DEPARTMENT OF CIVIL ENGINEERING

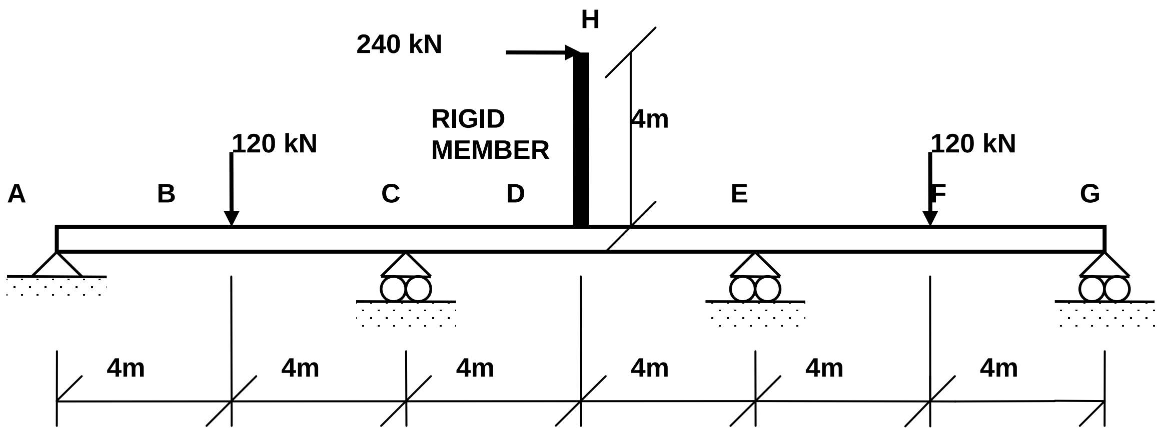
CE-222 STRUCTURAL MECHANICS I

Quiz-2 3/4/09

All problems equally weighted.

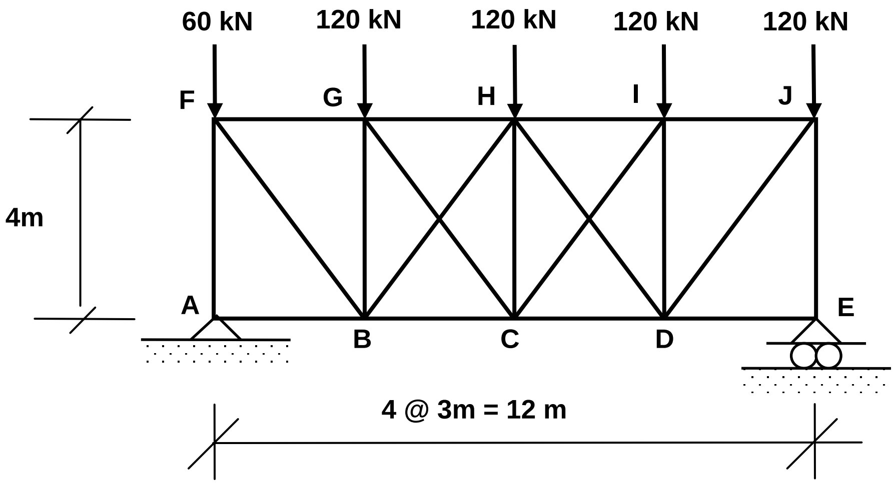
Problem 1

Draw the **Shear Force and Bending Moment Diagrams** and sketch the **Qualitative Deflected Shape** for the following beam. Support settlement is $0.0512\,\text{m}$ for **C** and **E** and $0.0512/3$ m for **A** and **G**. Use $E = 200\,\text{GPa}$, $I = 250 \times 10^6\,\text{mm}^4$. Member DH is rigid and is welded perpendicular to beam **AG** as shown.



Problem 2

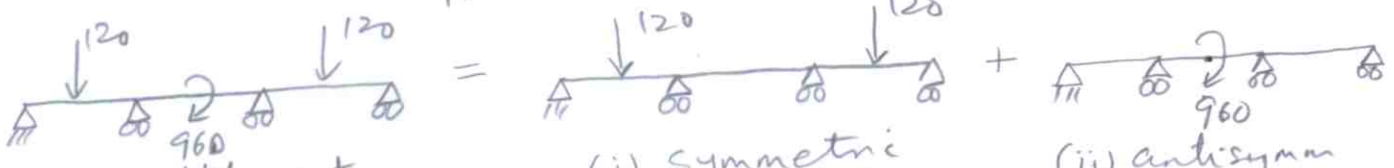
Find the **forces in members FG, GB, BH, HC.** Members **BH** and **DH** undergo a temperature rise of 60^0 F. Use $E = 200\,\text{GPa}$, $A = 400\,\text{mm}^2$, $\alpha = \frac{1}{1.5\times10^5}\ /^0F$.



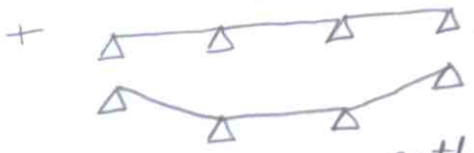
CE222 Quiz 2 Spring 2009

P1

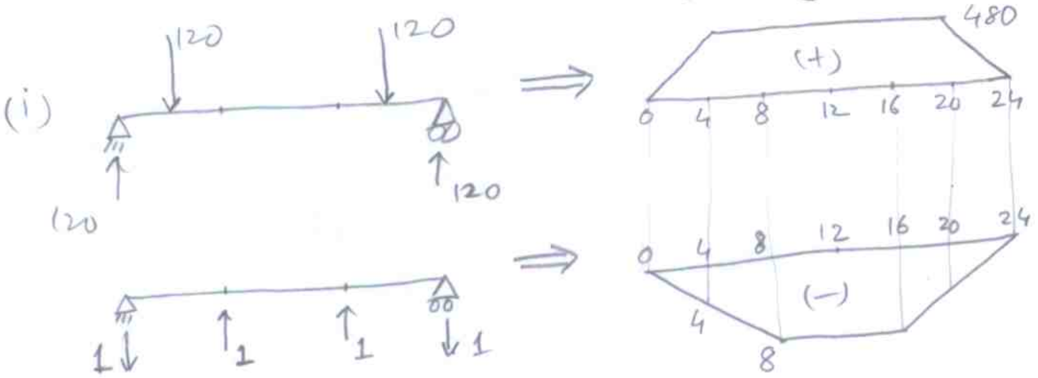
Axial load has no effect on BMD, SFD, so drop it.



+ settlement, (i) symmetric ($R_C = R_E$) (ii) antisymm ($R_C = -R_E$)



(iii) Symmetric settlement. ($R_C = R_E$)



BMD — symm

BMD (due to combined application of identical redundants)

$X_1 = R_C = R_E = 1$

$$\Delta_{10} = -\left(\tfrac{1}{3}(480)(4)(4) + \tfrac{1}{2}(480)(4+8)(4) + (480)(8)(4)\right) * \frac{2}{EI}$$

$$f_{11} = \left[\tfrac{1}{3}(8)(8)(8) + (8)(8)(4)\right] * \frac{2}{EI}$$

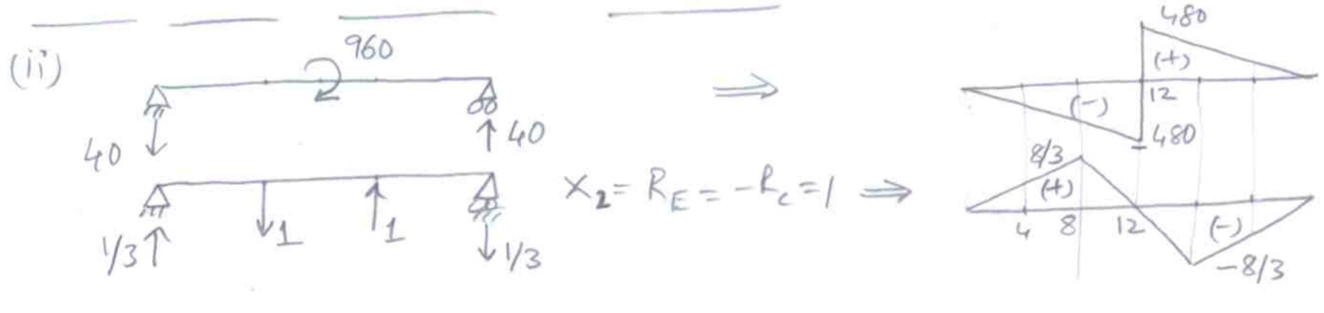
$$\Delta_{10} + f_{11} X_1 = 0 \Rightarrow \boxed{X_1 = 69\,kN}$$

(Note: we can apply two redundants at a time since they are the same value. It is like adding simultaneous equations as follows:

$$\left.\begin{aligned} \Delta_{10C} + X_1 f_{CC} + X_1 f_{CE} &= 0 \\ \Delta_{10E} + X_1 f_{EC} + X_1 \underbrace{f_{EE}}_{f_{CC}} &= 0 \end{aligned}\right\} \Rightarrow \underbrace{\Delta_{10C} + \Delta_{10E}}_{\Delta_{10}} + X_1 \underbrace{(2f_{CC} + 2f_{CE})}_{= \int \frac{m_C^2}{EI} + \int \frac{m_E^2}{EI} + 2\int \frac{m_C m_E}{EI} = \int \frac{(m_C + m_E)^2}{EI}}$$

ie combined application of identical redundants

Explanation not required.



BMD — antisymm — BMD

$$\Delta_{20} = -\left((480)\left(\tfrac{8}{3}\right)\left(\tfrac{1}{6}\right)(12+8)\right) \times \frac{2}{EI}$$

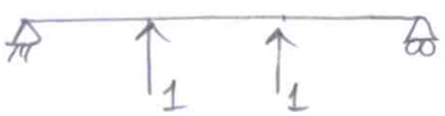
$$f_{22} = \left(\tfrac{1}{3}\left(\tfrac{8}{3}\right)^2(8+4)\right) \times \frac{2}{EI}$$

(same reasoning for simultaneous application of identical redundants)

$$\Delta_{20} + f_{22}X_2 = 0 \implies \boxed{X_2 = 150\,kN}$$

(iii) Settlement problem is symmetric.

$$\Delta_{3S} = \Delta_{3SC} + \Delta_{3SE} = 2\left(-\frac{0.0512}{3}\right)$$



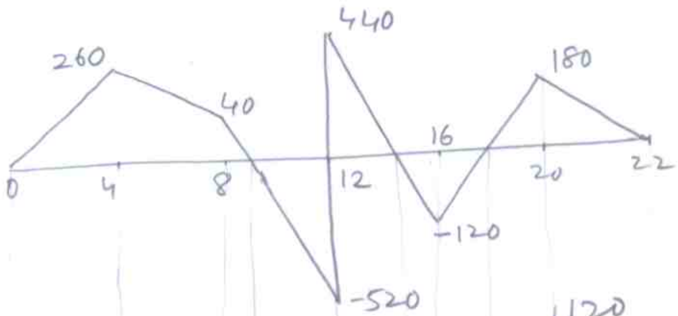
$$\Delta_3 = \Delta_{3C} + \Delta_{3E} = 2(-0.0512)$$

$X_3 = R_C = R_E = 1$

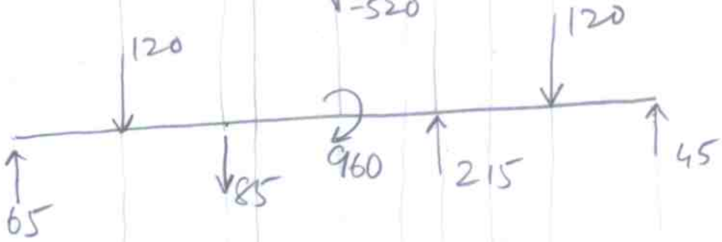
$$\Delta_{3S} + \underbrace{f_{33}}_{f_{11}} X_3 = \Delta_3 \implies \boxed{X_3 = -4\,kN}$$

$f_{33} = f_{11}$

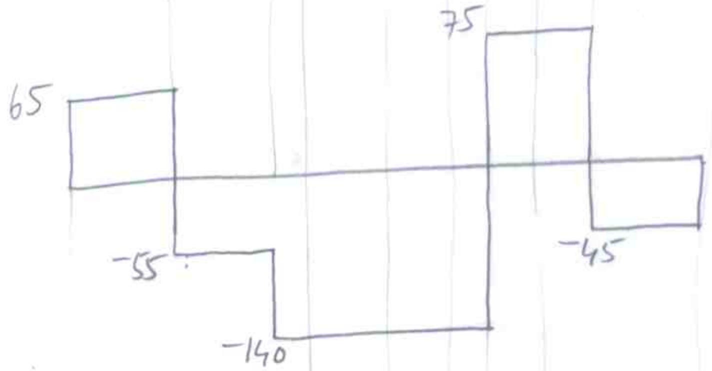
Superposing (i) + (ii) + (iii)



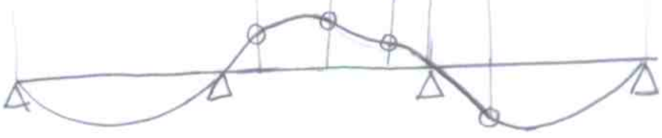
BMD



Loading & reactions

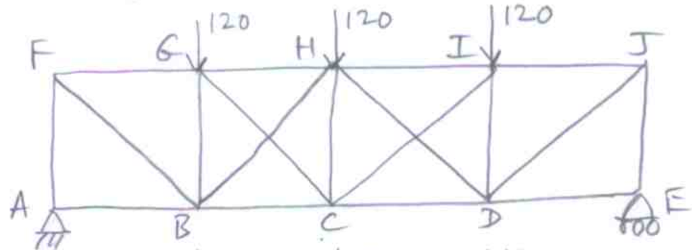


SFD



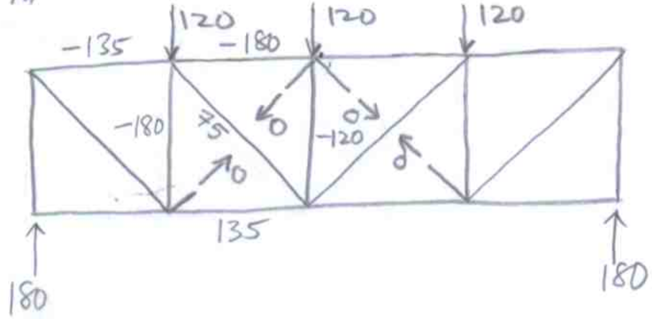
QDS
(inflexions indicated O)

P2 Loads at F, G directly transmitted to respective supports, so they don't affect other member forces except AF and EJ. So equivalent to solving the symmetric problem,



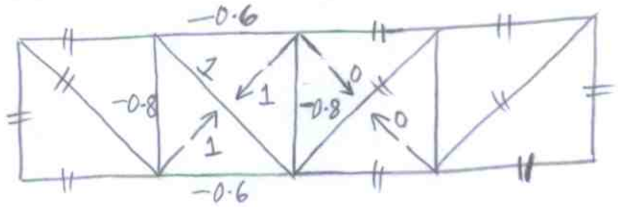
as far as other member forces (except AF, EJ) are concerned

$X_1 = BH$, $X_2 = DH$, $X_1 = X_2$



$\Rightarrow$ symmetric. Other member forces not shown ∵ not needed

$X_1 = X_2 = 0$



$X_1 = 1$, $X_2 = 0$

(mirror image for $X_1 = 0$, $X_2 = 1$ ∵ symmetric).

$$\Delta_{10} = \left[(-180+135)(-0.6)(3) + (-180-120)(-0.8)(4) + 75*5\right]\cdot\frac{1}{AE} = \frac{1416}{AE}$$

$$f_{11} = \left[(0.6)^2(3) + (0.8)^2(4) + (1)^2(5)\right]*2 = 17.28$$

$$f_{12} = (0.8)^2(4) = 2.56$$

$$\Delta_{1T} = (1)(\alpha\Delta T * 5)$$

$\Delta_{1T} + \Delta_{10} + f_{11}X_1 + f_{12}X_2 = 0$ and $X_1 = X_2$, give

$$X_1 = \left[-\frac{1416}{AE} - \alpha\Delta T(5)\right] \Big/ (19.84/AE)$$

$$= -\left(1416 + \frac{1}{1.5E5}(60)(5)\frac{(200E9)(400)}{10^3 * 10^6}\right) \Big/ 19.84 = -\frac{1576}{19.84}$$

i.e. AE in kN ∵ loading in kN.

$$= -79.435$$

$X_1 = X_2 = BH = DH = -79.435$ kN

$FG = -135$ kN

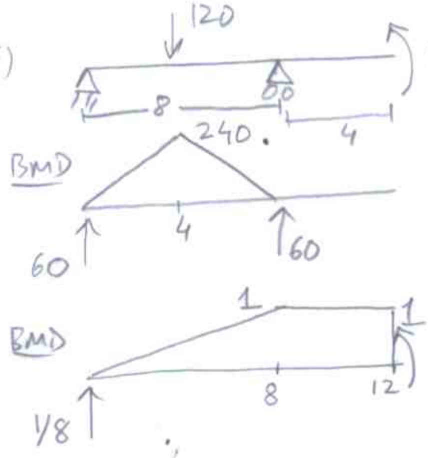
$GB = -180 + X_1(-0.8) = -116.4516$ kN

$HC = -120 + 2 * X_1(-0.8) = 7.0968$ kN

Other methods for P.1.

Method II

(i) symmetric vertical load problem



$X_1 \rightarrow N = 0$ (from given structure)

$X_1 = M_D$

$X_1 = 0$

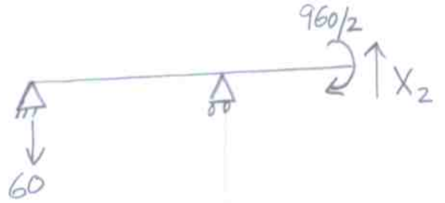
$X_1 = 1$

$$\Delta_{10} = \frac{1}{EI}\left(\frac{1}{6}(240)(1)(8+4)\right)$$

$$f_{11} = \frac{1}{EI}\left(\frac{1}{3}(1)(1)(8) + (1)(1)(4)\right)$$

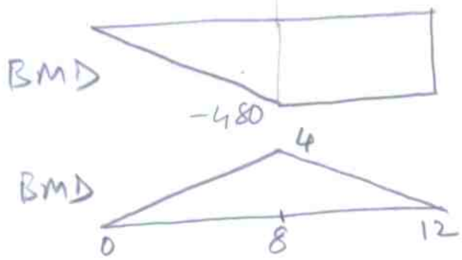
$$\Delta_{10} + f_{11}X_1 = 0 \Rightarrow \boxed{X_1 = M_D = -72}$$

(ii) antisymm applied torque problem



Here we split 960 equally since $\{M\} = \{\frac{M}{2}\ \frac{M}{2}\}$

ie applied infinitesimally apart on either side of $\not{C}$, so that problem is antisymmetric



$X_2 = 0$

$X_2 = 1$

$$\Delta_{20} = -\frac{1}{3}(480)(4)(8) - \frac{1}{2}(480)(4)(4)$$

$$f_{22} = \frac{1}{3}(4)(4)(8+4)$$

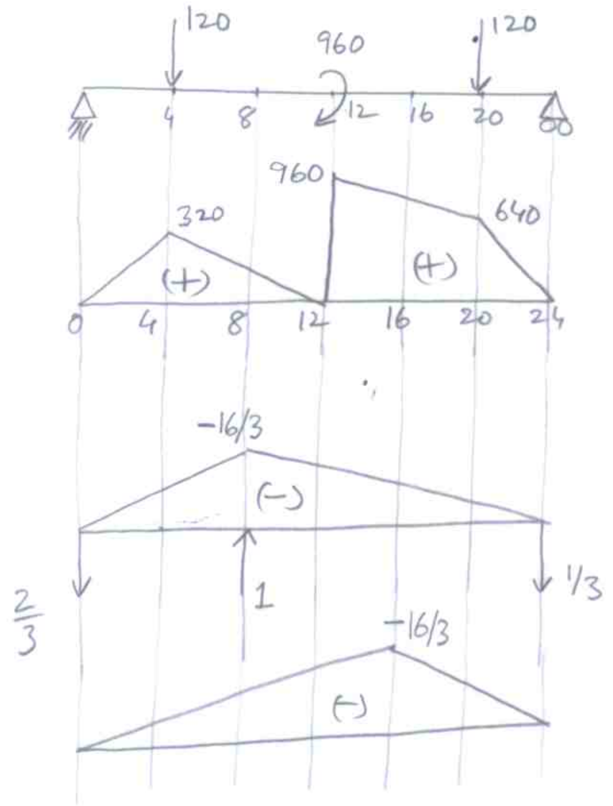
$$\Delta_{20} + f_{22}X_2 = 0 \Rightarrow \boxed{X_2 = V_D = 140}$$

check: You get same result for M_D, V_D from part (i), (ii) of the previous solution method (on pg. 1)

(iii) settlement – symmetric

same solution as previous one on pg. 1 or can also do as symm problem in terms of moment redundant X_1 as (i) above.

Method III Brute force 2-DOI approach. $X_1 = R_C$, $X_2 = R_E$



$X_1 = X_2 = 0$

$X_1 = 1,\ X_2 = 0$

$X_1 = 0,\ X_2 = 1$

$$-EI\Delta_{10} = \tfrac{1}{3}(320)\left(\tfrac{8}{3}\right)(4) + \tfrac{1}{6}\left[(320)\left(2*\tfrac{8}{3}+\tfrac{16}{3}\right) + (160)\left(\tfrac{8}{3}+2*\tfrac{16}{3}\right)\right]*4$$
$$+ \tfrac{1}{6}(160)\left(2*\tfrac{16}{3}+4\right)(4) + \tfrac{1}{6}\left[(960)\left(2*4+\tfrac{8}{3}\right) + (800)\left(4+2*\tfrac{8}{3}\right)\right]*4$$
$$+ \tfrac{1}{6}\left[(800)\left(2*\tfrac{8}{3}+\tfrac{4}{3}\right) + (640)\left(\tfrac{8}{3}+2*\tfrac{4}{3}\right)\right]*4 + \tfrac{1}{3}(640)\left(\tfrac{4}{3}\right)(4) = 75520/3$$

$$-EI\Delta_{20} = \tfrac{1}{3}(320)\left(\tfrac{4}{3}\right)(4) + \tfrac{1}{6}\left[(320)\left(2*\tfrac{4}{3}+4\right)\right]*8 + \tfrac{1}{6}\left[(960)\left(2*4+\tfrac{16}{3}\right) + (800)\left(4+2*\tfrac{16}{3}\right)\right]*4$$
$$+ \tfrac{1}{6}\left[(800)\left(2*\tfrac{16}{3}+\tfrac{8}{3}\right) + (640)\left(\tfrac{16}{3}+2*\tfrac{8}{3}\right)\right]*4 + \tfrac{1}{3}(640)\left(\tfrac{4}{3}\right)(4) = 101120/3$$

$$f_{11} = f_{22} = \tfrac{1}{3}\left[\left(\tfrac{16}{3}\right)^2(8) + \left(\tfrac{16}{3}\right)^2(16)\right]/EI = \frac{2048}{9EI}$$

$$EI f_{12} = 2*\tfrac{1}{3}\left(\tfrac{16}{3}\right)\left(\tfrac{8}{3}\right)(8) + \tfrac{1}{6}\left[\left(\tfrac{16}{3}\right)\left(2*\tfrac{8}{3}+\tfrac{16}{3}\right) + \left(\tfrac{8}{3}\right)\left(\tfrac{8}{3}+2*\tfrac{16}{3}\right)\right]*8 = \frac{1792}{9}$$

$$-\frac{1}{EI}\begin{Bmatrix} 75520/3 \\ 101120/3 \end{Bmatrix} + \begin{Bmatrix} -0.0512/3 \\ -0.0512/3 \end{Bmatrix} + \frac{1}{EI}\begin{bmatrix} 2048/9 & 1792/9 \\ 1792/9 & 2048/9 \end{bmatrix}\begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} = \begin{Bmatrix} -0.0512 \\ -0.0512 \end{Bmatrix}$$

Note: ∵ load applied in kN, EI should be used in kN m²

ie $EI = \dfrac{(200E9)(250E6)}{10^3 * 10^{12}}$

Get $X_1 = -85$, $X_2 = 215$ → checks out (see pg. 2 for final reactions at C, E)