TOPIC-3
INFLUENCE LINES.—
STATICALLY DETERMINATE STRUCTURES.

Influence line: Represents the variation of any load dependent quantity (i.e, reaction, shear, BM, displ.) at a particular point when the load is moved across the structure (ie load application point or patch is varied along structure's span).
It is used in design of bridges, cranes etc, ie in moving load structures. Applied load is taken as unit load
In contrast to AFD, SFD, BMD, which represent variation of internal force/moment quantity across span of structure due to fixed-in-space applied load, the IL does just the opposite. It represents variation of int. force or reaction at a fixed-in-space point due to applied load that moves across span of structure.

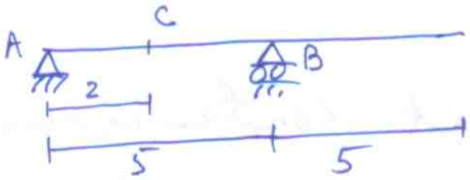
- IL for SD structure comprises of straight line segments (see examples). This is obvious (since reactions, SF, BM will vary linearly as point load moves over span).

Methods of finding IL's

Method 1) Determine end pts of the linear segment, ie key points. Place load at key pts and obtain the 'quantity' whose IL you seek. Join key pts by straight lines.

Method 2) IL function can also be obtained by placing load at variable position 'x' and obtaining 'quantity' $q(x)$.

Ex 1



Obtain IL for R_A, R_B, SF, BM at C

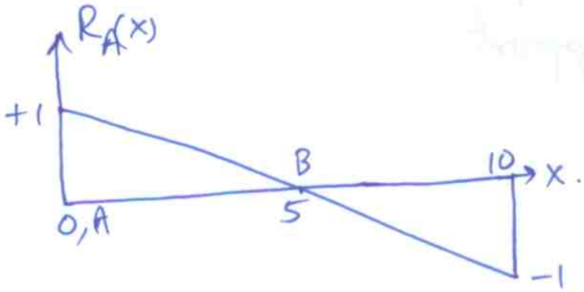
R_A, R_B +ve ($\uparrow$).

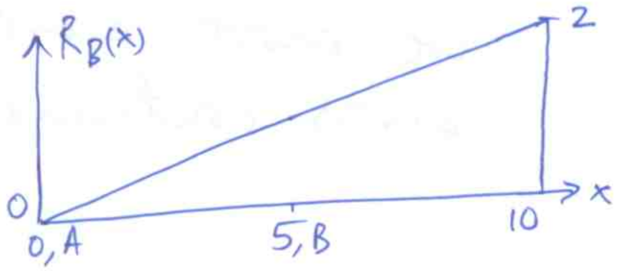
x	R_A
0	1
5	0
10	-1

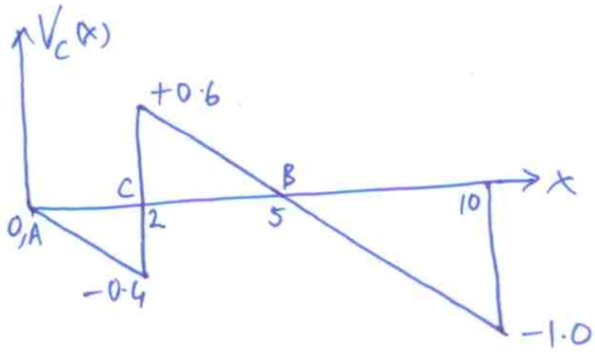
x	R_B
0	0
5	1
10	2

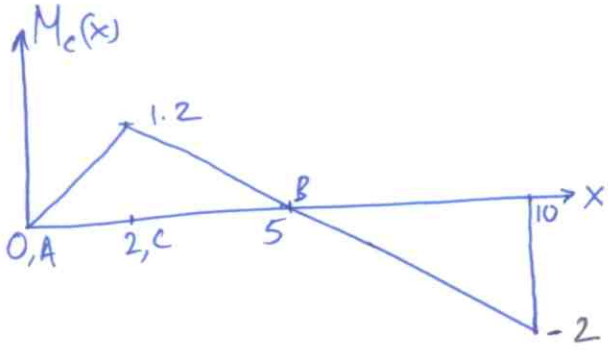
x	V_c
0	0
2^-	-0.4
2^+	0.6
5	0
10	-1

x	M_c
0	0
2	1.2
5	0
10	-2







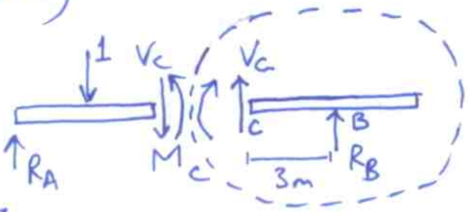


Alternatively, writing IL functions, we have,

$$R_A(x) = \frac{(1)(5-x)}{5} = 1 - \frac{x}{5} \quad (\text{from } \Sigma M_B = 0).$$

$$R_B(x) = \frac{(1)(x)}{5} = \frac{x}{5} \quad (\text{from } \Sigma M_A = 0)$$

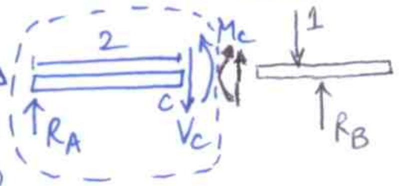
$$V_c = -R_B = -\frac{x}{5}, \quad 0 \le x < 2$$



$$= R_A = 1 - \frac{x}{5}, \quad 2 < x \le 10$$

$$M_c = (3)(R_B) = \frac{3x}{5}, \quad 0 \le x \le 2$$

$$= (2)(R_A) = 2(1 - x/5), \quad 2 \le x \le 10$$



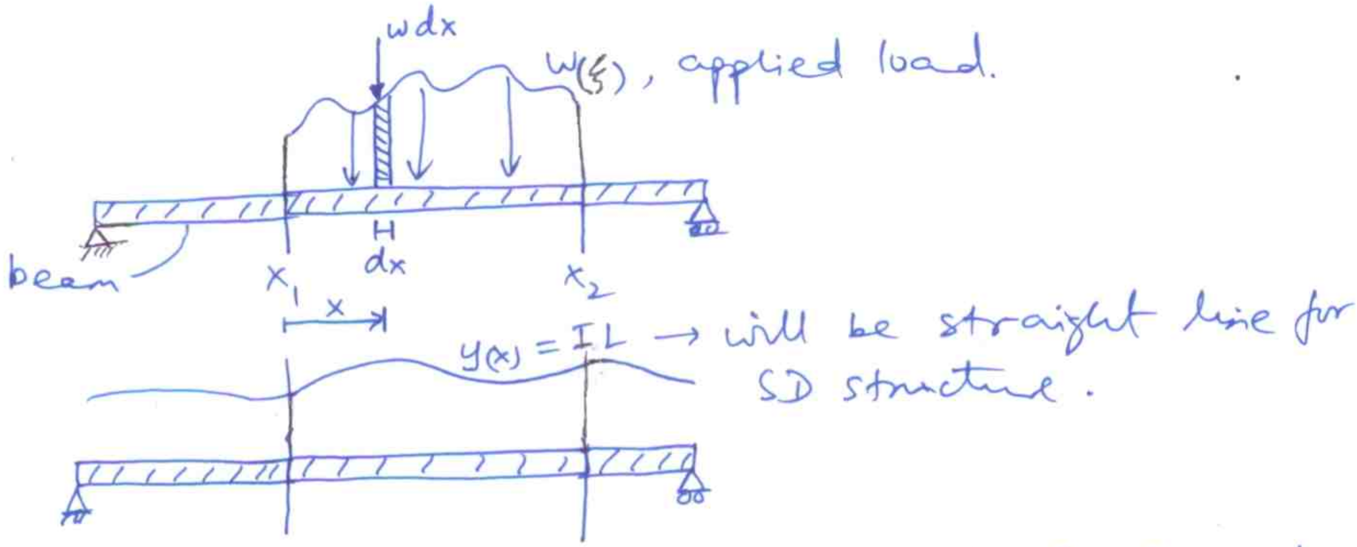
Points to note are :

1) IL of a reaction at continuous support is continuous across support.
2) IL of S.F. at a point is discontinuous as we move across that point.
3) IL of SF at a point is continuous as we move across a continuous support.
4) IL of BM at a point is continuous as we move across that point or move across continuous support.

IL for distributed load on beam

$w(\xi)$ applied over interval $[x_1, x_2]$, $\xi = x - x_1$, $0 \le \xi \le x_2 - x_1$

$y(x)$ = IL of a function (SF, BM, displ, reaction).



Represent $w(\xi)$ as series of point loads $w\,dx$ as shown. Value of influence function due to $w\,dx$ applied at x is $(w(\xi)dx)(y(x))$, ie y scaled by actual applied load $w\,dx$. Thus value of influence function due to total $w(\xi)$, applied over $[x_1, x_2]$, is sum of values due to individual $w\,dx$'s, ie,

$f(x_1, x_2)$ = Value of influence function due to distributed $w(x)$ load applied between $[x_1, x_2]$

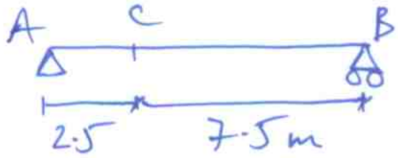
$$= \int_{x_1}^{x_2} w(\xi)\, y(x)\, dx = \int_{x_1}^{x_2} w(x - x_1)\, y(x)\, dx$$

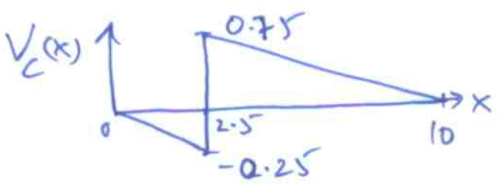
= area under the product of applied $w(\xi)$ and $y(x)$ in interval $[x_1, x_2]$.

If $w(\xi) = w_0$ = u.d.l applied over $[x_1, x_2]$ interval of beam

$f(x_1, x_2) = w_0 \int_{x_1}^{x_2} y\,dx = w_0 *$ area under IL function 'y' taken over the $[x_1, x_2]$ interval of beam.

Ex2 Find max positive shear at point C due to point load of 4000 N and moving load of 2000 N/m. Assume moving load patch of ∞ length and it can move off the beam (e.g. like in a bridge girder).





IL (due to point unit load).

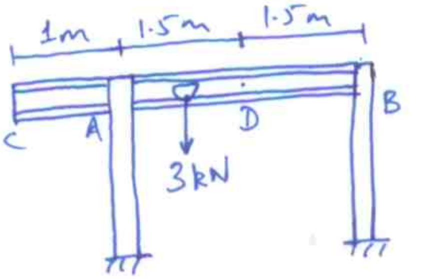
since udl can move off beam.

$(V_c)_{max} = 4000 * 0.75 + 2000 * (\text{area})_{max}$ under IL

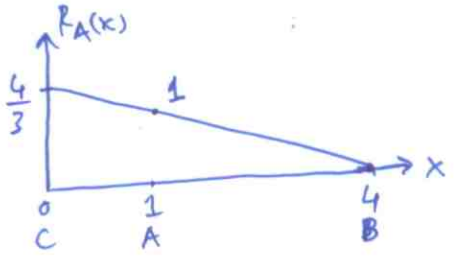
So $(V_c)_{max}$ will be for moving load patch acting over [2.5, 10], ie, −ve area wont participate and we get $(\text{area})_{max}$

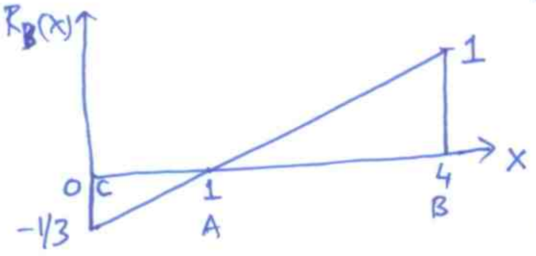
$(V_c)_{max} = 3000 + 2000 * \frac{1}{2}(0.75)(7.5) = 8625\text{ N}$

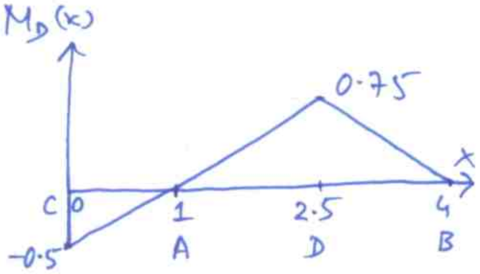
Ex 3.



Hoist supported on beam CB weighing 24 kg/m. It carries 3 kN load. Assume A is pin, B is roller. Find max reactions at A, B and moment at D.







$(R_A)_{max} = \frac{4}{3} * 3 + \frac{1}{2} \cdot \frac{4}{3} \cdot 4 \cdot 24 \frac{*10}{1000} = 4.64\text{ kN}$

$(R_B)_{max} = 1 * 3 + \frac{1}{2}\left(-\frac{1}{3} \cdot 1 + 1 * 3\right) \cdot 24 \frac{*10}{1000} = 3.32\text{ kN}$

$(M_D)_{max} = 0.75 * 3 + \frac{1}{2}\left(-0.5 * 1 + 0.75 * 3\right) * 24 * \frac{10}{1000} = 2.46\text{ kN.m.}$

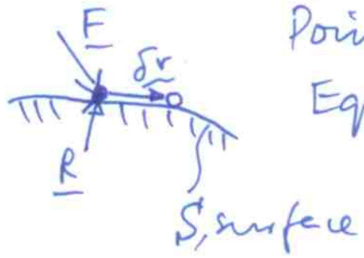
Müller-Breslau principle.

RECALL OF Principle of VW.

Recall that principle of virtual work states that for a system of bodies in equilibrium, if the system is ideal (ie, no friction losses), the VW due to active* forces is zero. Active forces are those that are capable of doing work when a virtual* displ is applied that is consistent with the constraints of the system. Thus support reactions do no VW.

Briefly recalling from CE102, this principle is derived for a point mass in equil, and then considering a system of bodies as a collection of point masses and noting that internal* forces do equal & opp VW.

(*displ → incl. rotations) (*forces → includes moments).



Point mass constrained to move on surface S.

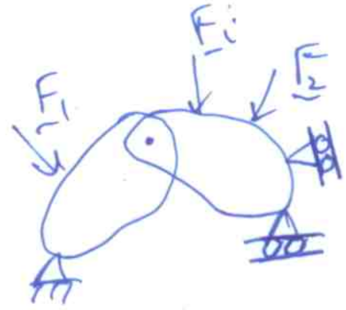
Equil $\Rightarrow \underline{F} + \underline{R} = 0$, $\underline{F}$ = applied forces

$\underline{R}$ = reactions

Let $\underline{\delta r}$ = virtual displ consistent with constraint.

$$VW = \delta W = (\underline{F} + \underline{R}) \cdot \underline{\delta r} = \underline{F} \cdot \underline{\delta r} = 0$$

($\because \underline{R} \cdot \underline{\delta r} = 0$ due to constraint and $\underline{F} + \underline{R} = \underline{0}$ due to equil).



$\sum$ VW due to all particles $= \sum_{i=1}^{N} \underline{F_i} \cdot \underline{\delta r_i} = 0$

where we used $\sum_{j=1}^{m} \underline{R_j} \cdot \underline{\delta r_j} = 0$,

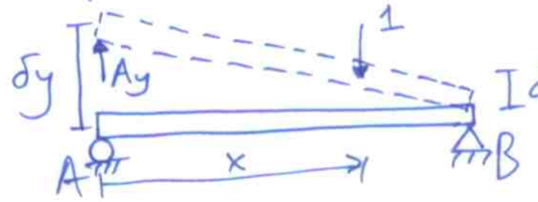
and internal forces between two particles do equal and opp VW $\because \underline{\delta r_i}$ is same but int. forces are equal & opp, so int. VW is zero sum.

$\underline{F_i} \rightarrow$ i^{th} active force

$\underline{R_j} \rightarrow$ j^{th} reaction

$\underline{\delta r_i}, \underline{\delta r_j} \rightarrow$ virtual displ where $\underline{F_i}$ or $\underline{R_j}$ present.

For IL of a reaction, release reaction, apply unit load at x,



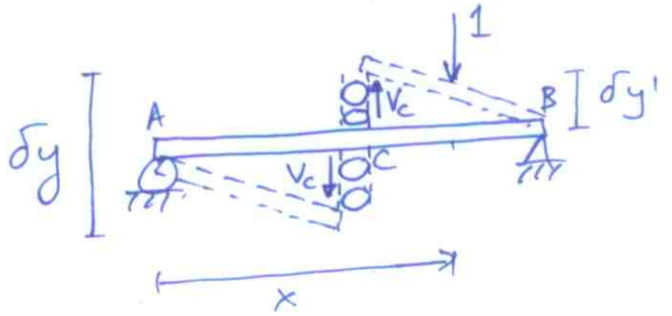
$A_y \underbrace{\delta y}_{=1} - 1\,\delta y'$

=1 (give unit virtual displ at A)

$\therefore\ A_y = \delta y'$, ie virtual displ at x (pt of load application) equals reaction A_y when unit virtual displ applied at A.

|| Hence deflected shape is same as IL for A_y when A_y released.

For IL of SF, release shear, apply unit load at x,



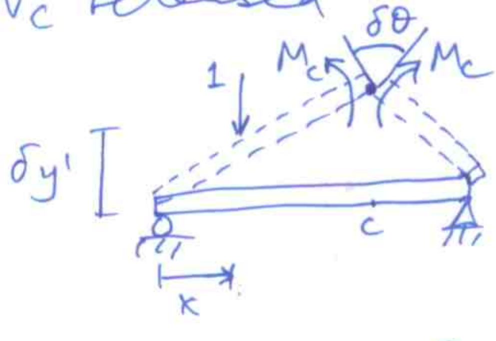
$VW = 0 = V_c \underbrace{\delta y}_{=1} - 1\,\delta y'$

=1, chosen for convenience, w/o loss of generality

$\therefore\ V_C = \delta y'$

|| Hence deflected shape is same as IL for V_c when V_C released

For IL of BM, release moment, apply unit load at x.



$VW = 0 = M_c \underbrace{\delta\phi}_{=1} - 1\,\delta y' = 0$

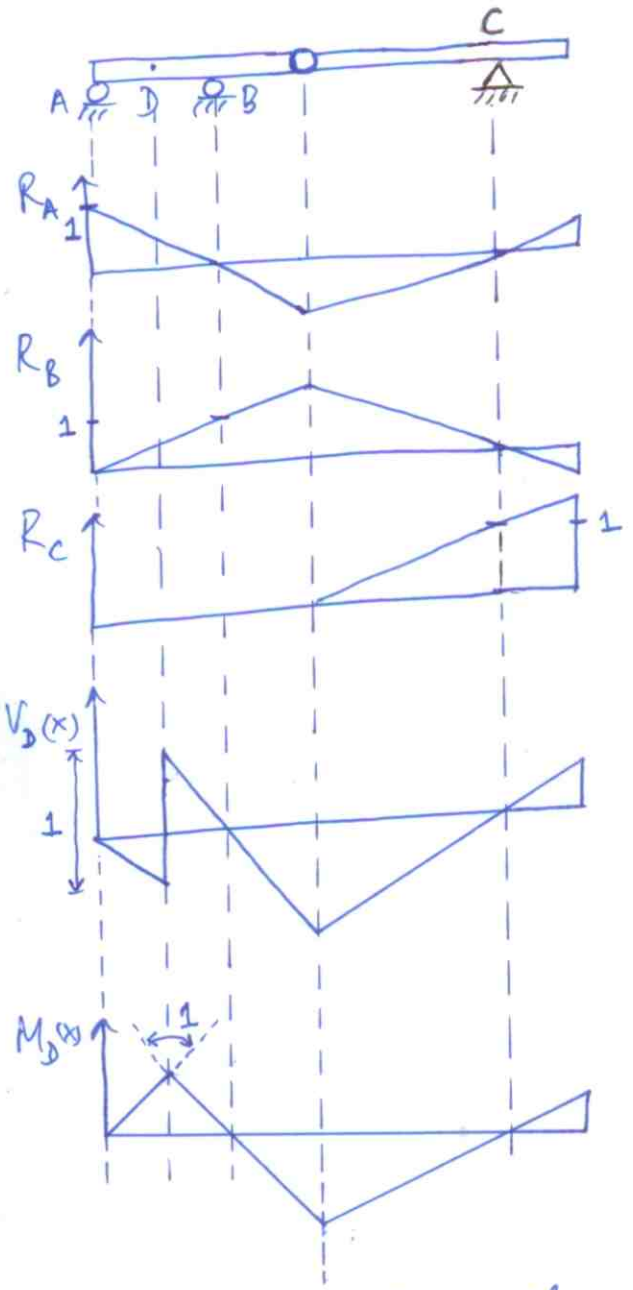
=1, for convenience, w/o loss of generality.

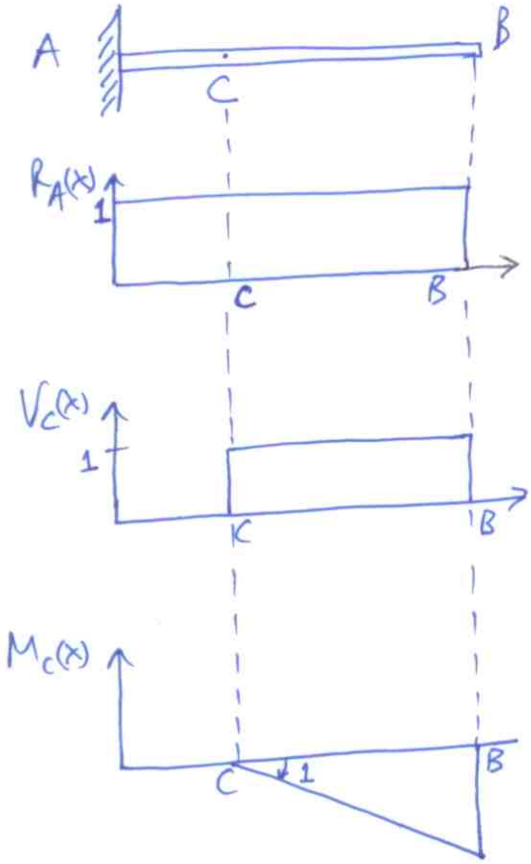
$\therefore\ M_C = \delta y'$

|| Hence deflected shape is same as IL for M_c when M_C released.

※ **MÜLLER-BRESLAU PRINCIPLE**: IL of a function (reaction, shear, moment) is a scaled version of the deflected shape of beam when the function is applied. We must release the capacity of beam to resist the function (reaction, SF, BM) so that beam will deflect when function is applied.

Ex 4 Sketch IL for reactions, SF, BM at point indicated.





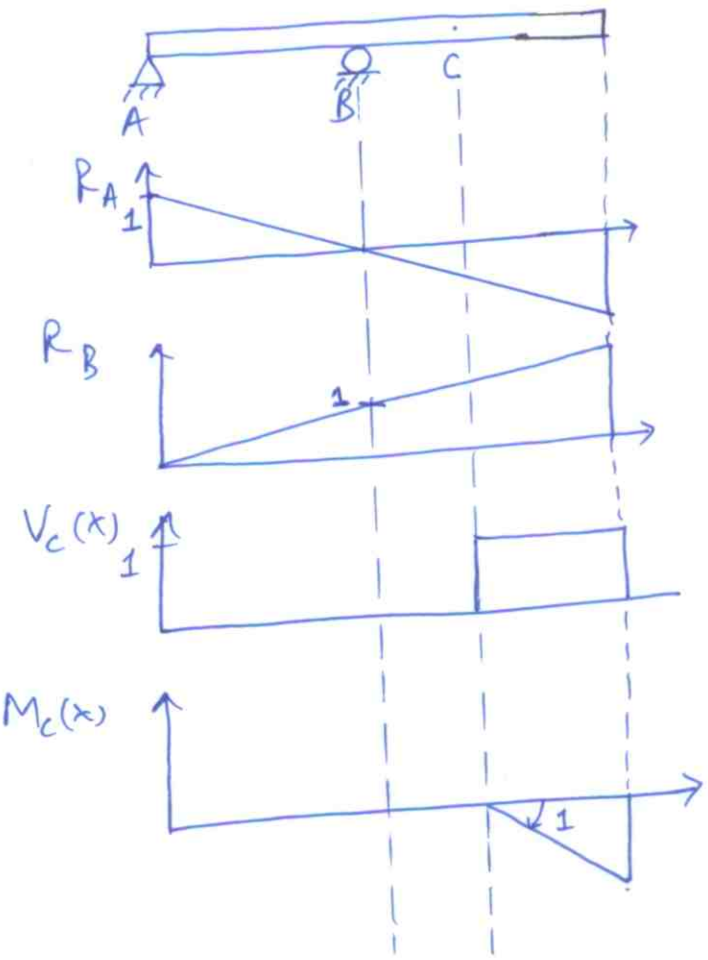
For R_A, release vertical reaction at A, keeping all other constraints at A intact (ie no rotation, horizontal translation).

For V_c, M_c, release SF, BM at C, keeping all constraints at A intact.

For R_A, R_B, R_c, release the support A, B, C, respectively and plot deflected shape. maintaining constraint at remaining supports.

For V_c, M_c, release SF, BM, respectively, plot deflected shape maintaining constraint at all supports.

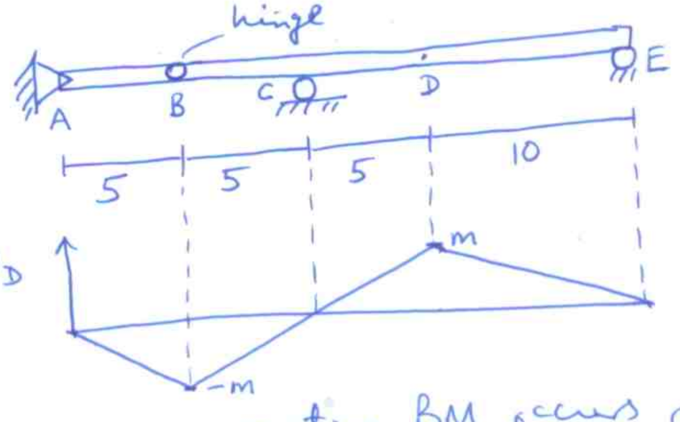
Ex 4 (contd.)



Results compare with Ex 1 for R_A, R_B.

For V_c, M_c, release SF, BM at C but ensure constraints intact at B, A, ie take care that no hinge forms at B $\because$ constraint at B also implies slope continuity at B.

Ex 5

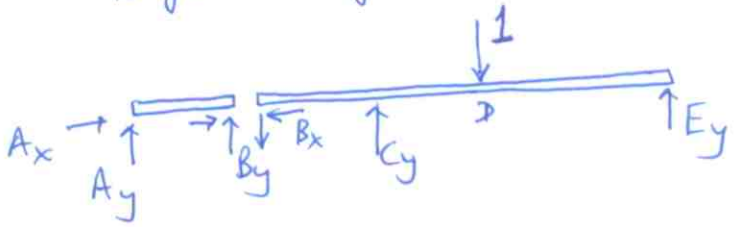


Moving point load of 4000 N, moving uniform load of 300 N/m applied. Self wt is 200 N/m.

Find $(BM)_{max}$ (positive) at D.

So max positive BM occurs for point load at D & moving udl between C & E (so that we get max +ve area under IL diagram).

To get IL function value at D, due to unit point load,



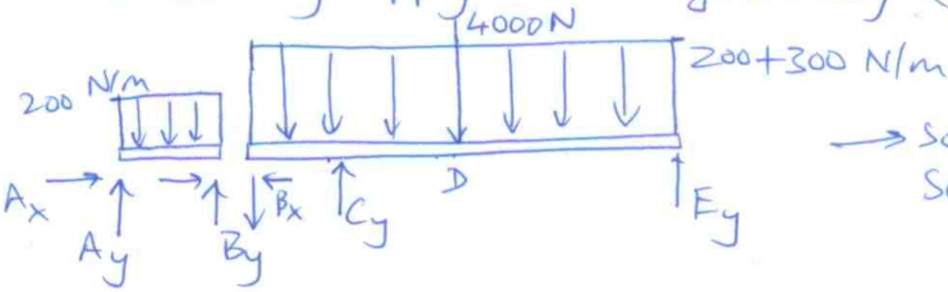
$B_y = 0$ (from $\Sigma M_A = 0$, LHFBD)

$E_y = \frac{1}{3}$ (from $\Sigma M_C = 0$, RHFBD).

$M_D = E_y * 10 = \frac{10}{3} = m$.

$$(M_D)_{max} = 4000 * \frac{10}{3} + 200 * \frac{1}{2}\left(\left(-\frac{10}{3}\right)(10) + \left(\frac{10}{3}\right)(15)\right) + 300 * \frac{1}{2} * \left(\frac{10}{3}\right)(15)$$
$$= 22500\,N.m$$

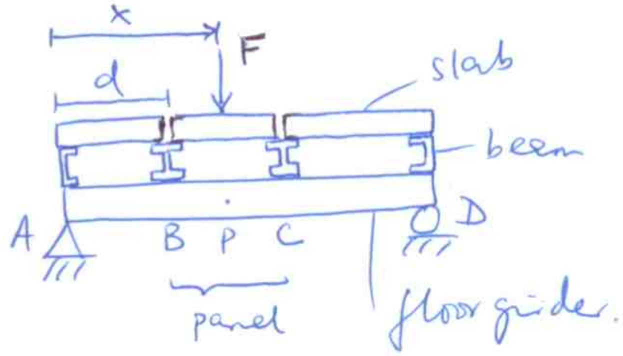
Alternately apply loads yielding $(M_D)_{max}$, i.e.,

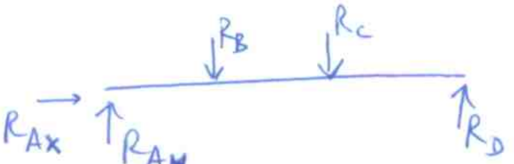


→ solve for M_D and get same result.

Floor Girders.

Fig 6.20(C) ≡





⇒ SF constant over a panel of the beam.
BM varies linearly over panel

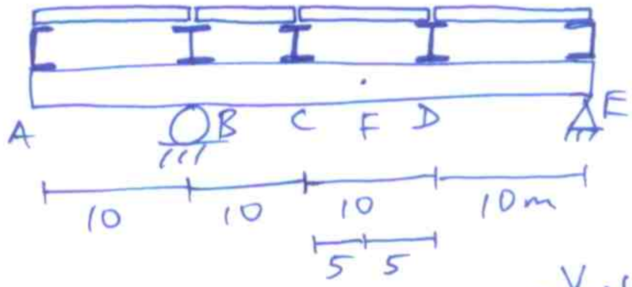
i.e, $V = R_{Ay} - R_B$ = const over panel BC

$M = R_{Ay}\,x - R_B(x-d)$ = linear variation over panel BC.

Thus V in panel BC of girder is constant and doesn't depend on location of point P in the panel. So for floor girders we determine "panel shear" instead of 'point shear'. For BM it remains 'point BM' as before.

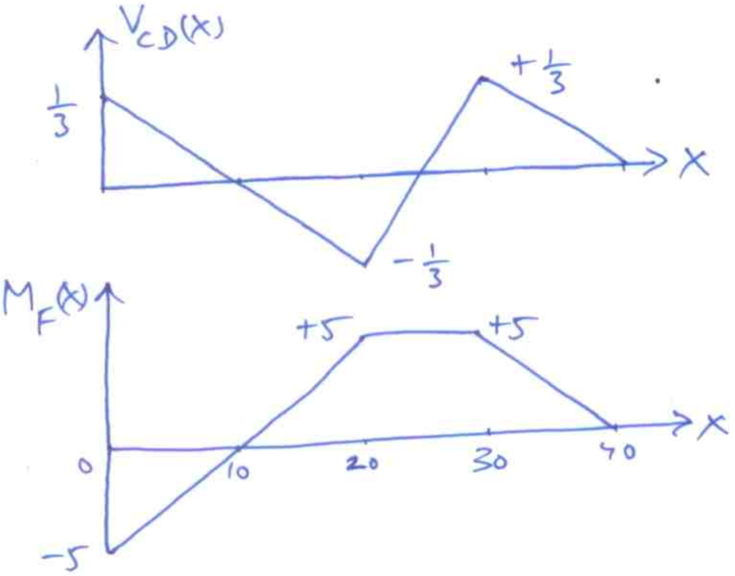
<u>Influence lines for panel **shear**</u> : Note that ext. (R_{Ay}, R_D) reactions vary linearly with x, and int reactions (R_B, R_C) vary piecewise linearly with x. Hence panel shear varies piecewise linearly with x, ie within each panel we have linear variation. <u>Hence key points for plotting IL function for shear</u> <u>are</u> the <u>end points of panels</u>. Using <u>same</u> arguements, <u>for BM</u> we have key pts as panel end-pts (i.e., floor beam locations) and piecewise linear variation of IL.

Ex 6.

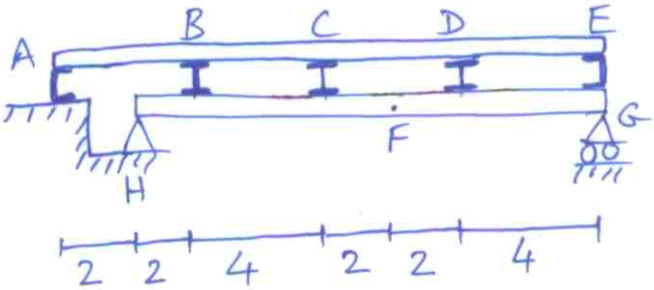


Draw IL for $(V)_{panel\ CD}$ & $(M)_F$

x	V_{CD}	M_D
0	1/3	$-\frac{1}{3}\cdot 15 = -5$
10	0	0
20	−1/3	$\frac{1}{3}\cdot 15 = 5$
30	1/3	$\frac{1}{3}\cdot 15 = 5$
40	0	0

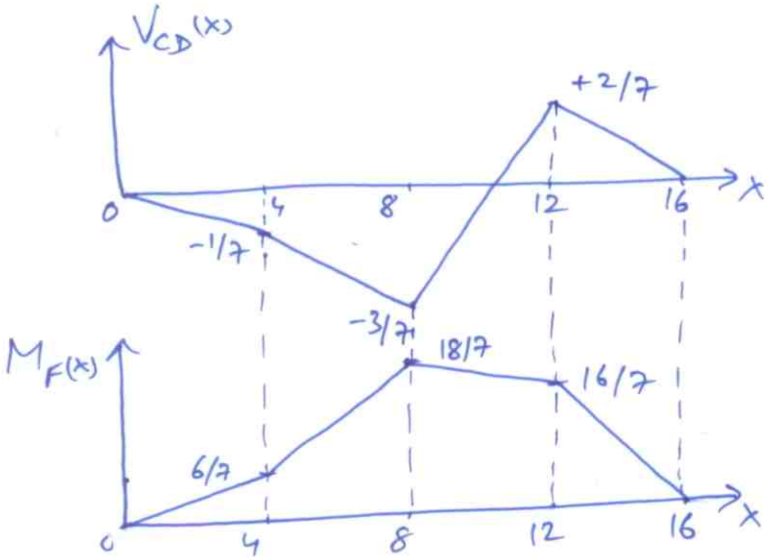


Ex 7



Draw IL for V_{CD} & M_F

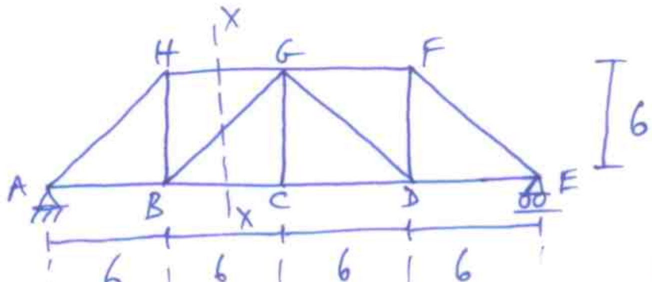
x	V_{CD}	M_F
0	0	0
4	−1/7	$\frac{1}{7}*6 = 6/7$
8	−3/7	$\frac{3}{7}*6 = 18/7$
12	2/7	$\frac{2}{7}*8 = 16/7$
16	0	0



Truss Girders.

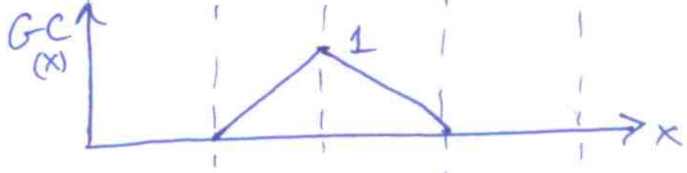
Same concept as in floor girders. Deck load transmitted via stringers and floor beams to joints of bottom chord or top chord (depending on whether deck supported on bottom chord or top chord). Hence the IL variation is piecewise linear between the key points which are joints of the respective chord on which load transmitted.
The IL here is for AF of a particular member (which is like panel shear as it is constant for the member).

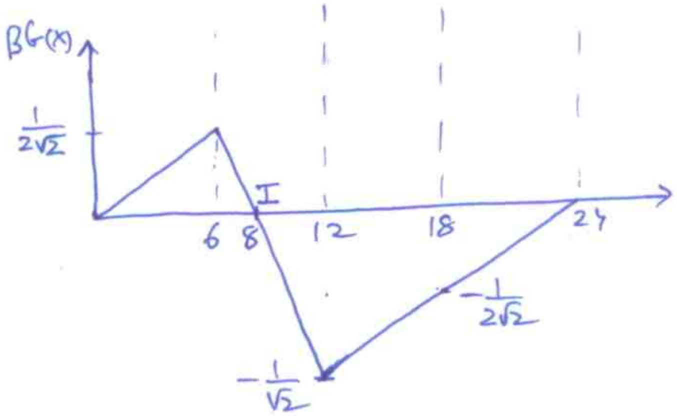
Ex 8



Find IL for members GB, and GC. Also find max tensile and max compressive force in these members due to 20 kN point load and 0.6 kN/m udl that are moving across deck. Deck supported on bottom chord.

GC = zero force member unless point load applied at C.





For BG, consider section XX

X	GB
0	0
6	$(-R_A+1)\sqrt{2} = (-\frac{3}{4}+1)\sqrt{2} = \frac{\sqrt{2}}{4}$
12	$-R_A\sqrt{2} = -\frac{1}{2}\sqrt{2} = -\frac{1}{\sqrt{2}}$
18	$-R_A\sqrt{2} = -\frac{1}{4}\sqrt{2} = -\frac{1}{2\sqrt{2}}$
24	0

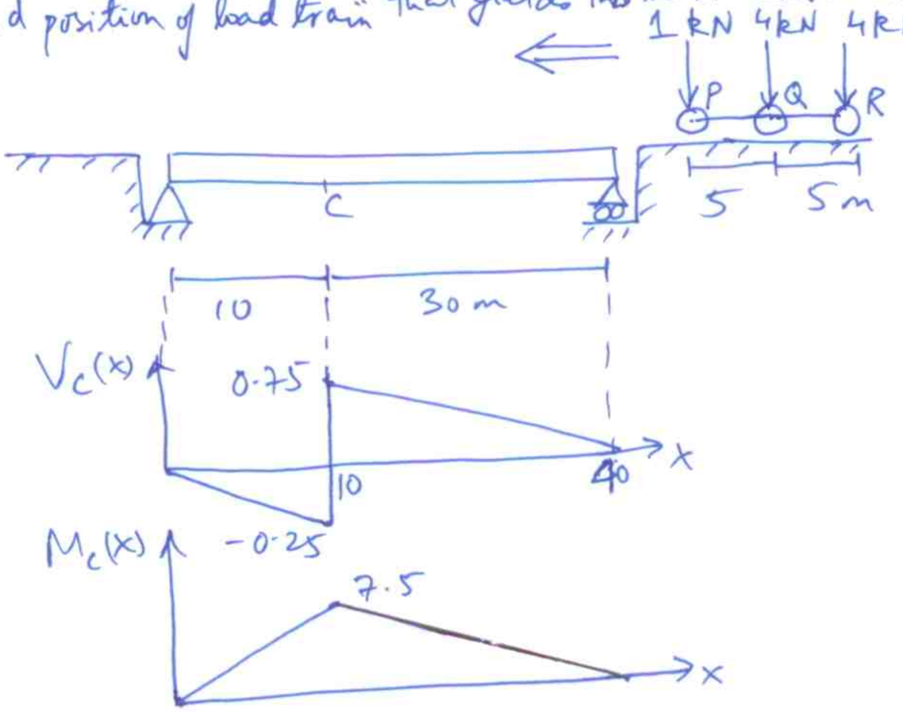
$(GC)_{max} = 20*1 + 0.6*1*6 = 23.6\,(T)$ → when udl at least over BD and pt load at C.

$(GB)_{max,\,Tensile} = 20*\frac{1}{2\sqrt{2}} + 0.6*\frac{1}{2}*8*\frac{1}{2\sqrt{2}} = 7.919\,(T)$ → udl over AI, pt load at B

$(GB)_{max.\,compr} = 20*1/\sqrt{2} + 0.6*\frac{1}{2}*16*1/\sqrt{2} = 17.536\,(C)$ → udl over IE, pt load at C.

Maximum Influence function at a given point due to train of loads.

We want to find max response (SF, BM, etc) at a given point when a series (train) of concentrated loads moves across the structure. Thus we need to find position of load train that yields this maximum. This is the essence of this section.



Due to piecewise-linear nature of IL, it is obvious that max will occur when one of the loads is at the point (station) where max value of IL occurs, i.e, at point C.

P at C $\rightarrow$ $V_c = (1)(0.75) + (4)\left(\left(\frac{0.75}{30}\right)(25+20)\right) = 5.25$ kN

$M_c = (1)(7.5) + (4)\left(\left(\frac{7.5}{30}\right)(25+20)\right) = 52.5$ kN.m

Q at C $\rightarrow$ $V_c = (4)(0.75) + (-1)\left(\frac{0.25}{10}\right)(5) + (4)\left(\frac{0.75}{30} * 25\right) = \underline{\underline{5.375}}$ Max

$M_c = (4)(7.5) + (4)\left(\frac{7.5}{30} * 25\right) + (1)\left(\frac{7.5}{10} * 5\right) = \underline{\underline{58.75}}$ Max.

R at C $\rightarrow$ $V_c = (-4)\left(\frac{0.25}{10} * 5\right) + (4)(0.75) = 2.5$

$M_c = (4)\left(\frac{7.5}{10} * 5\right) + (4)(7.5) = 45$

$\rightarrow$ Not required $\because$ we can see that this is obviously less than Q at C case.

Alternately, we can find change in SF (i.e, ΔV) or BM (i.e, ΔM) as we move from one load configuration to another. Then, if ΔV or ΔM change sign from (+) to (−) a <u>(local) maxima</u> occurs at the load config corresponding to the last (+ve) ΔV or ΔM, respectively. Due to piecewise-linear IL's,

$$\boxed{\begin{array}{l} \Delta V = P\,S\,(x_2 - x_1), \quad \Delta M = P\,S\,(x_2 - x_1) \\ P = \text{load}, \quad S = \text{slope of IL}, \quad x_2 - x_1 = \text{dist traversed by } P \end{array}}$$

when load traverses sloping part of IL, and,

$$\boxed{\begin{array}{l} \Delta V = P(y_2 - y_1) \\ P = \text{load}, \quad y_2 - y_1 = \text{jump in IL} \end{array}}$$

when load traverses a jump in SF IL

For the previous example, P at C → 1, Q at C → 2, R at C → 3,

$$\Delta V_{1-2} = (1)(-1) + (1)\left(\frac{0.25}{10}\right)(5) + (4)\cdot\left(\frac{0.75}{30}\right)(5+5) = 0.125 \text{ kN}$$

$$\Delta M_{1-2} = (1)\left(-\frac{7.5}{10}\right)(5) + (4)\left(\frac{7.5}{30}\right)(5+5) = 6.25 \text{ kN.m}$$

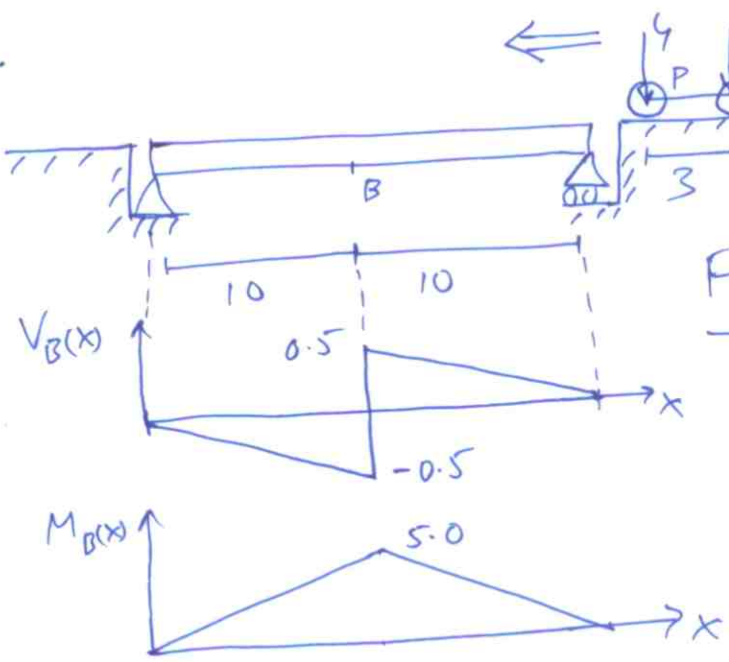
$$\Delta V_{2-3} = (4)(-1) + \left(\frac{0.25}{10}\right)(5)(1+4) + \left(\frac{0.75}{10}\right)(5)(4) = -1.875$$

$$\Delta M_{2-3} = \left(-\frac{7.5}{10}\right)(5)(1+4) + (4)\left(\frac{7.5}{30}\right)(5) = -13.75$$

⇒ Configuration '2' (Q at C) gives $(V_c)_{max}$, $(M_c)_{max}$

<u>NOTE</u>: If load train symmetric, or given pt. on beam at which max IL function required is at center, then you don't need to reverse load train. Else you need to reverse load train in opposite direction to get the true max.

Ex 9



Find $(V_B)_{max}$, $(M_B)_{max}$

P → B → 1
Q → B → 2
R → B → 3
S → B → 4.

$$\Delta V_{1-2} = (4)(-1) + \left(\frac{0.5}{10}\right)(3)(4+9+15) = 0.2 \text{ kN}$$

$$\Delta M_{1-2} = \left(\frac{5}{10}\right)(3)(-4+9+15) = 30 \text{ kN.m}$$

$$\Delta M_{2-3} = \left(\frac{5}{10}\right)(6)(-4-9+15) + (10)\left(\frac{5}{10}\right)\underline{\underline{(4)}} = 26 \text{ kN.m}$$

$$\Delta V_{2-3} = (9)(-1) + \left(\frac{0.5}{10}\right)(6)(4+9+15) + (10)\left(\frac{0.5}{10}\right)\underline{\underline{(4)}} = 1.4 \text{ kN}$$

$$\Delta V_{3-4} = (15)(-1) + \left(\frac{0.5}{10}\right)(6)(15+10) + (9)\left(\frac{0.5}{10}\right)\underline{\underline{(4)}} + (4)\left(\frac{0.5}{10}\right)\underline{\underline{(1)}} = -5.5 \text{ kN}$$

$$\Delta M_{3-4} = \left(\frac{5}{10}\right)\left((-15+10)(6) + (-9)\underline{\underline{(4)}} + (-4)\underline{\underline{(1)}}\right) = -35 \text{ kN.m.}$$

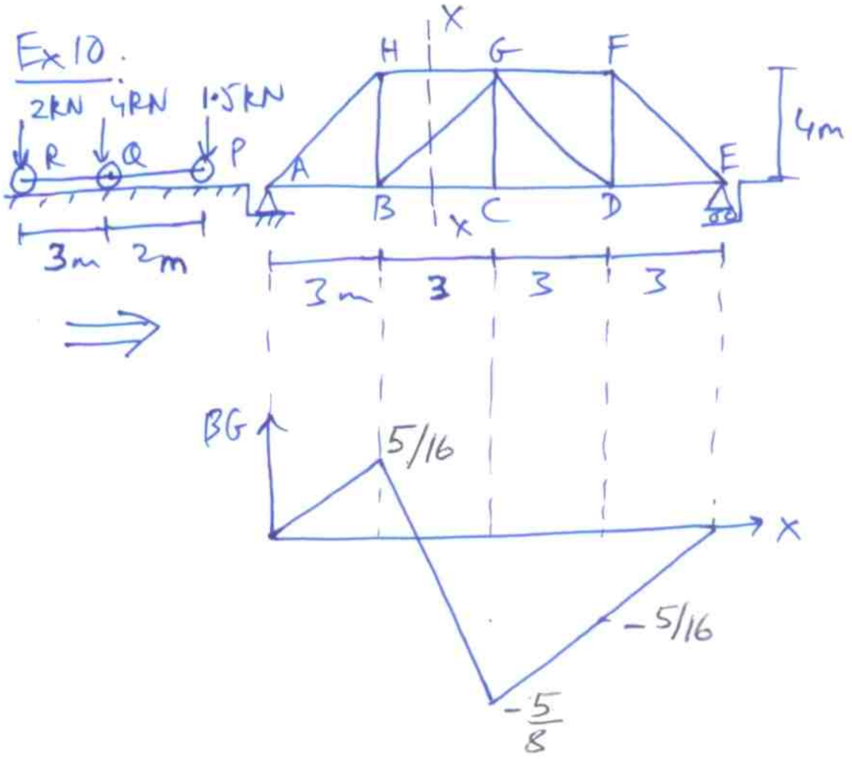
So max V_B, M_B at configuration '3' (R at B),

$$(V_B)_{max} = (4)\left(-\frac{0.5}{10}*1\right) + (9)\left(-\frac{0.5}{10}*4\right) + 15*0.5 + (10)\left(\frac{0.5}{10}*4\right)$$
$$= 7.5 \text{ kN}$$

$$(M_B)_{max} = (4)\left(\frac{5}{10}*1\right) + (9)\left(\frac{5}{10}*4\right) + 15*5 + (10)\left(\frac{5}{10}*4\right)$$
$$= 115 \text{ kN.m.}$$

Note: In 2→3, 10 kN moves only 4 m on beam,
in 3→4, 4 kN " " 1 m " " & 9 kN moves only
4 m on beam (see double-underlined terms).

Ex 10.



Find max compressive force in BG

X	GB
0	0
3	$(-R_A+1)\cdot\frac{4}{5} = (-\frac{3}{4}+1)\cdot\frac{5}{4} = \frac{5}{16}$
6	$-R_A * \frac{4}{5} = -\frac{1}{2}\cdot\frac{5}{4} = -\frac{5}{8}$
9	$-R_A * \frac{4}{5} = -\frac{1}{4}\cdot\frac{5}{4} = -\frac{5}{16}$
12	0

$P\to C\to 1$, $Q\to C\to 2$, $R\to C\to 3$.

$$\Delta BG_{1-2} = (1.5)(2)\left(\frac{5}{8}\cdot\frac{1}{6}\right) + (4)(2)\left(-\frac{15}{16}\cdot\frac{1}{3}\right) + (2)(2)\left(\frac{5}{16}\cdot\frac{1}{3}\right) = -1.7708$$

$$\Delta BG_{2-3} = (1.5)(3)\left(\frac{5}{8}\cdot\frac{1}{6}\right) + (4)(3)\left(\frac{5}{8}\cdot\frac{1}{6}\right) + (2)(3)\left(-\frac{15}{16}\cdot\frac{1}{3}\right) = -0.15625$$

Here, since we seek max compressive force in BG, we started with a configuration (ie, $P\to C\to 1$) that at least gave a compressive contribution due to one of the loads in the train.
Further, we will declare a (local) maximum for compressive force when ΔBG changes from (–) to (+).
So last config (ie $R\to C\to 3$) gives max compr BG.

$$(BG)_{max,\,compr} = (2)\left(-\frac{5}{8}\right) + (4)\left(-\frac{5}{8}\cdot\frac{1}{6}\cdot 3\right) + (1.5)\left(-\frac{5}{8}\cdot\frac{1}{6}\cdot 1\right)$$
$$= \underline{\underline{-2.65625\ kN}}\ (C)$$

NOTE: In general, if load train is symmetric or if given point/member for which max value of IL function is sought is symmetric (ie at center) then we dont need to reverse load train (ie move it in opposite direction). Otherwise, in general you should reverse its direction to get the true (global) maximum value.

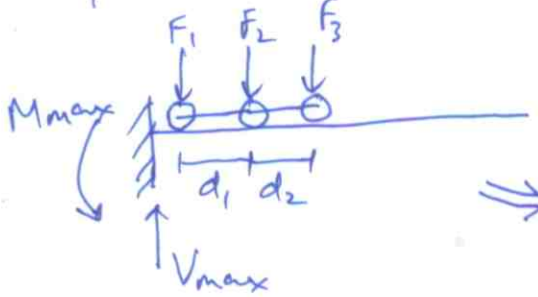
Absolute Maximum IL function due to load train.

In previous section we were given point at which max of IL function is sought, for which we had to find position of load train.

Now we want to find the location of that point and the corresponding load-train location, which yields the absolute max value of IL function.

For general support arrangement the only way is to choose closely spaced points on the beam, then find the max value of IL function for each of the selected points by method of previous section. Then plot these max values wrt points selected to get an envelope of max values. Then choose the "absolute max" value from this envelope. Procedure is suitable for computer programming and invariably would require it.

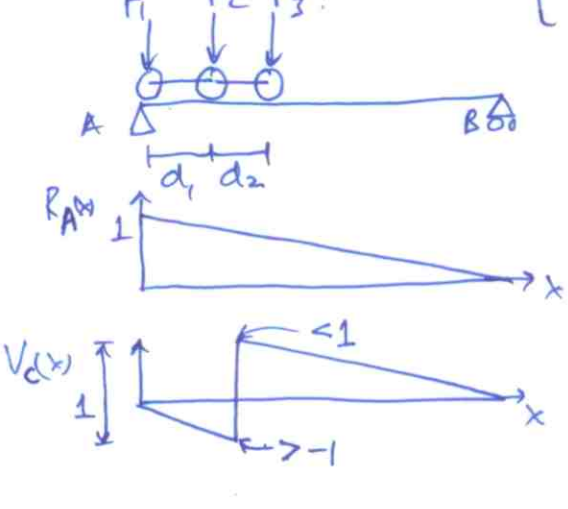
For cantilever & S.S (simply-supported) beams a manual procedure is possible.



$\Rightarrow$ V_{max}, M_{max} occurs at support.

V_{max} for any position of train.

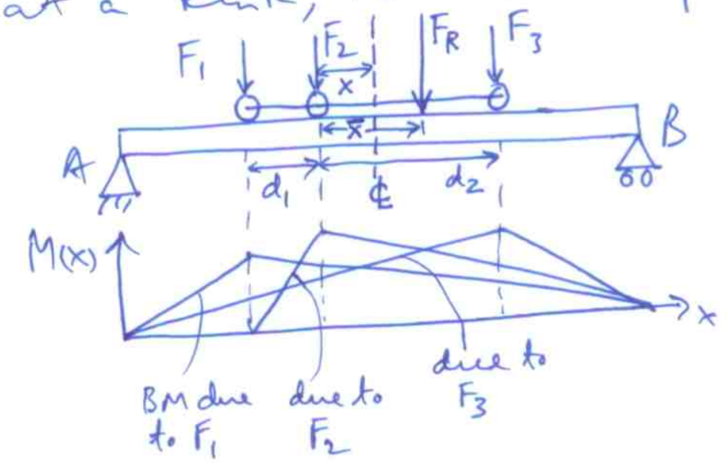
M_{max} for train positioned away from support with possibly one or more of the loads off the beam if permissible. Easily done by method of previous section.



Abs-Max shear force value at support

$\Rightarrow$ for one of the loads placed over support (possibly some of the loads off the beam). Easily done by previous section.

For S.S. beam, abs-max BM must occur under one of the loads. This is because, for any position of the train, the BM diagram is the sum of BM's due to each load. Now BM due to each load comprises two straight lines (one ascending from zero, other descending to zero). Thus BM due to train is sum of a series of straight lines, ie a piecewise linear curve with ends at zero. Hence max BM must occur at a kink, ie a load position (see sketch below).



$d_1, d_2 \rightarrow$ spacing between loads in the train

$F_R \rightarrow$ Resultant of loads in the train.

$x \rightarrow$ distance of load under which max BM required (assumed) to occur (F_2 in this case) and ℄ of beam.

$\bar{x} \rightarrow$ dist of load under which max BM required (assumed) to occur and F_R (ie dist between F_2 & F_R in this case).

$L \rightarrow$ length of beam.

So we assume, or demand, that max BM occurs under a chosen load. Then we find the position 'x' of that load wrt ℄ (ie in effect the position of the train) so that max BM occurs under that load. Then find the max BM for that position (ie BM under the chosen load). Then choose another load under which max BM should occur & repeat process. Finally choose absolute-max BM and corresponding load train position.

Say max BM occurs under F_2.

$$\Sigma M_B = 0 \Rightarrow A_y = \frac{1}{L} F_R\left[\frac{L}{2} - (\bar{x} - x)\right]$$

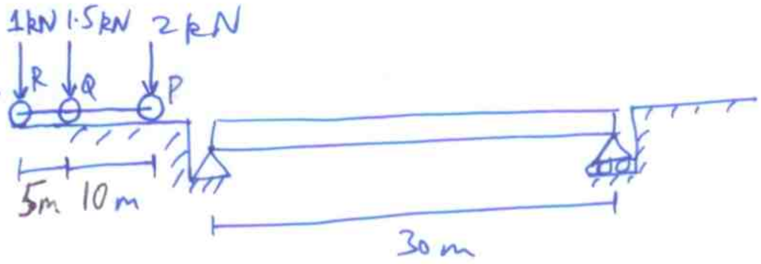
BM under $F_2 = M_2 = \left(\frac{L}{2} - x\right)\frac{1}{L}F_R\left(\frac{L}{2} - (\bar{x} - x)\right) - F_1 d_1$

For M_2 to be max, $\frac{dM_2}{dx} = 0 = \frac{1}{L}F_R\left((\bar{x} - x) - \cancel{\frac{L}{2}} + \cancel{\frac{L}{2}} - x\right) \Rightarrow \boxed{x = \frac{\bar{x}}{2}}$

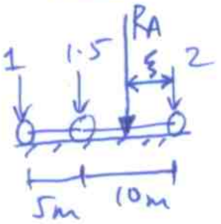
So if we position F_2 & F_R symmetrically about ℄ then max BM occurs under F_2. Find this max BM. Repeat process by positioning each of the other loads symmetrically about ℄ wrt F_R and find corresponding max BM's. Choose the absolute max BM & corresponding position of the train wrt ℄.

<u>Usually</u> abs-max BM occurs under one of the two loads adjacent to F_R (usually under the larger of the two loads adjacent to F_R).

Ex 11



Find abs-max BM.

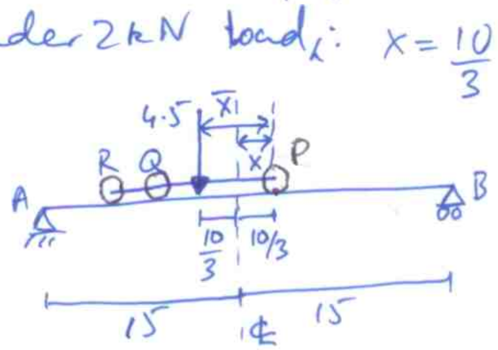


Resultant load position:

$$\xi = \frac{(1.5)(10) + (1)(15)}{4.5} = \frac{20}{3}$$

$$R_A = 4.5.$$

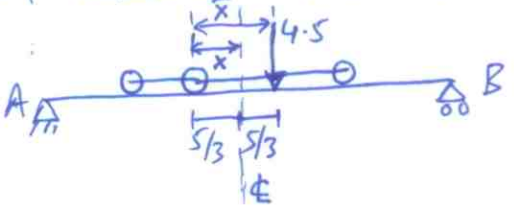
For max BM under 2kN load, ie P: $x = \frac{10}{3}$, $\bar{x} = \frac{20}{3}$



$$B_y = (4.5)(15 - 10/3)/30 = 1.75$$

$$M_P = 1.75 * (15 - 10/3) = 20.4167 \text{ kNm}$$

For max BM under 1.5kN load, ie Q: $\bar{x} = 10 - 20/3 = 10/3$, $x = 5/3$

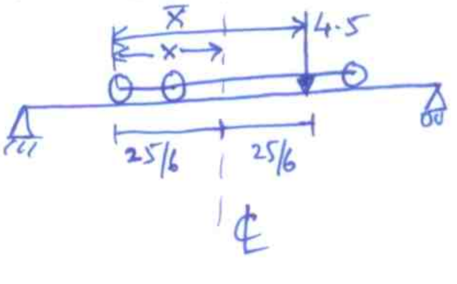


$$A_y = (4.5)(15 - 5/3)/30 = 2$$

$$\underline{M_Q} = (2)(15 - 5/3) = \underline{\underline{26.67 \text{ kN.m}}}$$

$\boxed{\text{ABS MAX}}$

For max BM under 1 kN load, ie R: $\bar{x} = \frac{10}{3} + 5 = \frac{25}{3}$, $x = \frac{25}{6}$



$$A_y = \frac{(4.5)}{30}\left(15 - \frac{25}{6}\right) = 1.625$$

$$M_R = 1.625\left(15 - \frac{25}{6}\right) = 17.604 \text{ kN.m}$$