DEPARTMENT OF CIVIL ENGINEERING
CE-222 STRUCTURAL MECHANICS I
Endsem 15/4/09

ALL QUESTIONS EQUAL MARKS.

Problem 1

Three identical rigid plates of size 0.8m x 0.8m each weighing 21 kN are connected by pin connections with each other as shown in **Fig. 1**. The plates are stabilized by three cables each having axial stiffness ***AE***. One cable is connected to each plate as shown. The three cables are parallel to each other and are inclined at 45°. Find the **vertical displacement of plate 1** (**the top most plate**) **at its centre of gravity**.

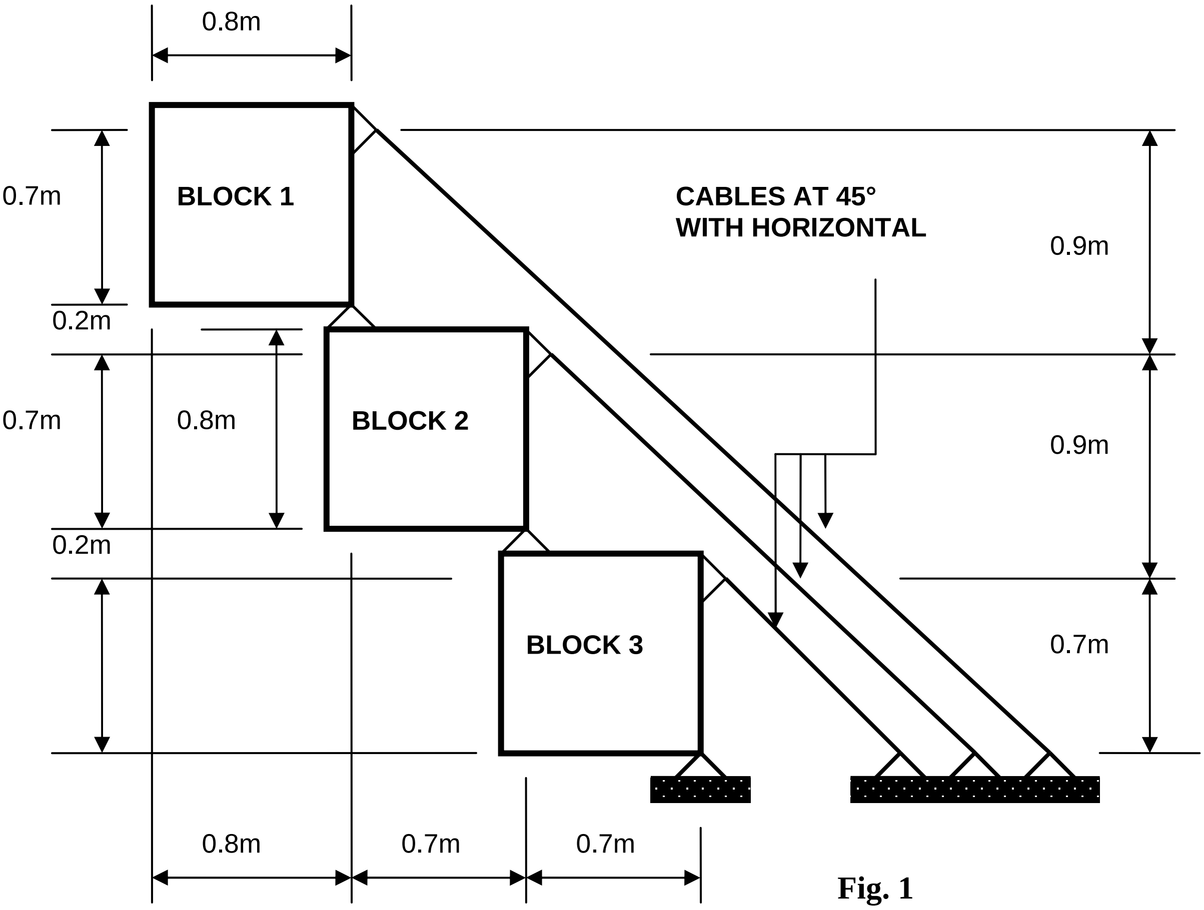


Fig. 1

Problem 2

The frame with internal hinges is loaded as shown in **Fig. 2**. Find the **internal forces at hinge *D***, and draw the **Bending Moment Diagram (BMD)** and sketch the **Qualititative Deflected Shape (QDS) for** the **frame**. Neglect effect of axial and shear forces on the deflection.

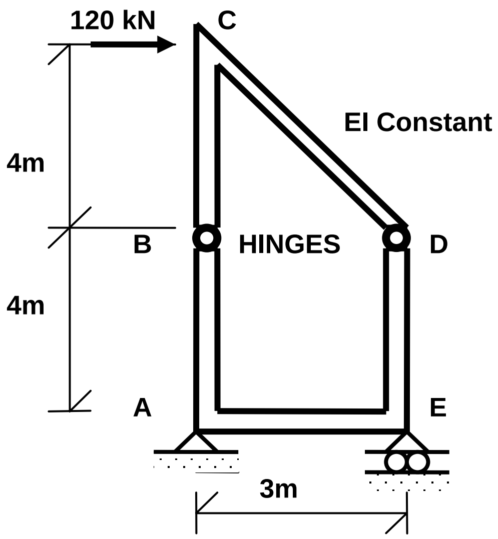


Fig. 2

Problem 3

The trussed-beam supports a load as shown in **Fig. 3**. The truss members are pin connected to the beam ***AF*** as shown. Determine the **forces in all the truss members**. Use $I = 100 \times 10^6\,\text{mm}^4$ for the beam, and $A = 200\ \text{mm}^2$ for all truss members, and $E = 200\,\text{GPa}$ for all members. For the deflection of the beam neglect effect of axial and shear forces.

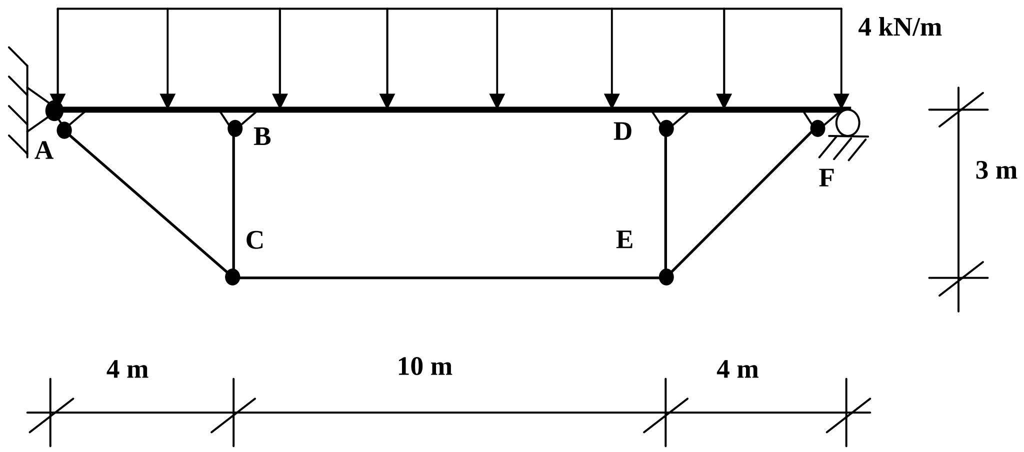


Fig. 3

Problem 4

The structural assembly supports the loading as shown in **Fig. 4**. Find the **force in** the **tie rod *CB***, and draw the **BMD** and sketch the **QDS for beams *AB*** and ***DE***. Use $I = 100 \times 10^6\,\text{mm}^4$ for the beams, and $A = 200\ \text{mm}^2$ for the tie rod, and $E = 200\,\text{GPa}$ for all members. For the deflection of the beams neglect effect of axial and shear forces.

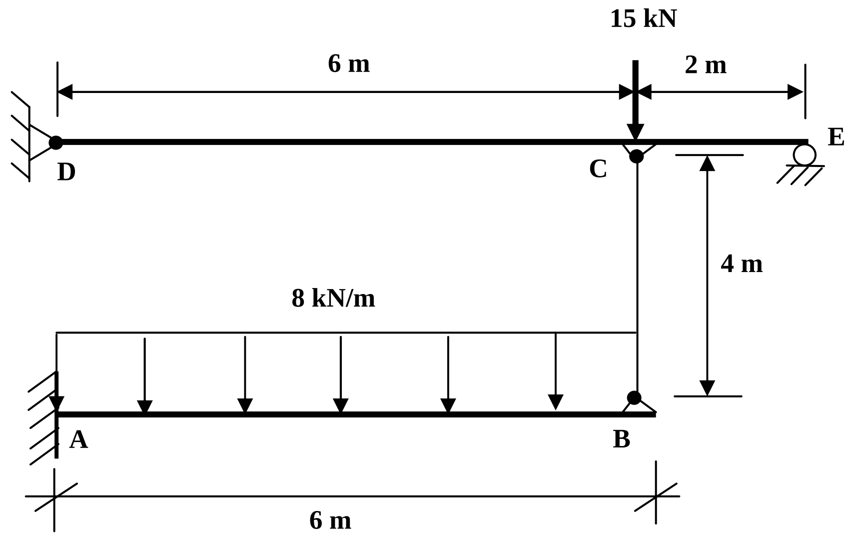


Fig. 4

Problem 5

Draw the **influence line for** the force in member ***FG*** for the truss shown in **Fig. 5**.

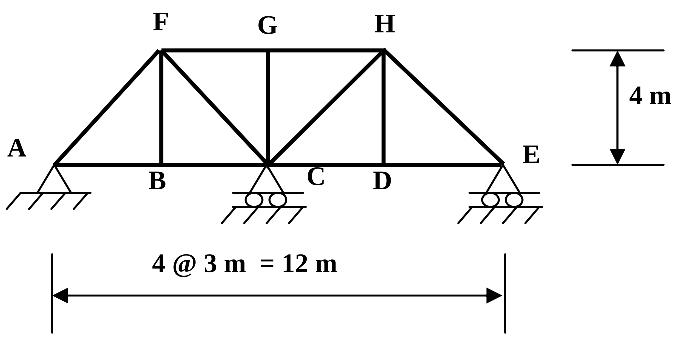


Fig. 5

P1 $\frac{T_1}{\sqrt{2}}(0.1+0.7) = 21(0.4) \Rightarrow T_1 = 10.5\sqrt{2}$

$\frac{T_2}{\sqrt{2}}(0.8) + \frac{T_1}{\sqrt{2}}(1.6-0.6) = 21(0.4+1.1) \Rightarrow T_2 = 26.25\sqrt{2}$

$\frac{T_3}{\sqrt{2}}(0.8) + \frac{T_2}{\sqrt{2}}(1.6-0.6) + \frac{T_1}{\sqrt{2}}(2.5-1.3) = 21(0.4+1.1+1.8)$

$\Rightarrow T_3 = 38.0625\sqrt{2}$

For unit ↓ load at CG of block 1:

$t_1 = T_1/21 = 1/\sqrt{2}$

$t_2 = \left[1(1.1) - \frac{t_1}{\sqrt{2}}(1)\right]\frac{\sqrt{2}}{0.8} = 0.75\sqrt{2}$

$t_3 = \left[1(1.8) - \frac{t_1}{\sqrt{2}}(1.2) - \frac{t_2}{\sqrt{2}}(1)\right]\frac{\sqrt{2}}{0.8} = 0.5625\sqrt{2}$

$\Delta = \sum T_i \frac{t_i L_i}{AE} = \frac{1}{AE}\left[10.5\left(\frac{2.5}{\sqrt{2}}\right) + 39.375\left(\frac{1.6}{\sqrt{2}}\right) + 42.8203125\left(\frac{0.7}{\sqrt{2}}\right)\right]$

$$\boxed{\Delta = \frac{84.3042}{AE}}$$

P3 SDOI → $X_1 = DE$

$\boxed{X_1=0}$: $M = \frac{wL}{2}x - w\frac{x^2}{2}$, $P_i = 0$

$\boxed{X_1=1}$: $m = x,\ 0 \le x \le 4$ → (used symmetry) $\quad p_{DE} = p_{CB} = 1$

$= 4,\ 4 \le x \le 14 \quad p_{EF} = p_{AC} = -5/3$

$= 4-(x-14),\ 14 \le x \le 18 \quad p_{CE} = -4/3$

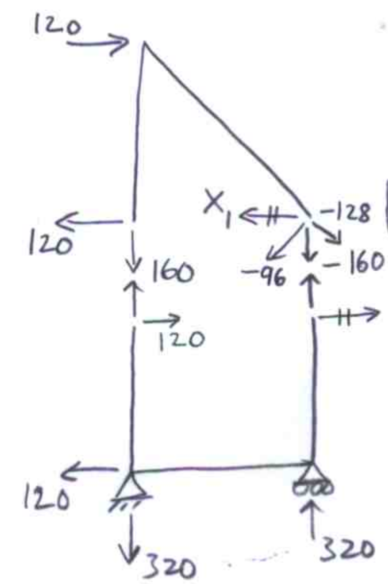
$f_{11} = \frac{1}{EI}\left[2*\frac{1}{3}(4)(4)(4) + (4)(4)(10)\right] + \frac{1}{AE}\left[(1)^2(3)*2 + \left(\frac{5}{3}\right)^2(5)*2 + \left(\frac{4}{3}\right)^2(10)\right] = \frac{608}{3EI} + \frac{464}{9AE}$

$\Delta_{10} = \frac{1}{EI}w\left[\frac{(18)}{2}\cdot\frac{4^3}{3} - \frac{4^4}{2*4} + 4\cdot\left\{\frac{18}{2}\left(\frac{9^2-4^2}{2}\right) - \frac{1}{2}\left(\frac{9^3-4^3}{3}\right)\right\}\right]*2 = \frac{5320}{3}\cdot\frac{w}{EI}$

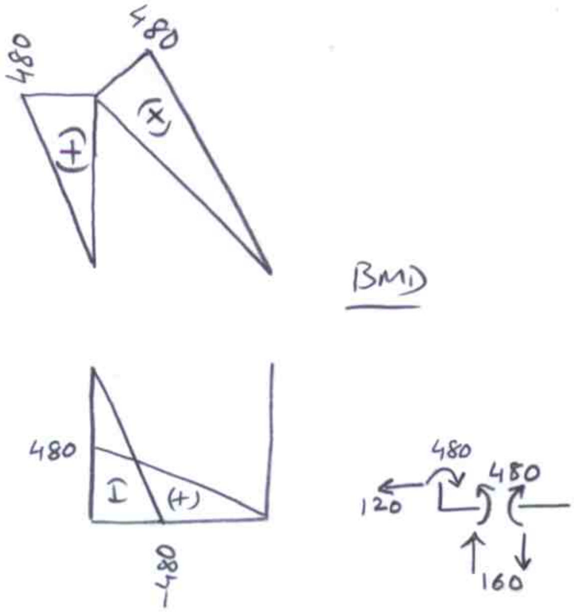
$X_1 f_{11} + \Delta_{10} = 0 \Rightarrow X_1 = -\frac{\left[\frac{5320}{3} * \frac{4}{200E9*10^{-3}*100E6*10^{-12}}\right]}{\left[\frac{608}{3*200*100} + \frac{464}{9*200E-6*200E9*10^{-3}}\right]} = \boxed{-31.0506\ kN}$

$$\boxed{DE = CB = -31.0506\ kN,\quad EF = AC = 51.7510\ kN,\quad CE = 41.4008\ kN}$$

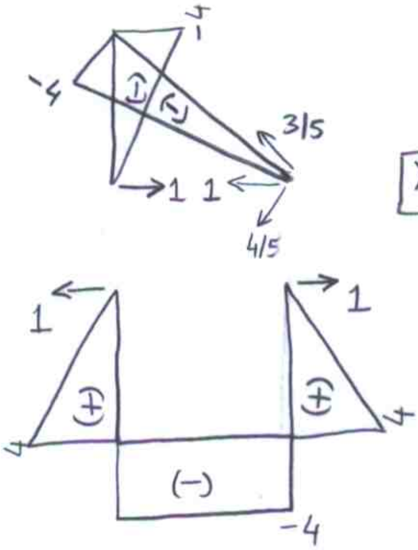
P2 X_1 = as shown



$\boxed{X_1 = 0}$



BMD



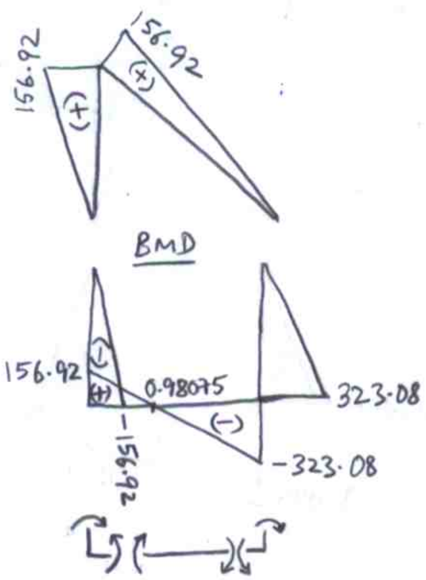
$\boxed{X_1 = 1}$

BMD

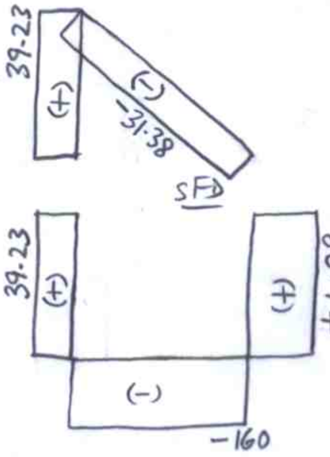
$$\Delta_{10} = -\left[\frac{1}{3}(4)(480)(4+5+4) + \frac{1}{2}(4)(480)(3) \right] \cdot \frac{1}{EI} = -\frac{11200}{EI}$$

$$f_{11} = \frac{1}{EI}\left[\frac{1}{3}(4)(4)(4) * 2 + (4)(4)(3) + \frac{1}{3}(4)(4)(4+5) \right] = \frac{416}{3EI}$$

$$X_1 f_{11} + \Delta_{10} = 0 \Rightarrow X_1 = 80.7692 \text{ kN}$$

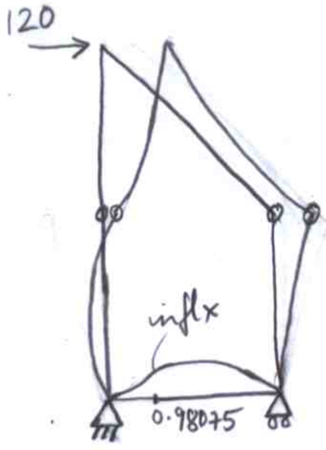


BMD



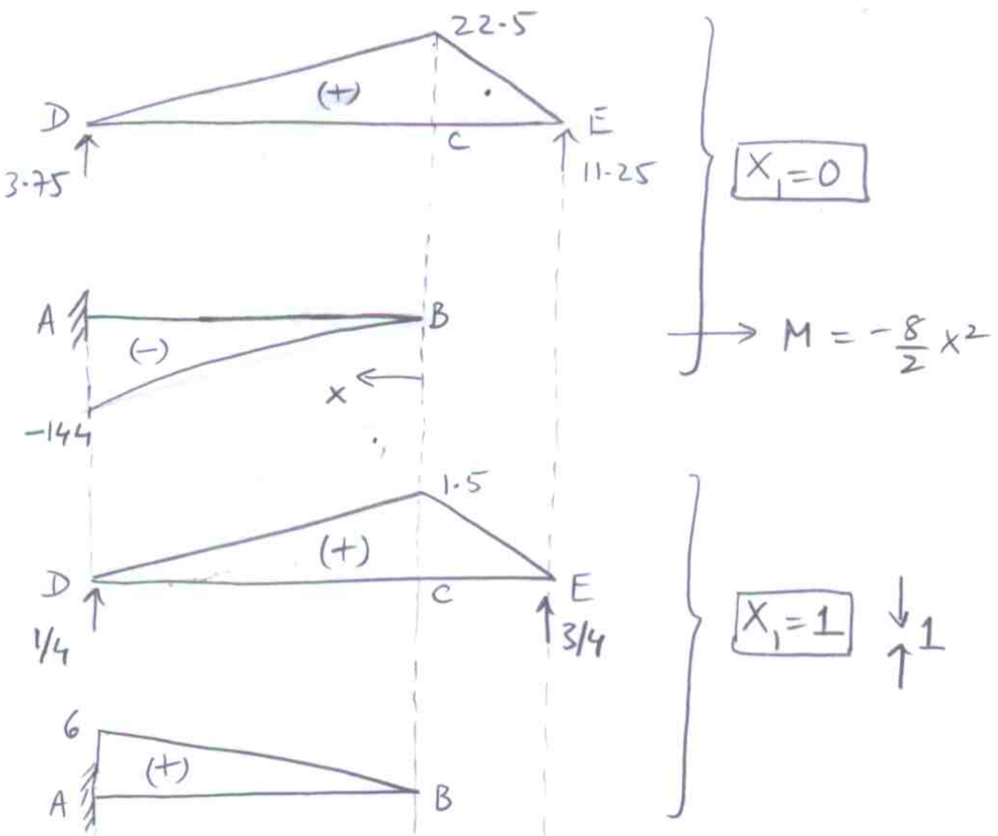
SFD

(Not reqd)



QDS

P4 $\quad CB = X_1$

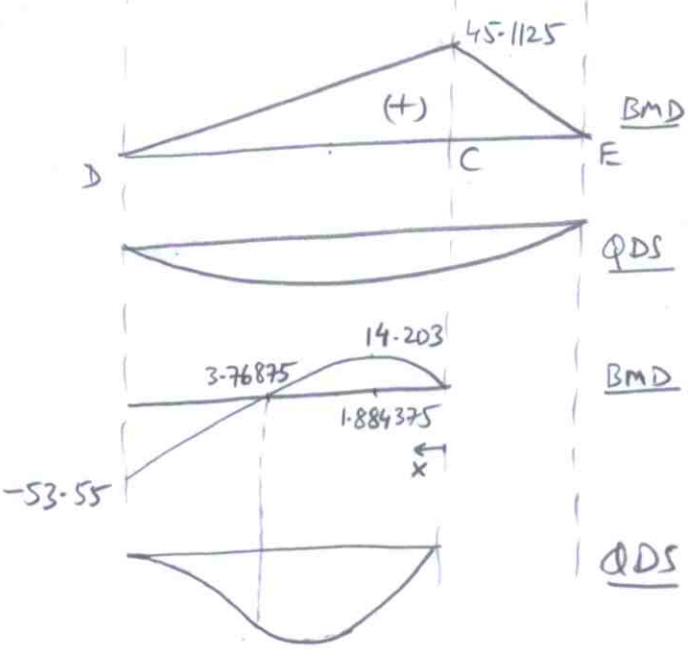


$$\Delta_{10} = \left[\tfrac{1}{3}(22.5)(1.5)(6+2) + \tfrac{1}{4}(144)(6)(6) - \tfrac{1}{2}(144)(6)(6)\right]\cdot\frac{1}{EI} = -\frac{1206}{EI}$$

$$f_{11} = \left[\tfrac{1}{3}(1.5)^2(6+2) + \tfrac{1}{3}(6)^2(6)\right]\cdot\frac{1}{EI} + \frac{(1)^2(4)}{AE} = \frac{78}{EI} + \frac{4}{AE}$$

$$X_1 f_{11} + \Delta_{10} = 0 \Rightarrow X_1 = \frac{\left(\dfrac{1206}{200E9 * 100E6 * 10^{-12} * 10^{-3}}\right)}{\left(\dfrac{222}{EI} + \dfrac{4}{200E\text{-}6 * 200E9 * 10^{-3}}\right)}$$

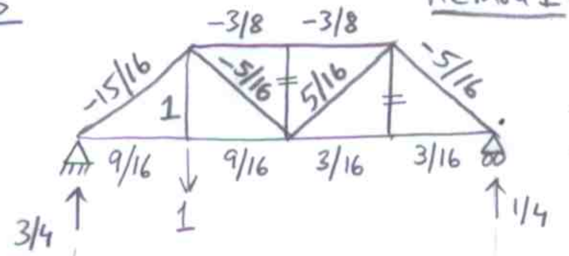
$$= \frac{0.0603}{\frac{78}{20000} + \frac{4}{(200)^2}} = \frac{0.0603}{4E\text{-}3} = \boxed{15.075 \text{ kN}}$$



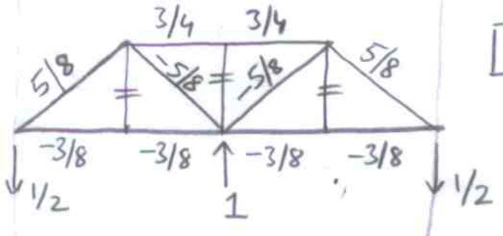
$$-\frac{8x^2}{2} + 15.075x = 0 \Rightarrow x = 3.76875$$

P5

Method 1: $\boxed{X_1 = R_C}$



$\boxed{X_1 = 0}$



$\boxed{X_1 = 1}$

$$\Delta_{10} = -\Big[2\cdot\left(\tfrac{3}{8}\right)\left(\tfrac{3}{4}\right)(3) + 2\cdot\left(\tfrac{3}{8}\right)\left(\tfrac{9}{16}\right)(3) + 2\cdot\left(\tfrac{3}{8}\right)\left(\tfrac{3}{16}\right)(3) + (2-1)\left(\tfrac{5}{8}\right)\left(\tfrac{5}{16}\right)(5) + \left(\tfrac{5}{8}\right)\left(\tfrac{5}{16}\right)(5)\Big]\cdot\frac{1}{AE}$$

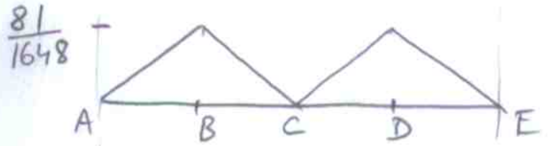
$$= \frac{233}{32}\cdot\frac{1}{AE}$$

$$f_{11} = \left[2\cdot\left(\tfrac{3}{4}\right)^2(3) + 4\cdot\left(\tfrac{5}{8}\right)^2(5) + 4\cdot\left(\tfrac{3}{8}\right)^2(3)\right]\cdot\frac{1}{AE}$$

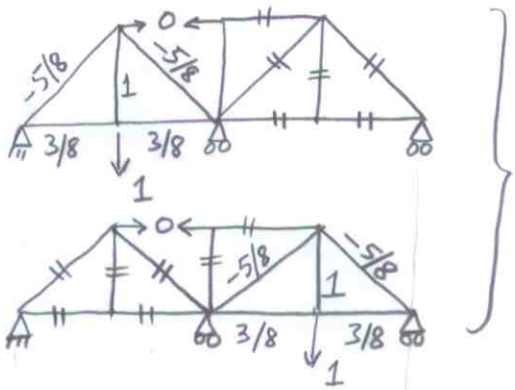
$$= \frac{103}{8}\cdot\frac{1}{AE}$$

$$\Delta_{10} + f_{11}X_1 = 0 \Rightarrow X_1 = \frac{233}{412}$$

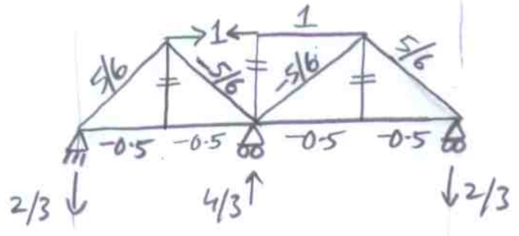
$$FG = -\frac{3}{8} + \frac{3}{4}\times\frac{233}{412} = \frac{81}{1648}$$ → same for unit load at B or D



Method 2: $\boxed{X_1 = FG}$



$\boxed{X_1 = 0}$

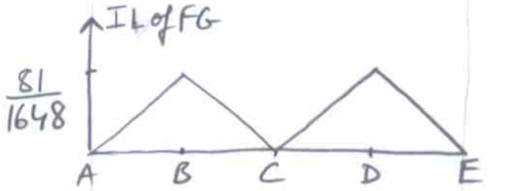


$\boxed{X_1 = 1}$

Unit load at B:

$$\Delta_{10B} = -\left[2\cdot\left(\tfrac{1}{2}\right)\left(\tfrac{3}{8}\right)(3)\right]\cdot\frac{1}{AE} = -\frac{9}{8}\cdot\frac{1}{AE}$$

Unit load at D: $\Delta_{10D} = \Delta_{10B}$ (can also see it from FG = GH if appl. load on lower chord)



$$f_{11} = \left[4\cdot\left(\tfrac{5}{6}\right)^2(5) + 2\cdot(1)^2(3) + 4\cdot(0.5)^2(3)\right]\cdot\frac{1}{AE}$$

$$= \frac{206}{9}\cdot\frac{1}{AE}$$

$$\Delta_{10B} + f_{11}X_1 = 0 \Rightarrow X_1 = FG = \frac{9}{8}\cdot\frac{9}{206} = \frac{81}{1648}\ \text{kN}$$