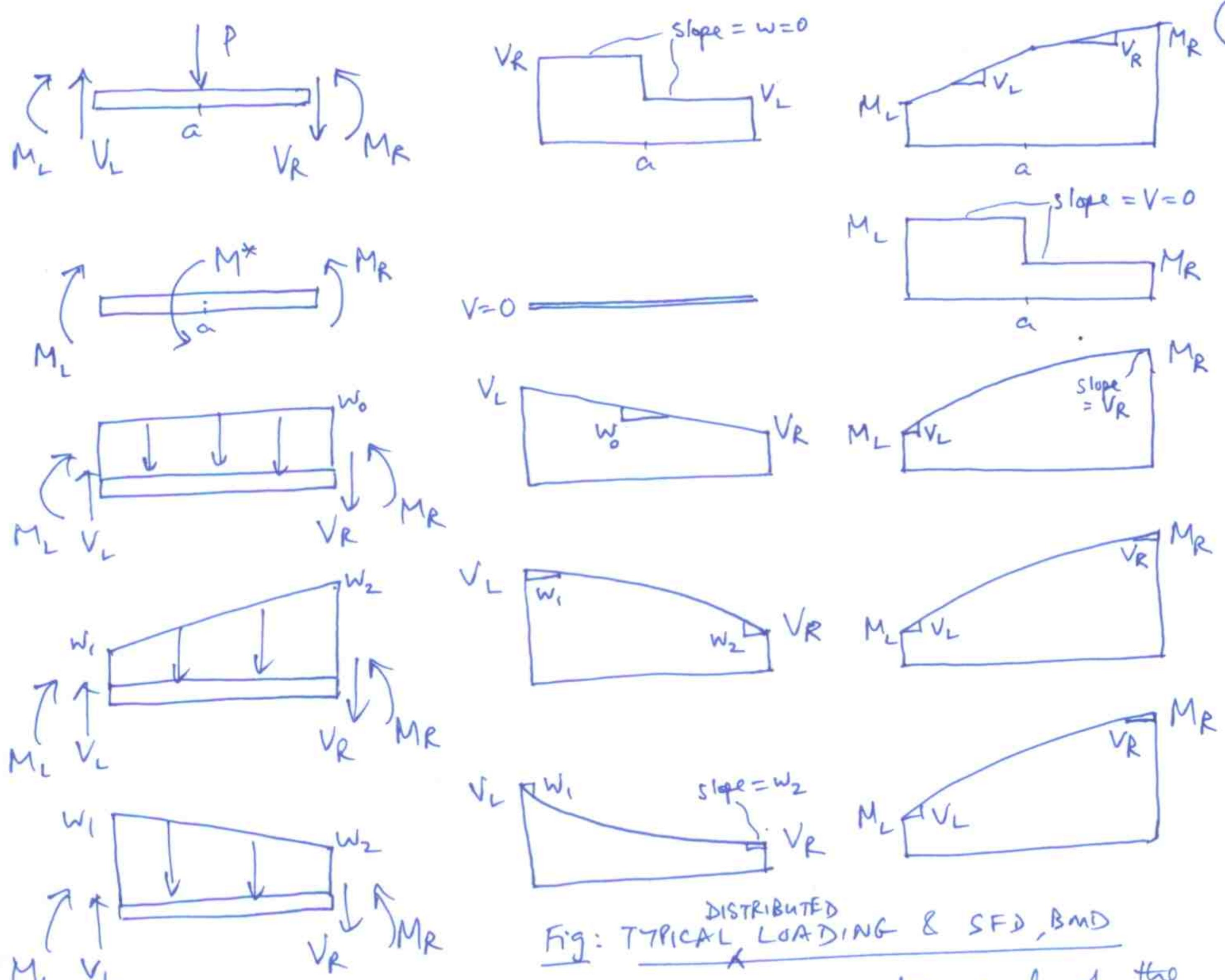
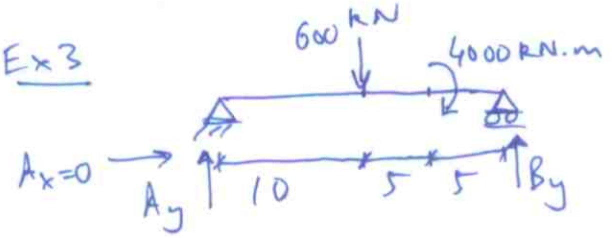


Fig: TYPICAL DISTRIBUTED LOADING & SFD, BMD

Procedure: 1) Find V_L, V_R, M_L, M_R, i.e. SF, BM at two ends of the segment. Each segment should contain no discontinuities in load or its slope.

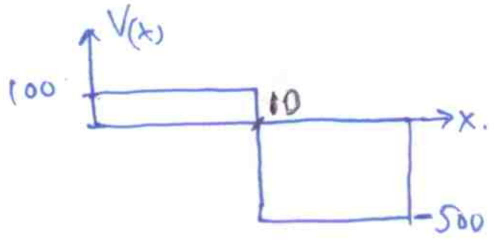
2) Use ①, ②, ③ to plot shape and values of SFD, BMD.

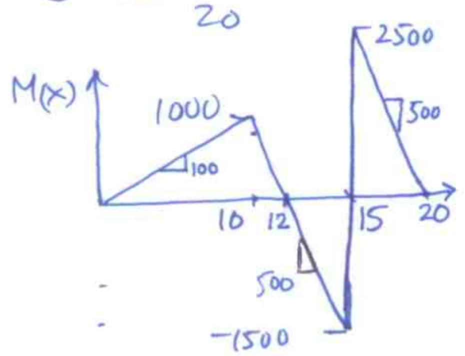
Ex 3



Draw SFD, BMD.

$$B_y = \frac{600*10 + 4000}{20} = 500, \quad A_y = 100\text{ kN}$$





M = area under SFD

$M(5) = 100*10 = 1000$

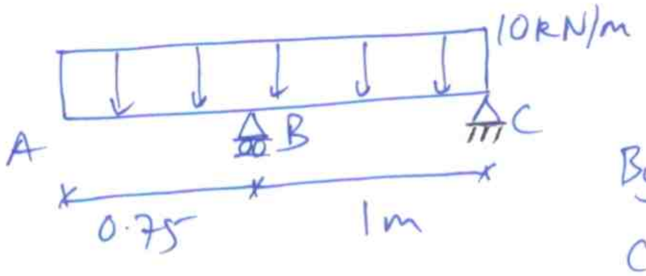
$M(15^-) = 1000 - 500*5$
$= -1500$

$M(15^+) = -1500 + 40000$
$= 2500$

$M(20) = 2500 - 500*5$
$= 0$ (checks out)

|| For $M(5)$, $M(15^-)$, $M(20)$, used Eq(2).
For $M(15^+)$ used Eq(3).

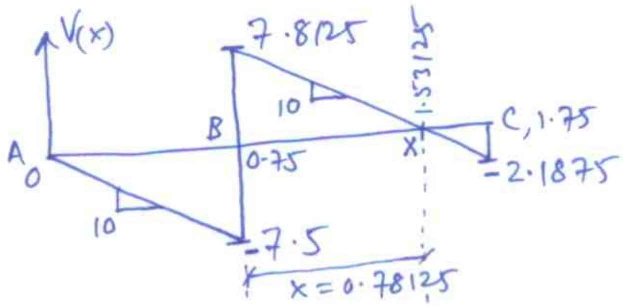
Ex 4



Draw SFD, BMD

$$B_y = 10 * \frac{1.75^2}{2} = 15.3125$$

$$C_y = 17.5 - B_y = 2.1875$$



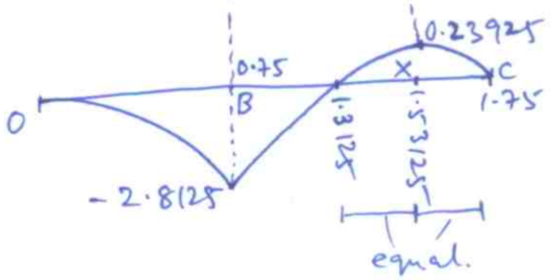
$$V_{B^-} = V_A - \int w\,dx = 0 - 10*7.5 = -7.5$$

$$V_{B^+} = V_{B^-} + B_y = -7.5 + 15.3125 = 7.8125$$

$$V_C = V_{B^+} - \int w\,dx = 7.8125 - 10*1 = -2.1875$$

$$V_x = V_{B^+} - \int w\,dx = 7.8125 - 10x = 0 \Rightarrow x = 0.78125$$

$$M_B = M_A + \int V\,dx = 0 + \frac{(-7.5)(0.75)}{2} = -2.8125$$



$$M_x = -2.8125 + \frac{7.8125\,x}{2}$$

$$= -2.8125 + 7.8125 * \frac{0.78125}{2}$$

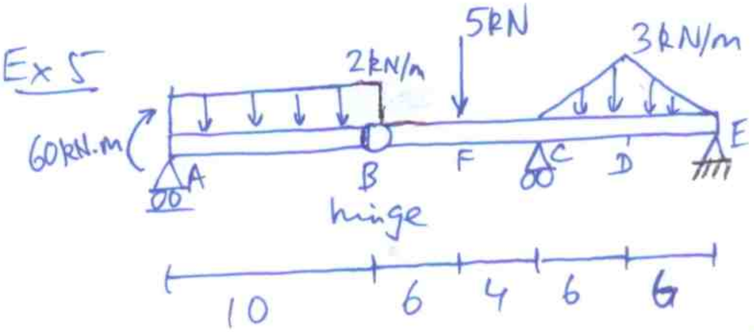
$$= 0.23925 \text{ kN.m.}$$

$$M_C = M_x - 2.1875 * \frac{(1 - 0.78125)}{2}$$

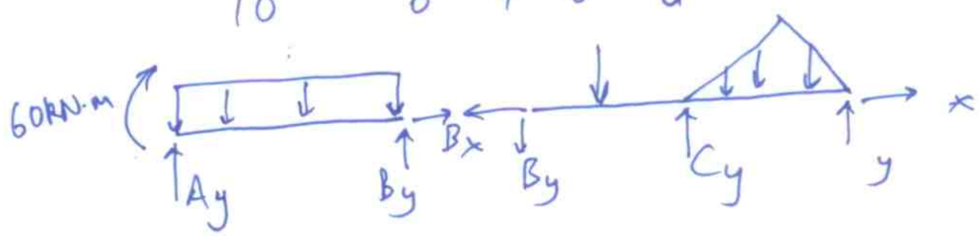
$$= 0 \quad (\text{checks out}).$$

|| For V_{B+} used Eq(3), all others used Eq(2)

Ex 5



Draw BMD, SFD.



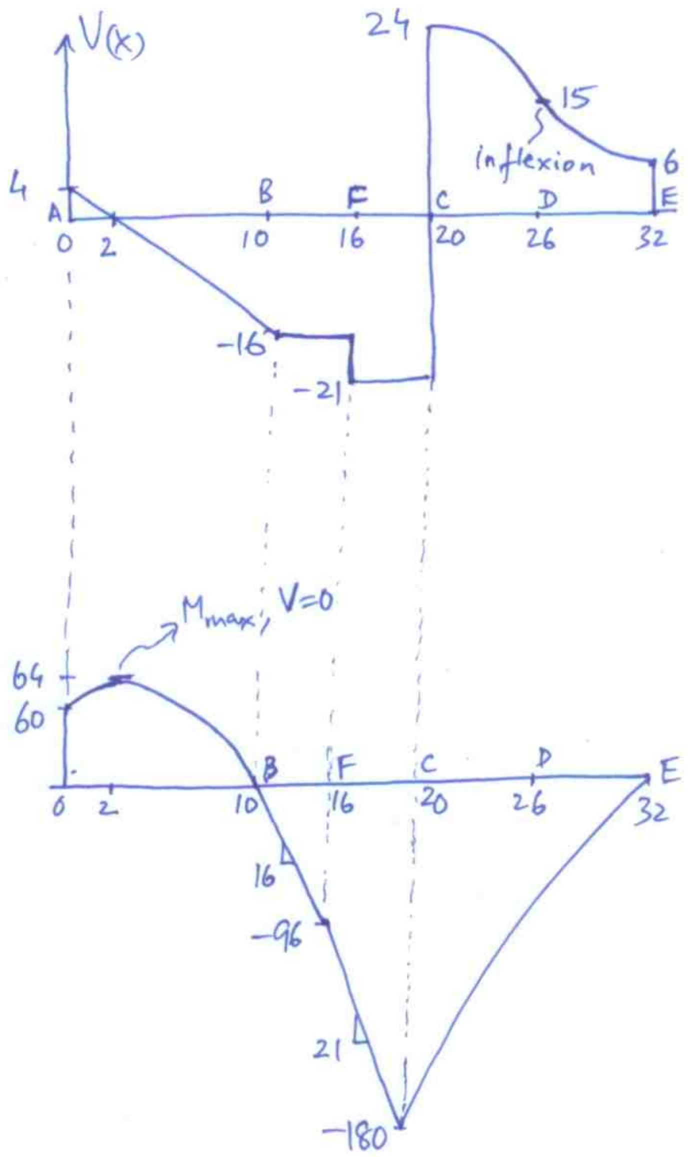
$$\Sigma M_B = 0 \Rightarrow A_y = \frac{2*\frac{10^2}{2} - 60}{10}$$

$$A_y = 4$$

$$B_y = 16$$

$$\Sigma M_E = 0 \Rightarrow \frac{(16*22 + 5*16 + 3*6*6)}{12} = C_y$$

$$C_y = 45$$



$V_B = V_A - 2*10 = 4-20 = -16$ (ie B_y as expected)

$V_{F-} = V_B = -16$

$V_{F+} = V_{F-} - 5 = -16-5 = -21$

$V_{C-} = V_{F+} = -21$

$V_{C+} = V_{C-} + C_y = -21+45 = 24$

$V_D = V_{C+} - \frac{3*6}{2} = 24-9 = 15$

$V_E = V_{C+} - 3*6 = 6$

$M_A = 60$

$M(2) = 60 + \frac{4*2}{2} = 64$

$M_B = 64 - \frac{8*16}{2} = 0$ (checks out)

$M_F = 0 - 16*6 = -96$

$M_C = -96 - 21*4 = -180$

To find M_D we need to find area $\int_{20}^{26} V dx$ which is not easy by inspection. So lets also write the $V(x)$, $M(x)$ functions.

$0 \le x \le 10$: $V(x) = V_A - \int_0^x w\,dx = 4 - \int_0^x 2\,dx = 4-2x \rightarrow \boxed{V_B = V(10) = -16 = -B_y \text{ checks out}}$

$M(x) = M_A + \int_0^x V\,dx = 60+4x-x^2 \rightarrow \boxed{M_B = M(10) = 0 \text{ checks out}}$

$10 \le x \le 16$: $V(x) = V(10) = -16$

$M(x) = M(10) + \int_{10}^x V\,dx = 0 - 16(x-10) = 160 - 16x$

alt. [or put $\bar{x} = x-10$,

$M(x) = M(10) + \int V\,d\bar{x} = -16\bar{x} = -16(x-10)$

$16 \le x \le 20$: $V(x) = -16 - 5 = -21$

$$M(x) = M(16) + \int_{16}^{x} V\,dx = -96 - 21(x-16) = 240 - 21x$$

alt. [or put $\bar{x} = x - 16$

$$M(x) = M(16) + \int V\,d\bar{x} = -96 - 21\bar{x} = 240 - 21x$$

$20 \le x \le 26$: $V(x) = -21 + C_y - \int_{20}^{x} w\,dx = 24 - \int_{20}^{x} (x-20)/2\,dx$

$$= 24 - \left[\frac{x^2}{4} - 10x\right]_{20}^{x} = -\frac{x^2}{4} + 10x - 76 \rightarrow \boxed{V_D = V(26) = 15 \text{ checks out}}$$

$$M(x) = M(20) + \int_{20}^{x} V\,dx = -180 + \left[\frac{-x^3}{12} + 5x^2 - 76x\right]_{20}^{x}$$

$$= \frac{-x^3}{12} + 5x^2 - 76x + \frac{20}{3}$$

alternative [$\underline{\text{or}}$ put $\bar{x} = x - 20$

$$V(x) = -21 + C_y - \int w\,d\bar{x} = 24 - \int \frac{\bar{x}}{2}\,dx$$

$$= 24 - 0.25\bar{x}^2 = 24 - 0.25(x-20)^2$$

$$= -0.25x^2 + 10x - 76$$

$$M(x) = M(20) + \int V\,d\bar{x} = -180 + \int (24 - 0.25\bar{x}^2)\,d\bar{x}$$

$$= -180 + 24\bar{x} - \frac{\bar{x}^3}{12} = -180 + 24(x-20) - \frac{(x-20)^3}{12}$$

$$= -76x - \frac{x^3}{12} + 5x^2 + \frac{20}{3}.$$

$26 \le x \le 32$: $V(x) = V(26) - \int_{26}^{x} w\,dx = 15 - \int_{26}^{x}\left[3 - \frac{(x-26)}{2}\right]dx = 15 - \left[16x - \frac{x^2}{4}\right]_{26}^{x}$

$$= \frac{x^2}{4} - 16x + 262 \longrightarrow \boxed{V(32) = 6 = V_E = E_y \text{ checks out}}$$

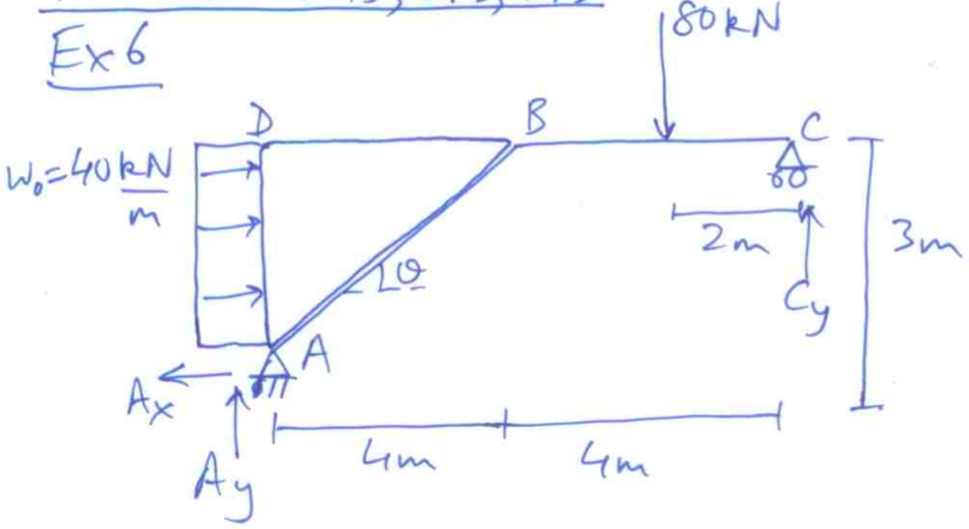
$$M(x) = M(26) + \int_{26}^{x} V\,dx = -54 + \left[\frac{x^3}{12} - 8x^2 + 262x\right]_{26}^{x}$$

$$= \frac{x^3}{12} - 8x^2 + 262x - 2922.667 \rightarrow \boxed{M(32) = 0 \text{ checks out}}$$

Alternative way will also give same result.

Frames – BMD, SFD, AFD

Ex 6

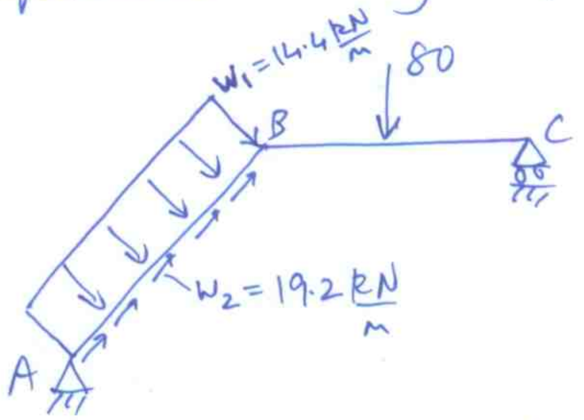


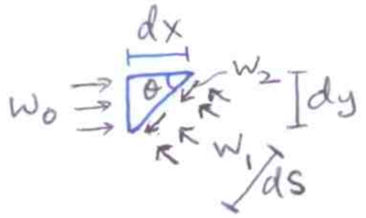
Frame ABC with wall/cladding ADB. Wind load $40\,\frac{kN}{m}$ applied on wall as shown. Find AFD, SFD, BMD.

$$C_y = \frac{80*6 + 40*3*1.5}{8} = 82.5\text{ kN}, \quad A_y = 80 - 82.5 = -2.5\text{ kN}$$

$$A_x = 40*3 = 120\text{ kN}$$

Equivalent loading on frame is,





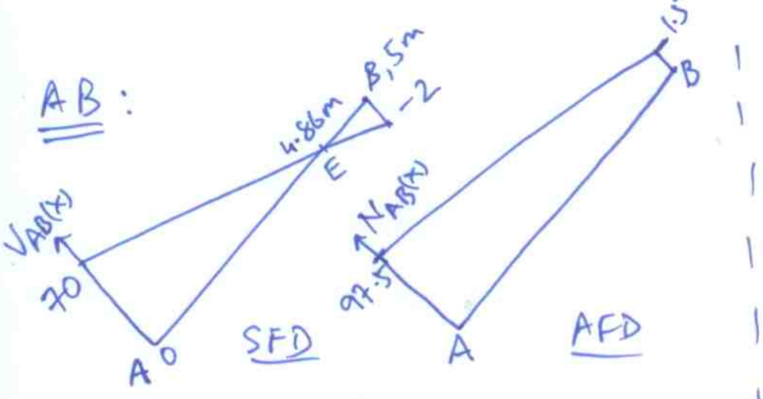
$$w_0\,dy\sin\theta = w_1\,ds = w_1\frac{dy}{\sin\theta}$$

$$w_1 = w_0\sin^2\theta = 40*\frac{9}{25}$$

$$w_0\,dy\cos\theta = w_2\,ds = w_2\frac{dy}{\sin\theta}$$

$$w_2 = w_0\cos\theta\sin\theta = 40*\frac{12}{25}$$

AB :



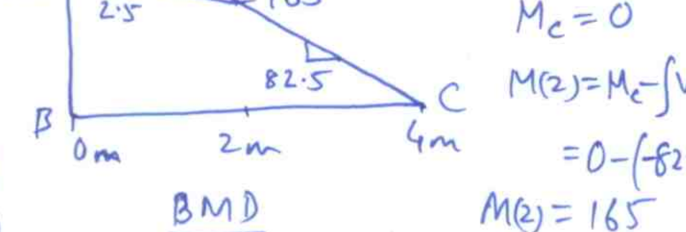
$$V_A = A_x\sin\theta + A_y\cos\theta = 120*\frac{3}{5} - 2.5*\frac{4}{5} = 70\text{ kN}$$

$$N_A = A_x\cos\theta - A_y\sin\theta = 120*\frac{4}{5} + 2.5*\frac{3}{5} = 97.5\text{ kN}$$

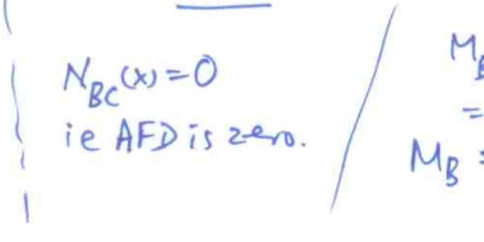
$$V_B = 70 - 14.4*5 = -2$$

$$N_B = 97.5 - 19.2*5 = 1.5.$$

BC :



SFD



BMD

$$M_c = 0$$

$$M(2) = M_c - \int V\,dx = 0 - (-82.5*2)$$

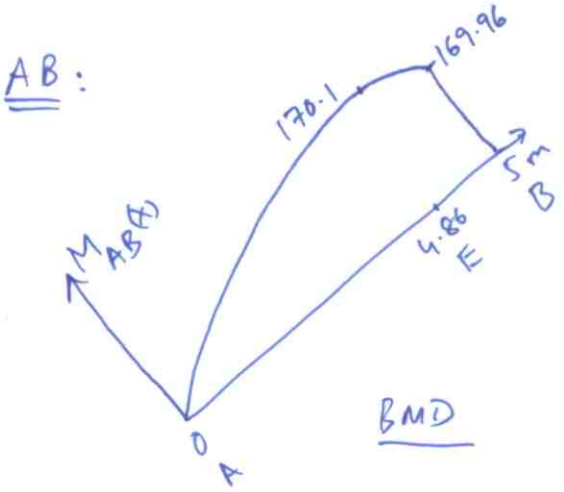
$$M(2) = 165$$

$$M_B = M(2) - \int V\,dx = 165 - (-2.5*2)$$

$$M_B = 170$$

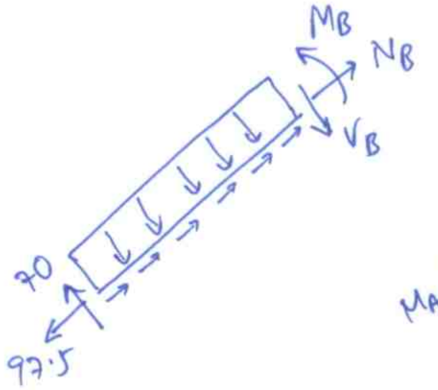
$N_{BC}(x) = 0$ ie AFD is zero.

AB:

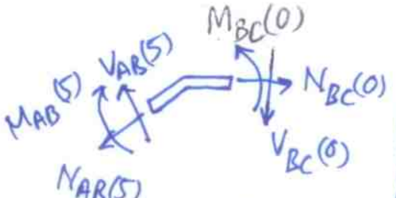


$$M_E = M_A + \int V dx$$
$$= 0 + \tfrac{1}{2}(70 * 4.86) = 170.1$$
$$M_B = M_E + \int V dx = 170.1 - \tfrac{1}{2}(5 - 4.86)(2)$$
$$= 169.96 \simeq 170$$

Check equilibrium of AB, and point B.



$$\Longrightarrow \quad \Sigma M_B = 0 \Rightarrow M_B + 14.4 * \frac{5^2}{2} - 70 * 5 = 0$$
$$\Rightarrow M_B = 170 \simeq 169.96 \text{ (checks out)}$$



$$M_{AB}(5) - M_{BC}(0) = 170 - 170 = 0$$
$$-N_{AB}(5)\cos\theta - V_{AB}(5)\sin\theta + N_{BC}(0) = -1.5 * \tfrac{4}{5} + 2 * \tfrac{3}{5} + 0 = 0$$
$$-N_{AB}(5)\sin\theta + V_{AB}(5)\cos\theta - V_{BC}(0) = -1.5 * \tfrac{3}{5} - 2 * \tfrac{4}{5} + 2.5 = 0$$

(Checks out)

We can also write $N[x]$, $V[x]$, $M(x)$ for AB without equivalent loading, as follows. Let y = vertical coord along AD, x = coord along incline AB

$$V(x) = V_A - 40y\sin\theta = 70 - 40(x\sin\theta)\sin\theta = 70 - 14.4x.$$

$$N(x) = N_A - 40y\cos\theta = 97.5 - 40(x\sin\theta)\cos\theta = 97.5 - 19.2x$$

$$M(x) = M_A + \int V dx = 0 + 70x - \frac{14.4}{2}x^2$$

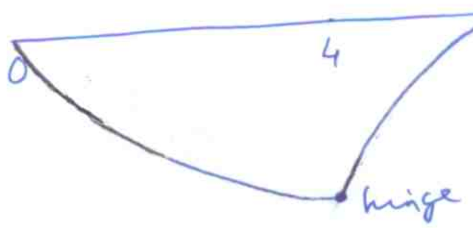
NOTE: We are drawing BMD on the compression side as per our convention. Another convention is to draw it on tension side, i.e., ( ) +ve, which immediately tells us that the side with +ve BMD requires reinforcement steel. In our case the side with −ve BMD requires the steel.

Deflected Shapes.

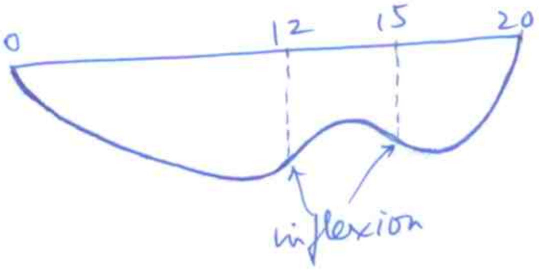
Use BM diagrams and support conditions.

(+) BM ⇒ concave upward, (−) BM ⇒ convex upward.

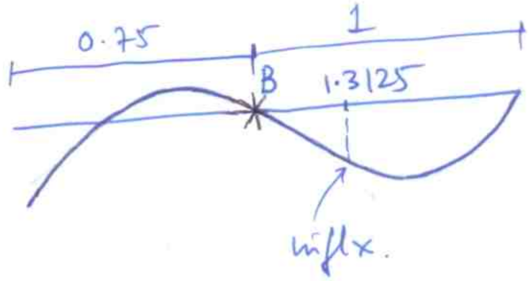
Deflected shapes for Ex 2−6



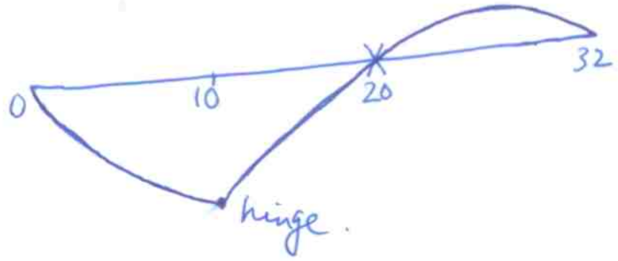
Ex 2



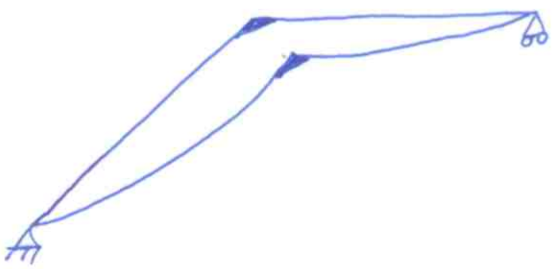
Ex 3



Ex 4



Ex 5



Ex 6.