DEPARTMENT OF CIVIL ENGINEERING

CE-222 STRUCTURAL MECHANICS I

Endsem 19/4/10

(Each question = 20 marks)

Problem 1

Beams *ABC* and *CDE* are hinged together at *C* and supported as shown in **Fig. 1**. Assume *EI* is same for all members.

(i) Obtain the **influence line** (i.e., draw its **sketch** and obtain its **equation**) **for shear at** hinge point ***C***.

(ii) A uniform patch load w kN/m of length $L/4$ m. moves across the connected beam structure. Obtain the **magnitude of** the **maximum shear at *C* and** the **corresponding location of** the **patch load**.

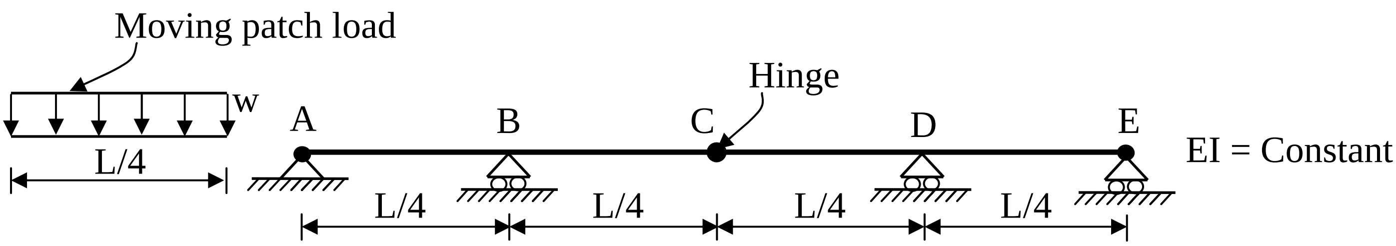


Fig. 1

Problem 2

Consider the closed rectangular frame *ABCD* with rigid (i.e., welded) joints at *A*, *B*, *C*, *D*. The frame rests on soil which exerts an upward uniform pressure load of 5 kN/m. In addition horizontal loads of 10 kN are applied at *B* and *C*, and a downward vertical load of 20 kN is applied at *O* which lies at the center of *BC*, as shown in **Fig. 2**. Assume *E* is same for all members, and *I* as shown in **Fig. 2**.

(i) Obtain the **axial force**, **shear force**, and **bending moment at** point ***O***. Clearly indicate their **directions** and **magnitude** in a sketch of section at *O*.

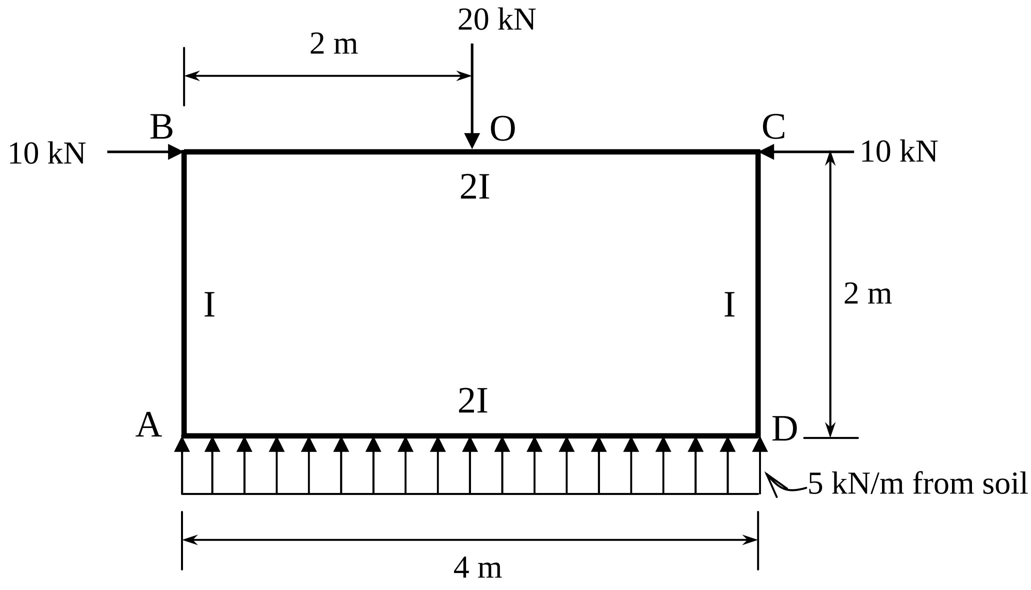


Fig. 2

Problem 3

The truss *ABCD* is pin jointed at *A*, *B*, *C*, *D*. Members *AD* and *BC* are not connected to each other. It is supported by a vertical roller at *A*, a pinned support at *C*, and a horizontal roller at *D*. The applied loads are 120 kN applied downward at *A*, 60 kN applied rightward at *B*, and 80 kN applied leftward at *D*, as shown in **Fig. 3**. Assume $AE = 40000$ kN for all members.

Given support settlement/movement: When the truss is loaded, the support at *D* settles downward by 0.5 cm, and support at *A* moves rightward by 0.3 cm.

(i) Find the **reaction at *A*** and **member force in *BC*** (**magnitude and direction**).

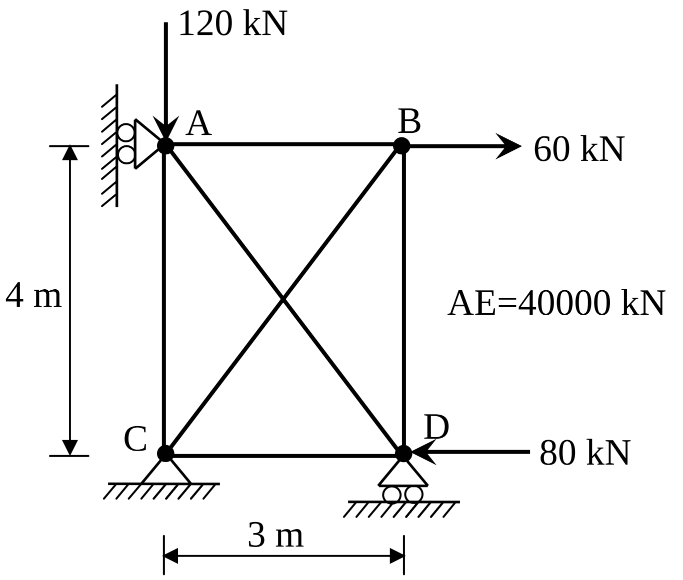


Fig. 3

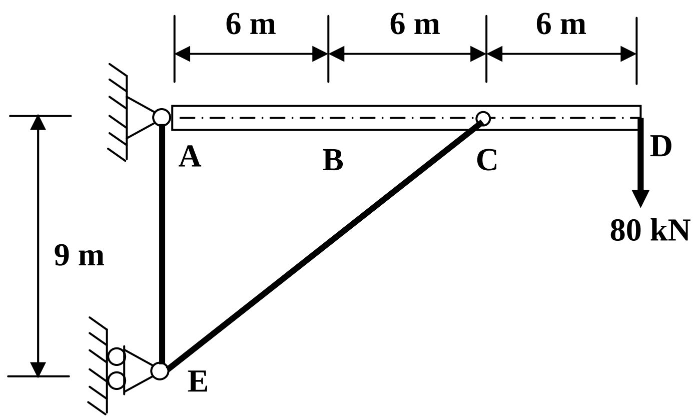


Fig. 5

Problem 4

The continuously supported beam *ABCD* is supported and loaded as shown in **Fig. 4**.

(i) Determine the **bending moments at all supports** (**magnitude and direction**).
(ii) Determine the **reactions at all supports** (**magnitude and direction**).

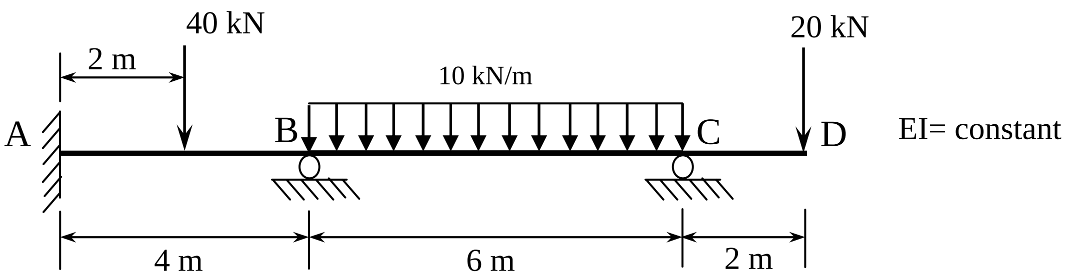


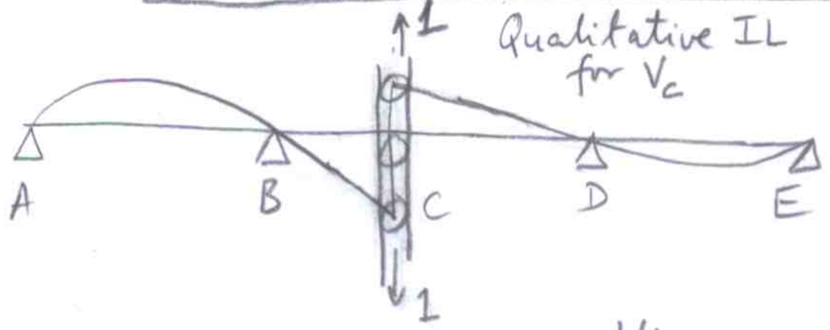
Fig. 4

Problem 5

The structure shown in **Fig. 5** comprises three members *ACD*, *CE*, and *AE*. Members *ACD* and *CE* are connected by a pin/hinge at *C*. Members *AE* and *ACD* are connected by a pin/hinge at *A*. Members *AE* and *CE* are connected by a pin/hinge at *E*. The support at *A* is a pin and the support at *E* is a roller. Assume *EA* is the axial stiffness and *EI* is the bending stiffness for all members.

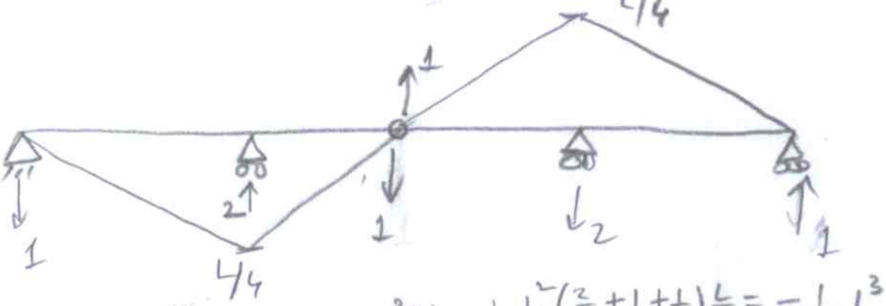
(i) Find the **vertical** component of **deflection of point *B* using Castigliano's theorem**.

P.1



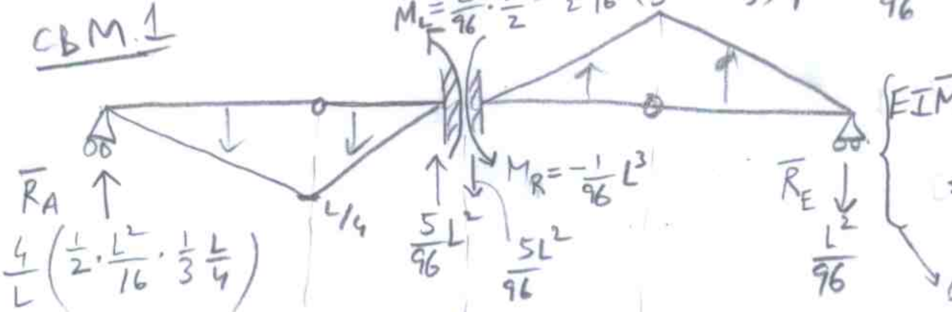
Given: Continuous beam as in Fig.

Find: IL for V_c and $(V_c)_{max}$ when patch u.d.l. 'w' of length $L/4$ travels over beam.



$$M_L = \frac{L^2}{96}\cdot\frac{L}{2} - \frac{1}{2}\cdot\frac{L^2}{16}\left(\frac{2}{3}+1+\frac{1}{3}\right)\frac{L}{4} = -\frac{1}{96}L^3$$

CBM 1



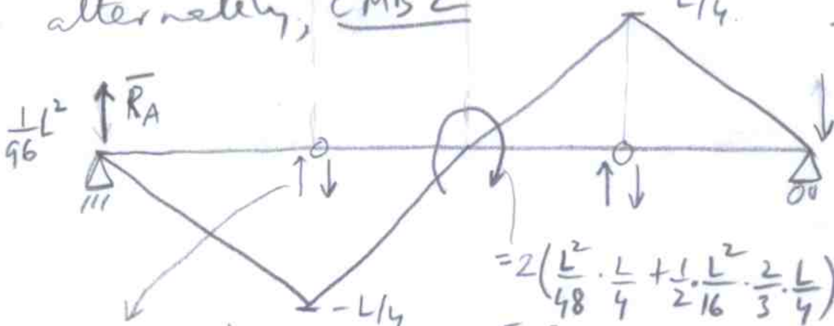
$\bar{R}_A = \frac{4}{L}\left(\frac{1}{2}\cdot\frac{L^2}{16}\cdot\frac{1}{3}\frac{L}{4}\right) = \frac{L^2}{96}$

$$EI\bar{M} = EIv_{AB} = \frac{L^2}{96}x - \frac{1}{2}x^2\cdot\frac{1}{3}x, \text{ in } AB$$

$$= EIv_{CB} = -\frac{L^3}{96} + \frac{5}{96}L^2x - \frac{1}{6}x^3, \text{ in } CB$$

antisymm in CDE.

alternately, CMB 2



$\bar{R}_E$: $\frac{1}{L}\left(-\frac{L^3}{48} - \frac{L^2}{16}\cdot\frac{L}{4} + \frac{L^2}{16}3\cdot\frac{L}{4}\right) = \frac{1}{96}L^2$

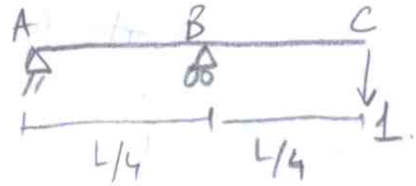
$= 2\left(\frac{L^2}{48}\cdot\frac{L}{4} + \frac{1}{2}\cdot\frac{L^2}{16}\cdot\frac{2}{3}\cdot\frac{L}{4}\right) = \frac{L^3}{48}$ (ie, same as $-(M_L+M_R)$ above)

$= \frac{4}{L}\left(\frac{1}{2}\cdot\frac{L^2}{16}\cdot\frac{2}{3}\frac{L}{4}\right) = \frac{L^2}{48}$

$\because \bar{R}_A, \bar{R}_E$ in CBM 1 & 2 match, as do the loading (triangular), hence $\bar{M}$ will match, so CBM 1 & 2 give identical results.

alternatively

This can also be done by integration, ie w/o CBM



$$EIv'' = M = -x \text{ in } AB \text{ and } CB$$

$$EI(v + c_1x + c_2) = -\frac{x^3}{6} \text{ in } AB \text{ and } CB$$

AB: $v_{AB} = 0$ at $x = 0, \frac{L}{4}$. $\Rightarrow c_{2,AB} = 0$, $c_{1,AB} = -\frac{L^2}{96}\cdot\frac{1}{EI}$

$$v_{AB} = \left(-\frac{x^3}{6} + \frac{L^2}{96}x\right)\cdot\frac{1}{EI} \quad \text{(same as above)}$$

CB: $v_{CB}\left[\frac{L}{4}\right] = 0$ and $v'_{CB}\left[\frac{L}{4}\right] = -v'_{AB}\left[\frac{L}{4}\right] \Rightarrow c_{1,CB}\frac{L}{4} + c_{2,CB} = -\frac{L^3}{384}\cdot\frac{1}{EI} \longrightarrow (i)$

$$\underbrace{-\frac{L^2}{32}\cdot\frac{1}{EI}+\frac{L^2}{96}\cdot\frac{1}{EI}}_{=\,v'_{AB}[L/4]} = -\underbrace{\left(-\frac{L^2}{32}\cdot\frac{1}{EI} - C_{1,CB}\right)}_{=\,-v'_{CB}[L/4]} \Rightarrow C_{1,CB} = -\frac{5}{96}\frac{L^2}{EI}$$

$$C_{2,CB} = \frac{1}{96}\frac{L^3}{EI}$$

$$v_{CB} = \left(-\frac{x^3}{6} + \frac{5}{96}L^2x - \frac{1}{96}L^3\right)\cdot\frac{1}{EI}$$ (Same as above)

Max shear at C

Patch over AB or DE, $V_1 = w\int_0^{L/4} v_{AB}\,dx = \frac{w}{EI}\left(\frac{L^2}{96}\cdot\frac{L^2}{32} - \frac{1}{6}\cdot\frac{1}{4}\cdot\frac{L^4}{256}\right)$

$$= \frac{w}{EI}\cdot\frac{1}{6144}$$

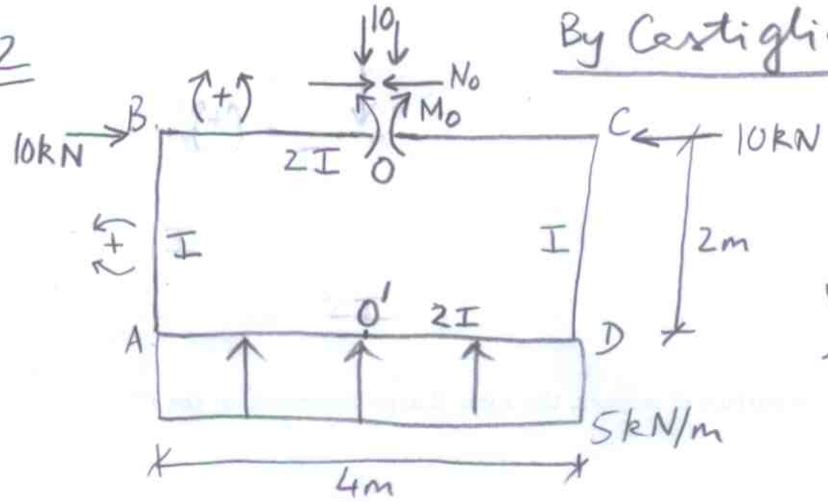
Patch over BC or CD, $V_2 = w\int_0^{L/4} v_{CB}\,dx = \frac{w}{EI}\left(-\frac{L^3}{96}\cdot\frac{L}{4} + \frac{5}{96}L^2\frac{L^2}{32} - \frac{1}{6}\cdot\frac{1}{4}\cdot\frac{L^4}{256}\right)$

$$= -\frac{w}{EI}\cdot\frac{7}{6144}$$

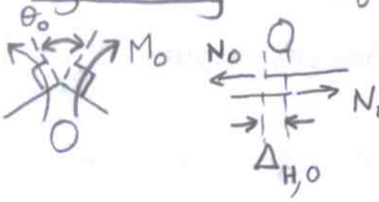
So max shear is $\frac{w}{EI}\cdot\frac{7}{6144}$ when patch over BC or CD.

P2

By Castigliano's Theorem.



Symmetry $\Rightarrow V_0 = 10 \downarrow$



$\Leftarrow$ Continuity ie Compatibility

$$M = M_0 - 10x, \quad OB, \quad (+) \qquad \Rightarrow \frac{\partial M}{\partial M_0} = 1, \quad \Rightarrow \frac{\partial M}{\partial N_0} = 0, \quad OB$$

$$= M_0 - 20 - 10x - N_0 x, \quad BA, \quad (+) \qquad = 1, \qquad = -x, \quad BA$$

$$= M_0 - 40 - 2N_0 - \frac{5x^2}{2} + 10x, \quad AO', \quad (+) \qquad = 1, \qquad = -2, \quad AO'$$

Compatibility $\Rightarrow \theta_0 = 0 = \dfrac{\partial U}{\partial M_0} = \dfrac{2}{EI} * \left[\dfrac{1}{2}\displaystyle\int_0^2 \left(M_0 - \cancel{10x} + M_0 - 40 - 2N_0 - \frac{5}{2}x^2 + \cancel{10x}\right)(1)\,dx\right.$

$$\left. + \int_0^2 (M_0 - 20 - 10x - N_0 x)(1)\,dx\right]$$

$$\Rightarrow 0 = \frac{1}{2}\left(4M_0 - 80 - 4N_0 - \frac{5}{6} * 8\right) + 2M_0 - 40 - \frac{10}{2} * 4 - \frac{N_0}{2} * 4$$

$$4M_0 - 4N_0 = \frac{310}{3}$$

Compatibility $\Rightarrow \Delta_{H,0} = 0 = \dfrac{\partial U}{\partial N_0} = \dfrac{2}{EI} * \left[\dfrac{1}{2}\displaystyle\int_0^2 \left(M_0 - 40 - 2N_0 - \frac{5}{2}x^2 + 10x\right)(-2)\,dx\right.$

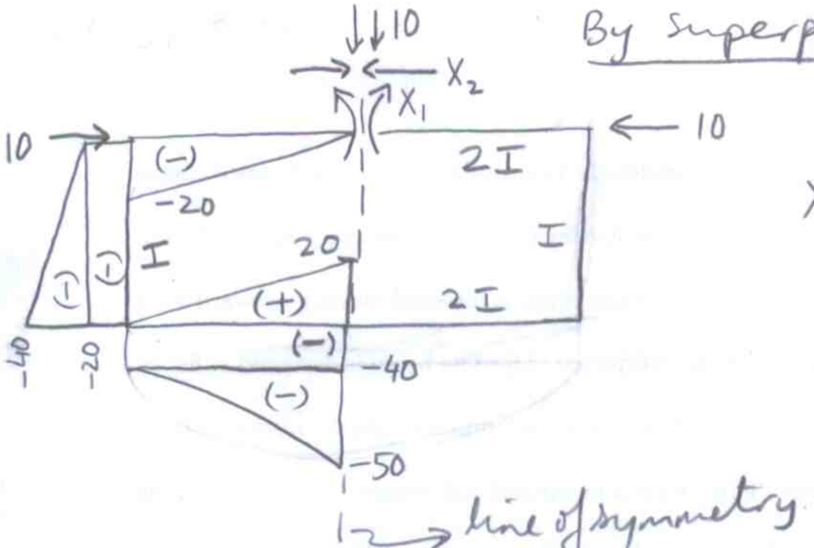
$$\left. + \int_0^2 (M_0 - 20 - 10x - N_0 x)(-x)\,dx\right]$$

$$\Rightarrow 0 = -\left(2M_0 - 80 - 4N_0 - \frac{5}{6} * 8 + \frac{10}{2} * 4\right) - \left(2M_0 - 40 - \frac{10}{3} * 8 - \frac{N_0}{3} * 8\right)$$

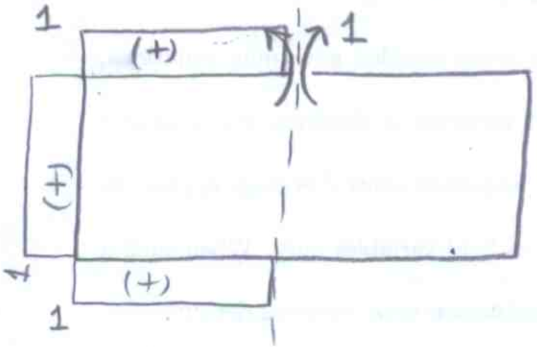
$$4M_0 - \frac{20}{3}N_0 = \frac{400}{3}$$

$$\Rightarrow \frac{8}{3}N_0 = -30 \Rightarrow N_0 = -\frac{90}{8} = -11.25, \quad M_0 = 14.5833$$

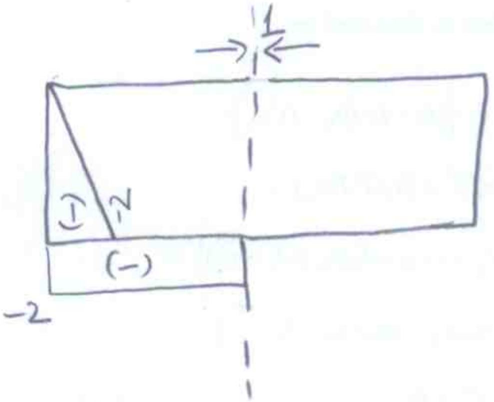
By superposition method (alternative solution) ④



$X_1 = X_2 = 0$



$X_1 = 1, \; X_2 = 0$



$X_2 = 1, \; X_1 = 0$

$$\Delta_{10} = \frac{1}{2}\cdot\frac{1}{2}(-20)(1)(2) + (-20)(1)(2) + \frac{1}{2}(-20)(1)(2) + \frac{1}{2}\left\{(-40)(1)(2) + \frac{2}{3}(10)(1)(2) + (-10)(1)(2) + \frac{1}{2}(20)(1)(2)\right\}$$
$$= -\frac{310}{3}$$

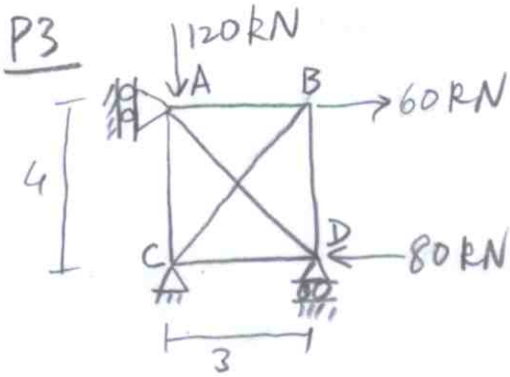
$$\Delta_{20} = \frac{1}{2}(20)(2)(2) + \frac{1}{3}(20)(2)(2) + \frac{1}{2}\left\{(40)(2)(2) + \frac{2}{3}(10)(-2)(2) + (10)(2)(2) + \frac{1}{2}(20)(-2)(2)\right\}$$
$$= \frac{400}{3}$$

$$f_{11} = \frac{1}{2}(1)(1)(2)*2 + (1)(1)(2) = 4 \; ; \; f_{22} = \frac{1}{3}(2)(2)(2) + \frac{1}{2}*(2)(2)(2) = \frac{20}{3}$$

$$f_{12} = \frac{1}{2}(1)(-2)(2) + \frac{1}{2}*(1)(-2)(2) = -4$$

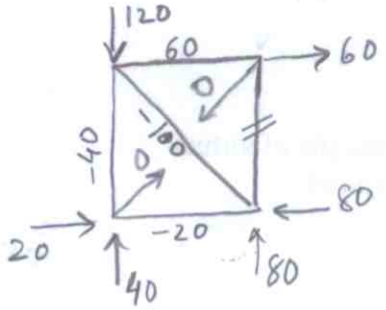
$$\begin{bmatrix} 4 & -4 \\ -4 & 20/3 \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} = \begin{Bmatrix} 310/3 \\ -\frac{400}{3} \end{Bmatrix} \Rightarrow \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} = \frac{3}{32}\begin{bmatrix} 20/3 & 4 \\ 4 & 4 \end{bmatrix} \begin{Bmatrix} 310/3 \\ -\frac{400}{3} \end{Bmatrix} = \begin{Bmatrix} 14.5833 \\ -11.25 \end{Bmatrix}$$

P3

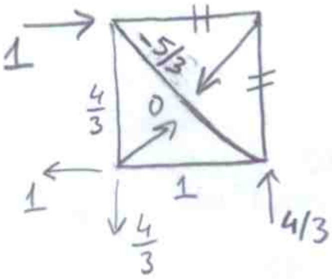


Given : Loading (in kN) and settlement
of D is 0.5 cm (↓)
of A is 0.3 cm (→)
area = 2 cm² for each member
E = 200 GPa
⇒ AE = 40000 kN.

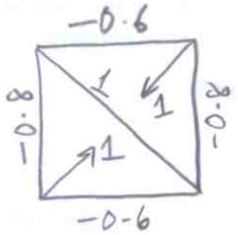
Find : R_A, F_{BC}



P_i :
$X_1 = X_2 = 0$
applied load.



p_{i1} :
$X_1 = 1$, $X_2 = 0$



p_{i2} :
$X_1 = 0$, $X_2 = 1$

	P_i	p_{i1}	p_{i2}	L_i	$P_i p_{i1} L_i$	$P_i p_{i2} L_i$	$p_{i1}^2 L_i$	$p_{i2}^2 L_i$	$p_{i1} p_{i2} L_i$
AB	60	0	-0.6	3	0	-108	0	1.08	0
CD	-20	1	-0.6	3	-60	36	3	1.08	-1.8
AC	-40	4/3	-0.8	4	-640/3	128	64/9	2.56	-64/15
BD	0	0	-0.8	4	0	0	0	2.56	0
AD	-100	-5/3	1	5	2500/3	-500	125/9	5	-25/3
BC	0	0	1	5	0	0	0	5	0
					$\Delta_{10} = \frac{560}{AE}$	$\Delta_{20} = \frac{-444}{AE}$	$f_{11} = \frac{24}{AE}$	$f_{22} = \frac{17.28}{AE}$	$f_{12} = \frac{-14.4}{AE}$

$$\Delta_{1S} = \frac{0.005}{3} \times 4 = \frac{0.02}{3}, \quad \Delta_{2S} = 0$$

$$\begin{Bmatrix} \frac{0.02}{3} + \frac{560}{AE} \\ \frac{-444}{AE} \end{Bmatrix} + \frac{1}{AE}\begin{bmatrix} 24 & -14.4 \\ -14.4 & 17.28 \end{bmatrix}\begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} = \begin{Bmatrix} 0.003 \\ 0 \end{Bmatrix}$$

$$\Rightarrow \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} = \frac{1}{207.36}\begin{bmatrix} 17.28 & 14.4 \\ 14.4 & 24 \end{bmatrix}\begin{Bmatrix} -2120/3 \\ 444 \end{Bmatrix} \Rightarrow \begin{aligned} X_1 &= R_A = \cancel{-28.06}\ {-505/18}\ (\leftarrow) \\ X_2 &= F_{CB} = 2.315\ (T) \end{aligned}$$

P4

Span A^-AB

$M_L = M_{A^-} = 0, \quad M_c = M_A, \quad M_R = M_B, \quad L_L = 0, \quad L_R = 4, \quad P_R = 40, \quad k_R = 0.5$

$$\Rightarrow 4M_B + 2M_A(4) = -40(4^2)(0.5 - 0.5^3)$$

$$8M_A + 4M_B = -240 \longrightarrow ①$$

Span ABC

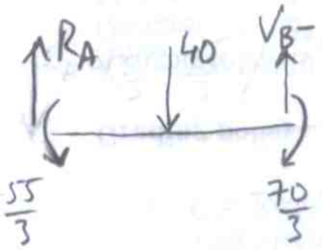
$M_L = M_A, \quad M_c = M_B, \quad M_R = M_c = -20(2), \quad L_L = 4, \quad L_R = 6,$

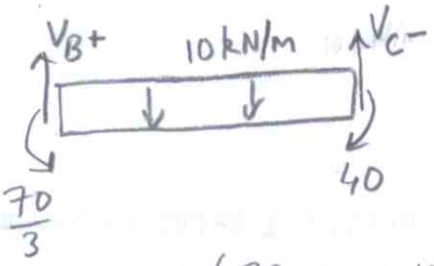
$P_L = 40, \quad k_L = 0.5, \quad w_R = 10$

$$\Rightarrow -40(6) + 4M_A + 2M_B(4+6) = -240 - \frac{10}{4}(6^3)$$

$$4M_A + 20M_B = -540$$

$$\Rightarrow M_B = -23.33 \text{ kN.m}, \quad M_A = -18.33 \text{ kN.m}$$





$$R_A = \frac{1}{4}\left(\frac{55}{3} - \frac{70}{3} + 40*2\right) \qquad V_{B+} = \frac{1}{6}\left(\frac{70}{3} - 40 + 10*\frac{6^2}{2}\right)$$

$$= 18.75 \text{ kN} \qquad\qquad = 27.22$$

$$V_{B^-} = 40 - 18.75 = 21.25 \qquad V_{C^-} = 60 - 27.22 = 32.78, \quad V_{C+} = 20$$

$R_A = 18.75 \uparrow$

$R_B = 21.25 + 27.22 = 48.47 \uparrow$

$R_c = 32.78 + 20 = 52.78 \uparrow$

$M_A = 18.33$ ↺, $\quad M_B = 23.33$ ↺